## CONTENTS

### PHYSICS

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- Once the camera detects the QR code, a url appears in the screen. Click the url and go to the content page.

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HOW TO USE THE BOOK

Scope of Physics
- Awareness on higher learning - courses, institutions and required competitive exams
- Financial assistance possible to help students to climb academic ladder

Learning Objectives:
- Overview of the unit
- Gives clarity on the goals and objective of the topics
- Additional facts related to the topics covered to facilitate curiosity driven learning

Example problems
- To ensure understanding, problems/illustrations are given at every stage before advancing to next level
- Visual representation of concepts with illustrations
- Videos, animations, and tutorials

ICT
- To harness the digital skills to class room learning and experimenting

Summary
- Recap of salient points of the lesson

Concept Map
- Schematic outline of salient learning of the unit

Evaluation
- Evaluate students’ understanding and get them acquainted with the application of physical concepts to numerical and conceptual questions

Books for Reference
- List of relevant books for further reading

Solved examples
- Solutions to exercise problems are accessible here. In addition, a few solved examples are given to facilitate students to apply the concepts learnt.

Competitive Exam corner
- Model Questions - To motivate students aspiring to take up competitive examinations such as NEET, JEE, Physics Olympiad, JIPMER etc

Practical
- List of practical and the description of each is appended for easy access.

Glossary
- Scientific terms frequently used with their Tamil equivalents

Content Focus
- Sequential understanding of the stationary charges, moving charges, electric current, magnetism and the interlink between electric and magnetic phenomena.

Back Wrapper: NIKOLA TESLA a Serbian-American Engineer
(10 July 1856 – 7 January 1943)

Nikola Tesla made breakthroughs in the production, transmission and application of electric power. He invented the first alternating current (AC) motor and developed AC generation and transmission technology. In 1884, he was hired by Edison (discoverer of DC dynamos) later on Tesla became his competitor in this field.

Tesla conducted a range of experiments with mechanical oscillators/generators, electrical discharge tubes, and early X-ray imaging. Tesla in 1890 itself conducted research for wireless lighting and worldwide wireless electric power distribution in his high – voltage, high-frequency power experiments in New York. Unfortunately Tesla could not put his ideas in practical use due to lack of funds.
Scope of Physics - Higher Education

Entrance Examinations After +2

- Physics Olympiad Exam
- NEET-National Eligibility cum Entrance Test
- JEE-Joint Entrance Examination (Main & Advanced)
- NEST-National Entrance Screening Test
- KVPY-Kishore Vaigyanik Protsahan Yojana
- JEE Main Paper II for B.Arch
- AIIMS - All Indian Institute of Medical Science's Examination
- Chennai Mathematical Institute Entrance Examination
- BITSAT-Bits Institute of Science And Technology Admission Test
- AIEEE - All India Engineering Entrance Exam
- CUCET - Central Universities Common Entrance Test
- JIPMER - Jawaharlal Institute of Postgraduate Medical Education & Research
- CLAT - Common Law Admission Test
- HSEE-Humanities and Social Sciences Entrance Examination
- AIPVT-All India Pre-Veterinary Test
- NDA - National Defence Academy Examination

After Graduation

- JAM-Joint Admission Test
- JEST-Joint Entrance Screening Test
- GATE-Graduate Aptitude Test in Engineering
- CAT-Common Admission Test (for MBA)
- Exams conducted by Respective Universities

After Post Graduation

- CSIR-National Eligibility Test for JRF and Lectureship

Education Career

After Completing +2

- Integrated MSc. Physics
- Central Universities through CUCET
- Central Research Institutes like IISER using KVPY, JEE Advanced, IISER Aptitude Test
- Top 1% students in State board are eligible for IISER Aptitude Test
- Admission in NISER through NEST
- B.Sc. Photonics
- B.Sc. Honours in Mathematics and Physics in CMI
- Five-Year Dual degree in IIIST (B.Tech + Master of Science)
- Master of Science (Astrophysics, Solid State Physics)

Note

- Students admitted to IISc, IIT's, NIT's IISER's, IIEST, will get a Scholarship equivalent to INSPIRE
- Assured placement in ISRO and other divisions for the students of IIEST
- Institutes and their ranking can be found in www.nirfindia.org

After B.Sc Physics

- M.Sc. Physics in IIT's and NIT's through JAM
- Integrated Phd in IISER's and IISc through JAM and JEST
- M.Sc. Physics in Central Universities through CUCET
- M.Sc. in Energy Physics, Applied Physics in IIT's through JAM
- Integrated Phd in IMSc, TIFR, JNCASR through JEST score
- Integrated Phd in TIFR through JEST and TIFR exam
- M.Sc. Photonics, Reactor physics, Nuclear Engineering
- M.Sc Medical Physics
- M.Sc Biophysics

Research Institutes in abroad like CERN, NASA, LIGO offer Summer Internship programmes for motivated Indian students pursuing Undergraduate course in physics

Indian Academy of Science & various other research institutes offer paid Summer Internship for science students to get an hands on experience in research.
Institutes in India to pursue research in Physics

**Famous Research Institutes for Physics in India**

- **Saha Institute of Nuclear Physics, Kolkata**  [www.saha.ac.in](http://www.saha.ac.in)
- **International Centre for Theoretical Sciences, Bangalore**  [www.icts.res.in](http://www.icts.res.in)
- **Harish Chandra Research Institute, Allahabad**  [www.hri.res.in](http://www.hri.res.in)
- **Aryabhatta Research Institute of Observational Sciences, Nainital**  [www.aries.res.in](http://www.aries.res.in)

**Scholarships**

- **INSPIRE Scholarship** - Scholarship for Higher Education (SHE) - Rs. 50,000 per annum for full-time B.Sc. or B.A. students

**Eligibility Criteria**

- Top 100 rank holders in JEE Main or NEET for M.Sc. or M.Tech.
- Students studying at IITs, IISERs, or Department of Atomic Energy, Centre for Basic Science and Technology

**Postgraduate Merit Scholarship**

- Rs. 10,000 per annum for Ph.D. students

**Jobs in Government Sector**

- Scientific Officer and Scientific Assistant Jobs
- CSR Labs
- DRDO - Defence Research and Development Organisation
- DAE - Department of Atomic Energy
- DoS - Department of Science
- IMR - Indian Meteorological Department
- ONGC - Oil and Natural Gas Corporation
- ATC - Air Traffic Controller
- Teaching faculty in schools and colleges through SET, NET, IETE
- Scientist posted in various research institutes in India

**Opportunities after B.Sc. Physics**

- After +2
- After B.Sc. Physics
- NEET - National Eligibility cum Entrance Test
- IIT JEE - Joint Entrance Examination (Mains & Advanced)
- INSPIRE Scholarship - Scholarship for Higher Education (SHE) - Rs. 50,000 per annum
- Vikram Sarabhai Space Centre (VSSC), Thiruvananthapuram  [www.vssc.gov.in](http://www.vssc.gov.in)

**Eligibility Criteria**

- Students who are pursuing M.Sc. or Ph.D.
- Students who studied Maths at UG or PG level
Opportunities after B.Sc. Physics in India to pursue research in physics

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<td>S.N. Bose Centre for Basic Sciences</td>
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Institutes and their ranking can be found in www.nirfindia.org

Eligibility Criteria:- Passing JEST, GATE, NET Exams

Note

Scientist post in various research institutes in India

Dr. K S Krishnan Research Associateship (KSKRA)

Post Graduate Merit Scholarship for University rank holders in UG level

Central Research Institutes like IISER using KVPY, JEE

KVPY-Kishore Vaigyanik Protsahan Yojana

Top 10000 rank holders in JEE or NEET

NEET-National Eligibility cum Entrance Test

Entrance Examinations

JEST – Joint Entrance Screening Test

CSIR Labs

IIT JEE-Joint Entrance Examination (Mains & Advanced)

Scientific Officer and Scientific Assistant Jobs

NEST- National Entrance Screening Test

Women Scientist Scheme (WOS -A)

ATC – Air Traffic Controller

HSEE- Humanities and Social Sciences Entrance Examination

Indira Gandhi Scholarship for single girl child for full time regular Master's Degree

Dr. K S Krishnan Research Associateship (KSKRA)

Post Graduate Merit Scholarship for University rank holders in UG level

Central Universities through CUCET

Promotion of Science Education (POSE) Scholarship Scheme

B.Sc Hons in Mathematics and Physics in CMI

B.Sc Hons in Mathematics and Computer Science in CMI

M.Sc Medical Physics

Integrated Phd in IISER's and IISc through JAM and JEST

Integrated Msc. Physics

JIPMER - Jawaharlal Institute of Postgraduate Medical Education & Research

Advanced, IISER

Summer internship programmes for motivated Indian students

Dhirubhai Ambani Scholarship Programme

BIOSCIENFI.C

Admission in NISER through NEST

Five-Year Dual degree In IIST (B.Tech + Master of Science)

National Institute of Technology(NIT) in various places

Pre Matric and Post Matric Scholarship for students with Disabilities to pursue their School and Collegiate Education by the Department of Empowerment of Persons with Disabilities, Government of India.

Pre Matric and Post Matric Scholarship for students belonging to minority communities to pursue their School and Collegiate education by the Ministry of Minority affairs, Government of India.
In this unit, student is exposed to
- Historical background of electricity and magnetism
- The role of electrostatic force in day-to-day life
- Coulomb’s law and superposition principle
- The concept of electric field
- Calculation of electric field for various charge configurations
- Electrostatic potential and electrostatic potential energy
- Electric dipole and dipole moment
- Electric field and electrostatic potential for a dipole
- Electric flux
- Gauss law and its various applications
- Electrostatic properties of conductors and dielectrics
- Polarisation
- Capacitors in series and parallel combinations
- Effect of a dielectric in a capacitor
- Distribution of charges in conductors, corona discharge
- Working of a Van de Graaff generator

1.1 INTRODUCTION

Electromagnetism is one of the most important branches of physics. The technological developments of the modern 21st century are primarily due to our understanding of electromagnetism. The forces we experience in everyday life are electromagnetic in nature except gravity.

In standard XI, we studied about the gravitational force, tension, friction, normal force etc. Newton treated them to be independent of each other with each force being a separate natural force. But what is the origin of all these forces? It is now understood that except gravity, all forces which we experience in every day life (tension in the string, normal force from the surface, friction etc.) arise from electromagnetic forces within the atoms. Some examples are
(i) When an object is pushed, the atoms in our hand interact with the atoms in the object and this interaction is basically electromagnetic in nature.

(ii) When we stand on Earth’s surface, the gravitational force on us acts downwards and the normal force acts upward to counter balance the gravitational force. What is the origin of this normal force? It arises due to the electromagnetic interaction of atoms on the surface of the Earth with the atoms present in the feet of the person. Though, we are attracted by the gravitational force of the Earth, we stand on Earth only because of electromagnetic force of atoms.

(iii) When an object is moved on a surface, static friction resists the motion of the object. This static friction arises due to electromagnetic interaction between the atoms present in the object and atoms on the surface. Kinetic friction also has similar origin.

From these examples, it is clear that understanding electromagnetism is very essential to understand the universe in a holistic manner. The basic principles of electromagnetism are dealt in XII physics volume 1. This unit deals with the behaviour and other related phenomena of charges at rest. This branch of electricity which deals with stationary charges is called Electrostatics.

1.1.1 Historical background of electric charges

Two millenniums ago, Greeks noticed that amber (a solid, translucent material formed from the resin of a fossilized tree) after rubbing with animal fur attracted small pieces of leaves and dust. The amber possessing this property is said to be ‘charged’. It was initially thought that amber has this special property. Later people found that not only amber but even a glass rod rubbed with silk cloth, attracts pieces of papers. So glass rod also becomes ‘charged’ when rubbed with a suitable material.

Consider a charged rubber rod hanging from a thread as shown in Figure 1.1. Suppose another charged rubber rod is brought near the first rubber rod; the rods repel each other. Now if we bring a charged glass rod close to the charged rubber rod, they attract each other. At the same time, if a charged glass rod is brought near another charged glass rod, both the rods repel each other.

From these observations, the following inferences are made

(i) The charging of rubber rod and that of glass rod are different from one another.

(ii) The charged rubber rod repels another charged rubber rod, which implies that ‘like charges repel each other’. We can also arrive at the same inference by observing that a charged glass rod repels another charged glass rod.

(iii) The charged amber rod attracts the charged glass rod, implying that the charge in the glass rod is not the same kind of charge present in the rubber. Thus unlike charges attract each other.

Therefore, two kinds of charges exist in the universe. In the 18th century, Benjamin Franklin called one type of charge as positive (+) and another type of charge as negative (-). Based on Franklin’s convention, rubber and amber rods are negatively charged while the glass rod is positively charged. If the net charge is zero in the object, it is said to be electrically neutral.
is another intrinsic and fundamental property of particles. The nature of charges is understood through various experiments performed in the 19th and 20th century. The SI unit of charge is coulomb.

(ii) Conservation of charges

Benjamin Franklin argued that when one object is rubbed with another object, charges get transferred from one to the other. Before rubbing, both objects are electrically neutral and rubbing simply transfers the charges from one object to the other. (For example, when a glass rod is rubbed against silk cloth, some negative charge are transferred from glass to silk. As a result, the glass rod is positively charged and silk cloth becomes negatively charged). From these observations, he concluded that charges are neither created or nor destroyed but can only be transferred from one object to other. This is called conservation of total charges and is one of the fundamental conservation laws in physics. It is stated more generally in the following way.

The total electric charge in the universe is constant and charge can neither be created nor be destroyed. In any physical

1.1.2 Basic properties of charges

(i) Electric charge

Most objects in the universe are made up of atoms, which in turn are made up of protons, neutrons and electrons. These particles have mass, an inherent property of particles. Similarly, the electric charge is another intrinsic and fundamental property of particles. The nature of charges is understood through various experiments performed in the 19th and 20th century. The SI unit of charge is coulomb.

(ii) Conservation of charges

Benjamin Franklin argued that when one object is rubbed with another object, charges get transferred from one to the other. Before rubbing, both objects are electrically neutral and rubbing simply transfers the charges from one object to the other. (For example, when a glass rod is rubbed against silk cloth, some negative charge are transferred from glass to silk. As a result, the glass rod is positively charged and silk cloth becomes negatively charged). From these observations, he concluded that charges are neither created or nor destroyed but can only be transferred from one object to other. This is called conservation of total charges and is one of the fundamental conservation laws in physics. It is stated more generally in the following way.

The total electric charge in the universe is constant and charge can neither be created nor be destroyed. In any physical

Figure 1.1 (a) Unlike charges attract each other (b) Like charges repel each other

Following the pioneering work of J. J. Thomson and E. Rutherford, in the late 19th century and in the beginning of 20th century, we now understand that the atom is electrically neutral and is made up of the negatively charged electrons, positively charged protons, and neutrons which have zero charge. The material objects made up of atoms are neutral in general. When an object is rubbed with another object (for example rubber with silk cloth), some amount of charge is transferred from one object to another due to the friction between them and the object is then said to be electrically charged. Charging the objects through rubbing is called triboelectric charging.
process, the net change in charge will always be zero.

(iii) Quantisation of charges
What is the smallest amount of charge that can be found in nature? Experiments show that the charge on an electron is \(-e\) and the charge on the proton is \(+e\). Here, \(e\) denotes the fundamental unit of charge. The charge \(q\) on any object is equal to an integral multiple of this fundamental unit of charge \(e\).

\[
q = ne
\]  

(1.1)

Here \(n\) is any integer (0, \(\pm 1\), \(\pm 2\), \(\pm 3\), \(\pm 4\),............). This is called quantisation of electric charge.

Robert Millikan in his famous experiment found that the value of \(e = 1.6 \times 10^{-19}\)C. The charge of an electron is \(-1.6 \times 10^{-19}\)C and the charge of the proton is \(+1.6 \times 10^{-19}\)C.

When a glass rod is rubbed with silk cloth, the number of charges transferred is usually very large, typically of the order of \(10^{10}\). So the charge quantisation is not appreciable at the macroscopic level. Hence the charges are treated to be continuous (not discrete). But at the microscopic level, quantisation of charge plays a vital role.

**EXAMPLE 1.1**

Calculate the number of electrons in one coulomb of negative charge.

**Solution**

According to the quantisation of charge

\[
q = ne
\]

Here \(q = 1\)C. So the number of electrons in 1 coulomb of charge is

\[
n = \frac{q}{e} = \frac{1\text{C}}{1.6\times10^{-19}} = 6.25 \times 10^{18} \text{ electrons}
\]

1.2

**COULOMB’S LAW**

In the year 1786, Coulomb deduced the expression for the force between two stationary point charges in vacuum or free space. Consider two point charges \(q_1\) and \(q_2\) at rest in vacuum, and separated by a distance of \(r\), as shown in Figure 1.2. According to Coulomb, the force on the point charge \(q_2\) exerted by another point charge \(q_1\) is

\[
\vec{F}_{21} = k \frac{q_1 q_2}{r^2} \hat{r}_{12}
\]

(1.2)

where \(\hat{r}_{12}\) is the unit vector directed from charge \(q_1\) to charge \(q_2\) and \(k\) is the proportionality constant.

**Figure 1.2** Coulomb force between two point charges

**Important aspects of Coulomb’s law**

(i) Coulomb’s law states that the electrostatic force is directly proportional to the product of the magnitude of the two point charges and is inversely proportional to the square of the distance between the two point charges.

(ii) The force on the charge \(q_i\) exerted by the charge \(q_j\) always lies along the line joining the two charges. \(\hat{r}_{ij}\) is the unit
vector pointing from charge \( q_1 \) to \( q_2 \). It is shown in the Figure 1.2. Likewise, the force on the charge \( q_1 \) exerted by \( q_2 \) is along \(-\hat{r}_{12}\) (i.e., in the direction opposite to \( \hat{r}_{12} \)).

(iii) In SI units, \( k = \frac{1}{4\pi \varepsilon_0} \) and its value is \( 9 \times 10^9 \) N m\(^2\) C\(^{-2}\). Here \( \varepsilon_0 \) is the permittivity of free space or vacuum and the value of \( \varepsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \text{C}^2 \text{N}^{-1} \text{m}^{-2} \).

(iv) The magnitude of the electrostatic force between two charges each of one coulomb and separated by a distance of 1 m is calculated as follows:

\[
|F| = \frac{9 \times 10^9 \times 1\times 1}{1^2} = 9 \times 10^9 N.
\]

This is a huge quantity, almost equivalent to the weight of one million ton. We never come across 1 coulomb of charge in practice. Most of the electrical phenomena in day-to-day life involve electrical charges of the order of \( \mu \text{C} \) (micro coulomb) or nC (nano coulomb).

(v) In SI units, Coulomb’s law in vacuum takes the form \( \vec{F}_{21} = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12} \). In a medium of permittivity \( \varepsilon \), the force between two point charges is given by \( \vec{F}_{21} = \frac{1}{4\pi \varepsilon} \frac{q_1 q_2}{r^2} \hat{r}_{12} \). Since \( \varepsilon > \varepsilon_0 \), the force between two point charges in a medium other than vacuum is always less than that in vacuum. We define the relative permittivity for a given medium as \( \varepsilon_r = \frac{\varepsilon}{\varepsilon_0} \). For vacuum or air, \( \varepsilon_r = 1 \) and for all other media \( \varepsilon_r > 1 \).

(vi) Coulomb’s law has same structure as Newton’s law of gravitation. Both are inversely proportional to the square of the distance between the particles. The electrostatic force is directly proportional to the product of the magnitude of two point charges and gravitational force is directly proportional to the product of two masses. But there are some important differences between these two laws.

- The gravitational force between two masses is always attractive but Coulomb force between two charges can be attractive or repulsive, depending on the nature of charges.
- The value of the gravitational constant \( G = 6.626 \times 10^{-11} \) N m\(^2\) kg\(^{-2}\). The value of the constant \( k \) in Coulomb law is \( k = 9 \times 10^9 \) N m\(^2\) C\(^{-2}\). Since \( k \) is much more greater than \( G \), the electrostatic force is always greater in magnitude than gravitational force for smaller size objects.
- The gravitational force between two masses is independent of the medium. For example, if 1 kg of two masses are kept in air or inside water, the gravitational force between two masses remains the same. But the electrostatic force between the two charges depends on nature of the medium in which the two charges are kept at rest.
- The gravitational force between two point masses is the same whether two masses are at rest or in motion. If the charges are in motion, yet another force (Lorentz force) comes into play in addition to coulomb force.

(vii) The force on a charge \( q_1 \) exerted by a point charge \( q_2 \) is given by

\[
\vec{F}_{12} = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21}.
\]

Here \( \hat{r}_{21} \) is the unit vector from charge \( q_2 \) to \( q_1 \).

But \( \hat{r}_{21} = -\hat{r}_{12} \).
\[ \vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} (\hat{r}_{12}) = -\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^3} (\hat{r}_{12}) \]

(or) \[ \vec{F}_{12} = -\vec{F}_{21} \]

Therefore, the electrostatic force obeys Newton's third law.

(viii) The expression for Coulomb force is true only for point charges. But the point charge is an ideal concept. However we can apply Coulomb’s law for two charged objects whose sizes are very much smaller than the distance between them. In fact, Coulomb discovered his law by considering the charged spheres in the torsion balance as point charges. The distance between the two charged spheres is much greater than the radii of the spheres.

**EXAMPLE 1.2**

Consider two point charges \( q_1 \) and \( q_2 \) at rest as shown in the figure.

They are separated by a distance of 1m. Calculate the force experienced by the two charges for the following cases:

(a) \( q_1 = +2 \mu \text{C} \) and \( q_2 = +3 \mu \text{C} \)
(b) \( q_1 = +2 \mu \text{C} \) and \( q_2 = -3 \mu \text{C} \)
(c) \( q_1 = +2 \mu \text{C} \) and \( q_2 = -3 \mu \text{C} \) kept in water (\( \varepsilon_r = 80 \))

(a) \( q_1 = +2 \mu \text{C} \), \( q_2 = +3 \mu \text{C} \), and \( r = 1 \text{m} \). Both are positive charges, so the force will be repulsive.

Force experienced by the charge \( q_2 \) due to \( q_1 \) is given by

\[ \vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^3} \hat{r}_{12} \]

Here \( \hat{r}_{12} \) is the unit vector from \( q_1 \) to \( q_2 \). Since \( q_2 \) is located on the right of \( q_1 \), we have

\[ \hat{r}_{12} = \hat{i} , \text{ so that} \]

\[ \vec{F}_{21} = \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 3 \times 10^{-6}}{1^3} \hat{i} = \frac{1}{4\pi \varepsilon_0} \times 9 \times 10^9 \]

\[ = 54 \times 10^{-3} \text{N} \hat{i} \]
According to Newton’s third law, the force experienced by the charge \( q_1 \) due to \( q_2 \) is 
\[
\vec{F}_{12} = -\vec{F}_{21}
\]

So that 
\[
\vec{F}_{12} = -54 \times 10^{-3} N \hat{i}.
\]

The directions of \( \vec{F}_{21} \) and \( \vec{F}_{12} \) are shown in the above figure in case (a).

(b) \( q_1 = +2 \ \mu C, \ q_2 = -3 \ \mu C, \) and \( r = 1 \text{m}. \) They are unlike charges. So the force will be attractive.

Force experienced by the charge \( q_2 \) due to \( q_1 \) is given by
\[
\vec{F}_{21} = \frac{9 \times 10^9 \times (2 \times 10^{-6}) \times (-3 \times 10^{-6})}{1^2} \vec{r}_{12}
\]
\[
= -54 \times 10^{-3} N \hat{i} \quad \text{(Using } \vec{r}_{12} = \hat{i})
\]

The charge \( q_2 \) will experience an attractive force towards \( q_1 \) which is in the negative x direction.

According to Newton’s third law, the force experienced by the charge \( q_1 \) due to \( q_2 \) is 
\[
\vec{F}_{12} = -\vec{F}_{21}
\]

so that 
\[
\vec{F}_{12} = 54 \times 10^{-3} N \hat{i}
\]

The directions of \( \vec{F}_{21} \) and \( \vec{F}_{12} \) are shown in the figure (case (b)).

(c) If these two charges are kept inside the water, then the force experienced by \( q_2 \) due to \( q_1 \) is
\[
\vec{F}_{21}^{\text{W}} = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} \vec{r}_{12}
\]

since \( \varepsilon = \varepsilon_r \varepsilon_0 \),

we have 
\[
\vec{F}_{21}^{\text{W}} = \frac{1}{4\pi \varepsilon_r \varepsilon_0} \frac{q_1 q_2}{r^2} \vec{r}_{12} = \frac{\vec{F}_{21}}{\varepsilon_r}
\]

Therefore,
\[
\vec{F}_{21}^{\text{W}} = \frac{54 \times 10^{-3}}{80} \hat{i} = -0.675 \times 10^{-3} N \hat{i}
\]

Note that the strength of the force between the two charges in water is reduced by 80 times compared to the force between the same two charges in vacuum.

When common salt (NaCl) is taken in water, the electrostatic force between Na and Cl ions is reduced due to the high relative permittivity of water (\( \varepsilon_r = 80 \)). This is the reason water acts as a good solvent.

**Example 1.3**

Two small-sized identical equally charged spheres, each having mass 1 mg are hanging in equilibrium as shown in the figure. The length of each string is 10 cm and the angle \( \theta \) is 7° with the vertical. Calculate the magnitude of the charge in each sphere.

(Take \( g = 10 \text{ ms}^{-2} \))

\[ L + q \]

\[ \theta \]

\[ L \]

\[ \theta \]

\[ q + a \]

\[ q \]

**Solution**

If the two spheres are neutral, the angle between them will be 0° when hanged
vertically. Since they are positively charged spheres, there will be a repulsive force between them and they will be at equilibrium with each other at an angle of $7^\circ$ with the vertical. At equilibrium, each charge experiences zero net force in each direction. We can draw a free body diagram for one of the charged spheres and apply Newton's second law for both vertical and horizontal directions.

The free body diagram is shown below.

In the x-direction, the acceleration of the charged sphere is zero.

Using Newton's second law ($F_{\text{net}} = m\ddot{a}$), we have

$$T\cos\theta - F_e \sin\theta = 0 \quad \Rightarrow \quad T\cos\theta = F_e \sin\theta$$

Here $T$ is the tension acting on the charge due to the string and $F_e$ is the electrostatic force between the two charges.

In the y-direction also, the net acceleration experienced by the charge is zero.

$$T\sin\theta - mg = 0 \quad \Rightarrow \quad T\sin\theta = mg$$

By dividing equation (1) by equation (2),

$$\tan\theta = \frac{F_e}{mg} \quad (3)$$

Since they are equally charged, the magnitude of the electrostatic force is

$$F_e = k\frac{q^2}{r^2} \quad \text{where} \quad k = \frac{1}{4\pi\varepsilon_0}$$

Here $r = 2a = 2L\sin\theta$. By substituting these values in equation (3),

$$\tan\theta = k\frac{q^2}{mg(2L\sin\theta)^2} \quad (4)$$

Rearranging the equation (4) to get $q$

$$q = \frac{2L\sin\theta}{k} \sqrt{\frac{mg\tan\theta}{2}}$$

$$= 2 \times 0.1 \times \sin7^\circ \times \sqrt{\frac{10^{-3} \times 10 \times \tan7^\circ}{9 \times 10^9}}$$

$$q = 8.9 \times 10^{-9} \text{ C} = 8.9 \text{ nC}$$

EXAMPLE 1.4

Calculate the electrostatic force and gravitational force between the proton and the electron in a hydrogen atom. They are separated by a distance of $5.3 \times 10^{-11}$ m. The magnitude of charges on the electron and proton are $1.6 \times 10^{-19}$ C. Mass of the electron is $m_e = 9.1 \times 10^{-31}$ kg and mass of proton is $m_p = 1.6 \times 10^{-27}$ kg.

**Solution**

The proton and the electron attract each other. The magnitude of the electrostatic force between these two particles is given by

$$F_e = \frac{kq^2}{r^2} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{(5.3 \times 10^{-11})^2}$$

$$= \frac{9 \times 2.56}{28.09} \times 10^{-7} = 8.2 \times 10^{-8} \text{ N}$$

**Unit 1 Electrostastics**
1.2.1 Superposition principle

Coulomb’s law explains the interaction between two point charges. If there are more than two charges, the force on one charge due to all the other charges needs to be calculated. Coulomb’s law alone does not give the answer. The superposition principle explains the interaction between multiple charges.

According to this superposition principle, the total force acting on a given charge is equal to the vector sum of forces exerted on it by all the other charges.

Consider a system of n charges, namely \( q_1, q_2, q_3, \ldots, q_n \). The force on \( q_1 \) exerted by the charge \( q_3 \) is

\[
\vec{F}_{13} = k \frac{q_1 q_3}{r_{31}^2} \hat{r}_{31}
\]

Here \( \hat{r}_{31} \) is the unit vector from \( q_3 \) to \( q_1 \) along the line joining the two charges and \( r_{31} \) is the distance between the charges \( q_1 \) and \( q_3 \). The electrostatic force between two charges is not affected by the presence of other charges in the neighbourhood.

The force on \( q_1 \) exerted by the charge \( q_k \) is

\[
\vec{F}_{1k} = k \frac{q_1 q_k}{r_{k1}^2} \hat{r}_{k1}
\]

By continuing this, the total force acting on the charge \( q_1 \) due to all other charges is given by

\[
\vec{F}_{1}^{\text{tot}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots + \vec{F}_{1n}
\]

\[
\vec{F}_{1}^{\text{tot}} = k \left( \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21} + \frac{q_1 q_3}{r_{31}^2} \hat{r}_{31} + \frac{q_1 q_4}{r_{41}^2} \hat{r}_{41} + \ldots \right)
\]

\[\ldots + \frac{q_1 q_n}{r_{n1}^2} \hat{r}_{n1} \right)
\]

(1.3)

The gravitational force between the proton and the electron is attractive. The magnitude of the gravitational force between these particles is

\[
F_G = \frac{Gm_p m_e}{r^2} = \frac{6.67 \times 10^{-11} \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-27}}{(5.3 \times 10^{-11})^2} = \frac{97.11}{28.09} \times 10^{-47} = 3.4 \times 10^{-47} \text{ N}
\]

The ratio of the two forces \( \frac{F_e}{F_G} = \frac{8.2 \times 10^{-8}}{3.4 \times 10^{-47}} = 2.41 \times 10^{39} \)

Note that \( F_e \approx 10^{39} F_G \)

The electrostatic force between a proton and an electron is enormously greater than the gravitational force between them. Thus the gravitational force is negligible when compared with the electrostatic force in many situations such as for small size objects and in the atomic domain. This is the reason why a charged comb attracts an uncharged piece of paper with greater force even though the piece of paper is attracted downward by the Earth. This is shown in Figure 1.3
Without the superposition principle, Coulomb’s law will be incomplete when applied to more than two charges. Both the superposition principle and Coulomb’s law form fundamental principles of electrostatics and explain all the phenomena in electrostatics. But they are not derivable from each other.

**Example 1.5**

Consider four equal charges $q_1, q_2, q_3$, and $q_4 = q = +1 \mu\text{C}$ located at four different points on a circle of radius 1 m, as shown in the figure. Calculate the total force acting on the charge $q_1$ due to all the other charges.

**Solution**

According to the superposition principle, the total electrostatic force on charge $q_1$ is the vector sum of the forces due to the other charges,

$$
\vec{F}_{1\text{tot}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}
$$

The following diagram shows the direction of each force on the charge $q_1$.

The charges $q_2$ and $q_4$ are equi-distant from $q_1$. As a result the strengths (magnitude) of the forces $\vec{F}_{12}$ and $\vec{F}_{14}$ are the same even though their directions are different. Therefore the vectors representing these two forces are drawn with equal lengths. But the charge $q_3$ is located farther compared to $q_2$ and $q_4$. Since the strength of the electrostatic force decreases as distance increases, the strength of the force $\vec{F}_{13}$ is lesser than that of forces $\vec{F}_{12}$ and $\vec{F}_{14}$. Hence the vector representing the force $\vec{F}_{13}$ is drawn with smaller length compared to that for forces $\vec{F}_{12}$ and $\vec{F}_{14}$.

From the figure, $r_{21} = \sqrt{2} m = r_{41}$ and $r_{31} = 2 m$

The magnitudes of the forces are given by

$$
F_{ij} = \frac{kq^2}{r_{ij}^2} = \frac{9 \times 10^9 \times 10^{-12}}{4}
$$
The interaction between two charges is determined by Coulomb's law. How does the interaction itself occur? Consider a point charge kept at a point in space. If another point charge is placed at some distance from the first point charge, it experiences either an attractive force or repulsive force. This is called 'action at a distance'. But how does the second charge know about existence of the first charge which is located at some distance away from it? To answer this question, Michael Faraday introduced the concept of field.

According to Faraday, every charge in the universe creates an electric field in the surrounding space, and if another charge is brought into its field, it will interact with the electric field at that point and will experience a force. It may be recalled that the interaction of two masses is similarly explained using the concept of gravitational field (Refer unit 6, volume 2, XI physics). Both the electric and gravitational forces are non-contact forces, hence the field concept is required to explain action at a distance.

Consider a source point charge \( q \) located at a point in space. Another point charge \( q_o \) (test charge) is placed at some point P which is at a distance \( r \) from the charge \( q \). The electrostatic force experienced by the charge \( q_o \) due to \( q \) is given by Coulomb's law.

\[
F = \frac{kq q_o}{r^2} = \frac{1}{4\pi \epsilon_o} \frac{qq_o}{r^2} \hat{r}
\]

where \( k = \frac{1}{4\pi \epsilon_o} \)

**1.3 Electric Field**

The interaction between two charges is determined by Coulomb's law. How does the interaction itself occur? Consider a point charge kept at a point in space. If another point charge is placed at some distance from the first point charge, it experiences either an attractive force or repulsive force. This is called 'action at a distance'. But how does the second charge know about existence of the first charge which is located at some distance away from it? To answer this question, Michael Faraday introduced the concept of field.

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Consider a source point charge \( q \) located at a point in space. Another point charge \( q_o \) (test charge) is placed at some point P which is at a distance \( r \) from the charge \( q \). The electrostatic force experienced by the charge \( q_o \) due to \( q \) is given by Coulomb's law.

\[
F = \frac{kq q_o}{r^2} = \frac{1}{4\pi \epsilon_o} \frac{qq_o}{r^2} \hat{r}
\]

where \( k = \frac{1}{4\pi \epsilon_o} \)
The charge q creates an electric field in the surrounding space. The electric field at the point P at a distance r from the point charge q is the force experienced by a unit charge and is given by

$$\vec{E} = \vec{F} = \frac{kq}{r^2}r = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$  \hspace{1cm} (1.4)

Here $\hat{r}$ is the unit vector pointing from q to the point of interest P. The electric field is a vector quantity and its SI unit is Newton per Coulomb (NC\(^{-1}\)).

**Important aspects of Electric field**

(i) If the charge $q$ is positive then the electric field points away from the source charge and if $q$ is negative, the electric field points towards the source charge $q$. This is shown in the Figure 1.4.

(ii) If the electric field at a point P is $\vec{E}$, then the force experienced by the test charge $q_0$, placed at the point P is

$$\vec{F} = q_0 \vec{E}$$  \hspace{1cm} (1.5)

This is Coulomb’s law in terms of electric field. This is shown in Figure 1.5

(iii) The equation (1.4) implies that the electric field is independent of the test charge $q_0$ and it depends only on the source charge $q$.

(iv) Since the electric field is a vector quantity, at every point in space, this field has unique direction and magnitude as shown in Figures 1.6(a) and (b). From equation (1.4), we can infer that as distance increases, the electric field decreases in magnitude.

Note that in Figures 1.6 (a) and (b) the length of the electric field vector is shown for three different points. The strength or magnitude of the electric field at point P is stronger than at the points Q and R because the point P is closer to the source charge.
There are two kinds of the electric field: uniform (constant) electric field and non-uniform electric field. Uniform electric field will have the same direction and constant magnitude at all points in space. Non-uniform electric field will have different directions or different magnitudes or both at different points in space. The electric field created by a point charge is basically a non-uniform electric field. This non-uniformity arises, both in direction and magnitude, with the direction being radially outward (or inward) and the magnitude changes as distance increases. These are shown in Figure 1.7.

**Figure 1.6** (a) Electric field due to positive charge (b) Electric field due to negative charge

In the definition of electric field, it is assumed that the test charge \( q_0 \) is taken sufficiently small, so that bringing this test charge will not move the source charge. In other words, the test charge is made sufficiently small such that it will not modify the electric field of the source charge.

The expression (1.4) is valid only for point charges. For continuous and finite size charge distributions, integration techniques must be used. These will be explained later in the same section. However, this expression can be used as an approximation for a finite-sized charge if the test point is very far away from the finite sized source charge. Note that we similarly treat the Earth as a point mass when we calculate the gravitational field of the Sun on the Earth (refer unit 6, volume 2, XI physics).

**EXAMPLE 1.6**

Calculate the electric field at points P, Q for the following two cases, as shown in the figure.
### Case (b)

The magnitude of the electric field at point P

$$|\vec{E}_P| = \frac{kq}{r^2} = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{4}$$

$$= 4.5 \times 10^3 \text{ N C}^{-1}$$

Since the source charge is negative, the electric field points towards the charge. So the electric field at the point P is given by

$$\vec{E}_P = -4.5 \times 10^3 \hat{i} \text{ NC}^{-1}$$

For the point Q, $|\vec{E}_Q| = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{36}$

$$= 0.5 \times 10^3 \text{ N C}^{-1}$$

$$\vec{E}_Q = 0.56 \times 10^3 \hat{j} \text{ NC}^{-1}$$

At the point Q the electric field is directed along the positive x-axis.

### Solution

Case (a)

The magnitude of the electric field at point P is

$$E_p = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} = \frac{9 \times 10^9 \times 1 \times 10^{-6}}{4}$$

$$= 2.25 \times 10^3 \text{ NC}^{-1}$$

Since the source charge is positive, the electric field points away from the charge. So the electric field at the point P is given by

$$\vec{E}_p = 2.25 \times 10^3 \hat{i} \text{ NC}^{-1}$$

For the point Q

$$|\vec{E}_Q| = \frac{9 \times 10^9 \times 1 \times 10^{-6}}{16} = 0.56 \times 10^3 \text{ NC}^{-1}$$

Hence

$$\vec{E}_Q = 0.56 \times 10^3 \hat{j} \text{ NC}^{-1}$$
1.3.2 Electric field due to the system of point charges

Suppose a number of point charges are distributed in space. To find the electric field at some point P due to this collection of point charges, superposition principle is used. The electric field at an arbitrary point due to a collection of point charges is simply equal to the vector sum of the electric fields created by the individual point charges. This is called superposition of electric fields.

Consider a collection of point charges $q_1, q_2, q_3, \ldots, q_n$ located at various points in space. The total electric field at some point P due to all these n charges is given by

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots + \vec{E}_n \quad (1.6)$$

where $r_{1P}, r_{2P}, r_{3P}, \ldots, r_{nP}$ are the distance of the charges $q_1, q_2, q_3, \ldots, q_n$ from the point P respectively. Also $\hat{r}_{1P}, \hat{r}_{2P}, \hat{r}_{3P}, \ldots, \hat{r}_{nP}$ are the corresponding unit vectors directed from $q_1, q_2, q_3, \ldots, q_n$ to P.

Equation (1.7) can be re-written as,

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \left( \frac{q_i}{r_{ip}^2} \hat{r}_{ip} \right) \quad (1.8)$$

For example in Figure 1.8, the resultant electric field due to three point charges $q_1, q_2, q_3$ at point P is shown. Note that the relative lengths of the electric field vectors for the charges depend on relative distances of the charges to the point P.

**EXAMPLE 1.7**

Consider the charge configuration as shown in the figure. Calculate the electric field at point A. If an electron is placed at points A, what is the acceleration experienced by this electron? (mass of the electron $= 9.1 \times 10^{-31}$ kg and charge of electron $= -1.6 \times 10^{-19}$ C)

Equation (1.7) can be used to find the electric field at point A.

For example in Figure 1.8, the resultant electric field due to three point charges $q_1, q_2, q_3$ at point P is shown. Note that the relative lengths of the electric field vectors for the charges depend on relative distances of the charges to the point P.
etc., it is very difficult to look at individual charges in these charged bodies. Therefore, it is assumed that charge is distributed continuously on the charged bodies and the discrete nature of charges is not considered here. The electric field due to such continuous charge distributions is found by invoking the method of calculus.

Consider the following charged object of irregular shape as shown in Figure 1.9. The entire charged object is divided into a large number of charge elements \( \Delta q_1, \Delta q_2, \Delta q_3, \ldots, \Delta q_n \) and each charge element \( \Delta q \) is taken as a point charge.

The electric field at a point \( P \) due to a charged object is approximately given by the sum of the fields at \( P \) due to all such charge elements:

\[
\vec{E}_P \approx \frac{1}{4 \pi \varepsilon_0} \left( \frac{\Delta q_1}{r_{1P}^2} \hat{r}_{1P} + \frac{\Delta q_2}{r_{2P}^2} \hat{r}_{2P} + \ldots \right)
\]

\[
\approx \frac{1}{4 \pi \varepsilon_0} \sum_{i=1}^{n} \frac{\Delta q_i}{r_{iP}^2} \hat{r}_{iP}
\]  

(1.9)

The magnitude of electric field

\[
|\vec{E}_A| = \sqrt{(2.25 \times 10^9)^2 + (2.25 \times 10^9)^2} = 2.25 \sqrt{2} \times 10^9 \text{ N} \text{C}^{-1}
\]

The direction of \( \vec{E}_A \) is given by

\[
\frac{\vec{E}_A}{|\vec{E}_A|} = \frac{2.25 \times 10^9 (\hat{i} + \hat{j})}{2.25 \sqrt{2} \times 10^9} = \frac{\hat{i} + \hat{j}}{\sqrt{2}},
\]

which is the unit vector along \( OA \) as shown in the figure.

The acceleration experienced by an electron placed at point \( A \) is

\[
a_A = \frac{\vec{E}_A m_e}{q_1} = \frac{q_1}{2 \mu \text{C}} \times \left( 9.1 \times 10^{-31} \right)
\]

\[
= -3.95 \times 10^{20} (\hat{i} + \hat{j}) \text{N}
\]

The electron is accelerated in a direction exactly opposite to \( \vec{E}_A \).

### 1.3.3 Electric field due to continuous charge distribution

The electric charge is quantized microscopically. The expressions (1.2), (1.3), (1.4) are applicable to only point charges. While dealing with the electric field due to a charged sphere or a charged wire
Here $\Delta q_i$ is the $i^{th}$ charge element, $r_{ iP}$ is the distance of the point P from the $i^{th}$ charge element and $\hat{r}_{ iP}$ is the unit vector from $i^{th}$ charge element to the point P. However the equation (1.9) is only an approximation. To incorporate the continuous distribution of charge, we take the limit $\Delta q \to 0 (= dq)$. In this limit, the summation in the equation (1.9) becomes an integration and takes the following form

$$\vec{E} = \frac{1}{4\pi \varepsilon_0} \int \frac{dq \hat{r}_i}{r^2}$$  \hspace{1cm} (1.10)

Here $r$ is the distance of the point P from the infinitesimal charge $dq$ and $\hat{r}$ is the unit vector from $dq$ to point P. Even though the electric field for a continuous charge distribution is difficult to evaluate, the force experienced by some test charge $q$ in this electric field is still given by $\vec{F} = q\vec{E}$.

(a) If the charge $Q$ is uniformly distributed along the wire of length $L$, then linear charge density (charge per unit length) is $\lambda = \frac{Q}{L}$. Its unit is coulomb per meter (Cm$^{-1}$).

The charge present in the infinitesimal length $dl$ is $dq = \lambda dl$. This is shown in Figure 1.10 (a).

(b) If the charge $Q$ is uniformly distributed on a surface of area $A$, then surface charge density (charge per unit area) is $\sigma = \frac{Q}{A}$. Its unit is coulomb per square meter (C m$^{-2}$).

The charge present in the infinitesimal area $dA$ is $dq = \sigma dA$. This is shown in the figure 1.10 (b).

The electric field due to a of total charge $Q$ is given by

$$\vec{E} = \frac{1}{4\pi \varepsilon_0} \int \frac{\sigma dA \hat{r}}{r^2} = \frac{1}{4\pi \varepsilon_0} \sigma \int \frac{dA \hat{r}}{r^2}$$

This is shown in Figure 1.10(b).

(c) If the charge $Q$ is uniformly distributed in a volume $V$, then volume charge density (charge per unit volume) is given by $\rho = \frac{Q}{V}$. Its unit is coulomb per cubic meter (C m$^{-3}$).

The charge present in the infinitesimal volume element $dV$ is $dq = \rho dV$. This is shown in Figure 1.10(c).

The electric field due to a volume of total charge $Q$ is given by

$$\vec{E} = \frac{1}{4\pi \varepsilon_0} \int \frac{\rho dV \hat{r}}{r^2} = \frac{1}{4\pi \varepsilon_0} \rho \int \frac{dV \hat{r}}{r^2}$$

---

**EXAMPLE 1.8**

A block of mass $m$ and positive charge $q$ is placed on an insulated frictionless inclined plane as shown in the figure. A uniform electric field $E$ is applied parallel to the inclined surface such that the block is at rest. Calculate the magnitude of the electric field $E$. 

---

**Figure 1.10** Line, surface and volume charge distribution

The electric field due to the line of total charge $Q$ is given by

$$\vec{E} = \frac{1}{4\pi \varepsilon_0} \int \frac{\lambda dl \hat{r}}{r^2}$$

**UNIT 1 Electrostatics**
A convenient inertial coordinate system is located in the inclined surface as shown in the figure. The mass \( m \) has zero net acceleration both in \( x \) and \( y \)-direction.

Along \( x \)-direction, applying Newton’s second law, we have

\[
mg \sin \theta \hat{i} - qE \hat{i} = 0
\]

or,

\[
E = \frac{mg \sin \theta}{q}
\]

Note that the magnitude of the electric field is directly proportional to the mass \( m \) and inversely proportional to the charge \( q \). It implies that, if the mass is increased by keeping the charge constant, then a strong electric field is required to stop the object from sliding. If the charge is increased by keeping the mass constant, then a weak electric field is sufficient to stop the mass from sliding down the plane.

The electric field also can be expressed in terms of height and the length of the inclined surface of the plane.

\[
E = \frac{mg h}{qL}
\]

### 1.3.4 Electric field lines

Electric field vectors are visualized by the concept of electric field lines. They form a set of continuous lines which are the visual representation of the electric field in some region of space. The following rules are followed while drawing electric field lines for charges.

- The electric field lines start from a positive charge and end at negative charges or at infinity. For a positive
point charge the electric field lines point radially outward and for a negative point charge, the electric field lines point radially inward. These are shown in Figure 1.11 (a) and (b).

For a positive point charge, the field lines are directed radially outward.

For a negative point charge, the field lines are directed radially inward.

(a)

(b)

Figure 1.11 Electric field lines for isolated positive and negative charges

Note that for an isolated positive point charge the electric field line starts from the charge and ends only at infinity. For an isolated negative point charge the electric field lines start at infinity and end at the negative charge.

- The electric field vector at a point in space is tangential to the electric field line at that point. This is shown in Figure 1.12

Figure 1.12 Electric field at a point P

- The electric field lines are denser (more closer) in a region where the electric field has larger magnitude and less dense in a region where the electric field is of smaller magnitude. In other words, the number of lines passing through a given surface area perpendicular to the lines is proportional to the magnitude of the electric field in that region. This is shown in Figure 1.13

Figure 1.13 Electric field has larger magnitude at surface A than B

Figure 1.13 shows electric field lines from a positive point charge. The magnitude of the electric field for a point charge decreases
as the distance increases \( \left| \mathbf{E} \right| \propto \frac{1}{r^2} \). So the electric field has greater magnitude at the surface A than at B. Therefore, the number of lines crossing the surface A is greater than the number of lines crossing the surface B. Note that at surface B the electric field lines are farther apart compared to the electric field lines at the surface A.

- No two electric field lines intersect each other. If two lines cross at a point, then there will be two different electric field vectors at the same point, as shown in Figure 1.14.

As a consequence, if some charge is placed in the intersection point, then it has to move in two different directions at the same time, which is physically impossible. Hence, electric field lines do not intersect.

- The number of electric field lines that emanate from the positive charge or end at a negative charge is directly proportional to the magnitude of the charges.

For example in the Figure 1.15, the electric field lines are drawn for charges +q and -2q. Note that the number of field lines emanating from +q is 8 and the number of field lines ending at -2q is 16. Since the magnitude of the second charge is twice that of the first charge, the number of field lines drawn for -2q is twice in number than that for charge +q.

**Example 1.9**

The following pictures depict electric field lines for various charge configurations.
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(i) In figure (a) identify the signs of two charges and find the ratio \( \frac{q_1}{q_2} \).

(ii) In figure (b), calculate the ratio of two positive charges and identify the strength of the electric field at three points A, B, and C.

(iii) Figure (c) represents the electric field lines for three charges. If \( q_2 = -20 \text{ nC} \), then calculate the values of \( q_1 \) and \( q_3 \).

**Solution**

(i) The electric field lines start at \( q_2 \) and end at \( q_1 \). In figure (a), \( q_2 \) is positive and \( q_1 \) is negative. The number of lines starting from \( q_2 \) is 18 and the number of the lines ending at \( q_1 \) is 6. So \( q_2 \) has greater magnitude. The ratio of \( \frac{q_1}{q_2} \) is \( \frac{6}{18} = \frac{1}{3} \). It implies that \( |q_2| = 3|q_1| \).

(ii) In figure (b), the number of field lines emanating from both positive charges are equal (N=18). So the charges are equal. At point A, the electric field lines are denser compared to the lines at point B. So the electric field at point A is greater in magnitude compared to the field at point B. Further, no electric field line passes through C, which implies that the resultant electric field at C due to these two charges is zero.

(iii) In the figure (c), the electric field lines start at \( q_1 \) and \( q_3 \) and end at \( q_2 \). This implies that \( q_1 \) and \( q_3 \) are positive charges. The ratio of the number of field lines is \( \frac{q_1}{q_2} = \frac{8}{16} = \frac{1}{2} \), implying that \( q_1 \) and \( q_3 \) are half of the magnitude of \( q_2 \). So \( q_1 = q_3 = +10 \text{ nC} \).

**1.4 Electric Dipole and Its Properties**

1.4.1 Electric dipole

Two equal and opposite charges separated by a small distance constitute an electric dipole. In many molecules, the centers of positive and negative charge do not coincide. Such molecules behave as permanent dipoles. Examples: CO, water, ammonia, HCl etc.

Consider two equal and opposite point charges (+q, -q) that are separated by a distance 2a as shown in Figure 1.16(a).

The electric dipole moment is defined as

\[ \vec{p} = q\vec{r}_+ - q\vec{r}_- \]

Here \( \vec{r}_+ \) is the position vector of +q from the origin and \( \vec{r}_- \) is the position vector of -q from the origin. Then, from Figure 1.16 (a),
\[ \mathbf{p} = \sum_{i=1}^{n} q_i \mathbf{r}_i \]  
(1.12) 

where \( \mathbf{r}_i \) is the position vector of charge \( q_i \) from the origin.

**Example 1.10**

Calculate the electric dipole moment for the following charge configurations.

**Solution**

**Case (a)** The position vector for the +q on the positive x-axis is \( a \mathbf{i} \) and position vector for the +q charge the negative x-axis is \(-a \mathbf{i}\). So the dipole moment is, 
\[ \mathbf{p} = (+q)(a \mathbf{i}) + (+q)(-a \mathbf{i}) = 0 \]

**Case (b)** In this case one charge is placed at the origin, so its position vector is zero. Hence only the second charge +q with position vector \( a \mathbf{i} \) contributes to the dipole moment, which is \( \mathbf{p} = qa \mathbf{i} \).

From both cases (a) and (b), we can infer that in general the electric dipole moment depends on the choice of the origin and charge configuration. But for one special case, the electric dipole moment is independent of the origin. If the total
charge is zero, then the electric dipole moment will be the same irrespective of the choice of the origin. It is because of this reason that the electric dipole moment of an electric dipole (total charge is zero) is always directed from -q to +q, independent of the choice of the origin.

Case (c) \( \vec{p} = (-2q)a \hat{j} + q(2a)(-\hat{j}) = -4qa \hat{j} \). Note that in this case \( \vec{p} \) is directed from -2q to +q.

Case (d) \( \vec{p} = -2qa(-\hat{i}) + qa \hat{j} + qa(-\hat{j}) = 2qa \hat{i} \).

The water molecule (H\(^2\)O) has this charge configuration. The water molecule has three atoms (two H atom and one O atom). The centers of positive (H) and negative (O) charges of a water molecule lie at different points, hence it possess permanent dipole moment. The O-H bond length is \( 0.958 \times 10^{-10} \) m due to which the electric dipole moment of water molecule has the magnitude \( p = 6.1 \times 10^{-30} \) Cm. The electric dipole moment \( \vec{p} \) is directed from center of negative charge to the center of positive charge, as shown in the figure.

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### 1.4.2 Electric field due to a dipole

**Case (i) Electric field due to an electric dipole at points on the axial line**

Consider an electric dipole placed on the x-axis as shown in Figure 1.17. A point C is located at a distance of \( r \) from the midpoint O of the dipole along the axial line.

![Figure 1.17 Electric field of the dipole along the axial line](image)

The electric field at a point C due to +q is

\[
\vec{E}_+ = \frac{1}{4\pi\varepsilon_0} \frac{q}{(r-a)^2} \hat{p}
\]

Since the electric dipole moment vector \( \hat{p} \) is from -q to +q and is directed along BC, the above equation is rewritten as

\[
\vec{E}_+ = \frac{1}{4\pi\varepsilon_0} \frac{q}{(r-a)^2} \hat{p}
\]

where \( \hat{p} \) is the electric dipole moment unit vector from -q to +q.

The electric field at a point C due to -q is

\[
\vec{E}_- = -\frac{1}{4\pi\varepsilon_0} \frac{q}{(r+a)^2} \hat{p}
\]

Since +q is located closer to the point C than -q, \( \vec{E}_+ \) is stronger than \( \vec{E}_- \). Therefore, the length of the \( \vec{E}_+ \) vector is drawn larger than that of \( \vec{E}_- \) vector.

The total electric field at point C is calculated using the superposition principle of the electric field.

\[
\vec{E}_{tot} = \vec{E}_+ + \vec{E}_-
\]

\[
= \frac{1}{4\pi\varepsilon_0} \frac{q}{(r-a)^2} \hat{p} - \frac{1}{4\pi\varepsilon_0} \frac{q}{(r+a)^2} \hat{p}
\]
If the point C is chosen on the left side of the dipole, the total electric field is still in the direction of $\vec{p}$. We infer this result by examining the electric field lines of the dipole shown in Figure 1.16(b).

### Case (ii) Electric field due to an electric dipole at a point on the equatorial plane

Consider a point C at a distance r from the midpoint O of the dipole on the equatorial plane as shown in Figure 1.19.

Since the point C is equi-distant from $+q$ and $-q$, the magnitude of the electric fields of $+q$ and $-q$ are the same. The direction of $\vec{E}_+$ is along BC and the direction of $\vec{E}_- is along CA. $\vec{E}_+$ and $\vec{E}_-$ are resolved into two components: one component parallel to the dipole axis and the other perpendicular to it. The perpendicular components $\vec{E}_+ \sin \theta$ and $\vec{E}_- \sin \theta$ are oppositely directed and cancel each other. The magnitude of the total electric field at point C is the sum of the parallel components of $\vec{E}_+$ and $\vec{E}_-$ and its direction is along $-\hat{p}$ as shown in the Figure 1.19.

$$\vec{E}_{\text{tot}} = -|\vec{E}_+| \cos \theta \hat{p} - |\vec{E}_-| \cos \theta \hat{p}$$ (1.18)
The magnitudes \( \vec{E}_+ \) and \( \vec{E}_- \) are the same and are given by

\[
|\vec{E}_+| = |\vec{E}_-| = \frac{q}{4\pi\varepsilon_0 (r^2 + a^2)}
\] (1.19)

By substituting equation (1.19) into equation (1.18), we get

\[
\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_0} \frac{2q\cos\theta}{(r^2 + a^2)^{\frac{3}{2}}} \hat{p}
\]

since \( \cos\theta = \frac{a}{\sqrt{r^2 + a^2}} \)

\[
\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_0} \frac{2q}{(r^2 + a^2)^{\frac{3}{2}}} \hat{p}
\]

since \( \hat{p} = 2q\hat{a} \) (1.20)

At very large distances \( (r>>a) \), the equation (1.20) becomes

\[
\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_0} \frac{\hat{p}}{r^2} \quad (r >> a) \] (1.21)

**Important inferences**

(i) From equations (1.17) and (1.21), it is inferred that for very large distances, the magnitude of the electric field at points on the dipole axis is twice the magnitude of the electric field at points on the equatorial plane. The direction of the electric field at points on the dipole axis is directed along the direction of dipole moment vector \( \hat{p} \) but at points on the equatorial plane it is directed opposite to the dipole moment vector, that is along \( -\hat{p} \).

(ii) At very large distances, the electric field due to a dipole varies as \( \frac{1}{r^2} \). Note that for a point charge, the electric field varies as \( \frac{1}{r^2} \). This implies that the electric field due to a dipole at very large distances goes to zero faster than the electric field due to a point charge. The reason for this behavior is that at very large distance, the two charges appear to be close to each other and neutralize each other.

(iii) The equations (1.17) and (1.21) are valid only at very large distances \( (r>>a) \). Suppose the distance \( 2a \) approaches zero and \( q \) approaches infinity such that the product of \( \frac{2a}{qa} = \hat{p} \) is finite, then the dipole is called a point dipole. For such point dipoles, equations (1.17) and (1.21) are exact and hold true for any \( r \).
that experienced by \(-q\). In addition to the torque, there will be net force acting on the dipole. This is shown in Figure 1.21.

Figure 1.21 The dipole in a non-uniform electric field

\[ \tau = qE \cdot 2a \sin \theta \]  

(1.23)

where \(\theta\) is the angle made by \(\vec{p}\) with \(\vec{E}\). Since \(p = 2aq\), the torque is written in terms of the vector product as

\[ \tau = \vec{p} \times \vec{E} \]  

(1.24)

The magnitude of this torque is \(\tau = pE \sin \theta\) and is maximum when \(\theta = 90^\circ\).

This torque tends to rotate the dipole and align it with the electric field \(\vec{E}\). Once \(\vec{p}\) is aligned with \(\vec{E}\), the total torque on the dipole becomes zero.

If the electric field is not uniform, then the force experienced by \(+q\) is different from

EXAMPLE 1.11

A sample of HCl gas is placed in a uniform electric field of magnitude \(3 \times 10^4 \ \text{N} \ \text{C}^{-1}\). The dipole moment of each HCl molecule is \(3.4 \times 10^{-30} \ \text{Cm}\). Calculate the maximum torque experienced by each HCl molecule.

Solution

The maximum torque experienced by the dipole is when it is aligned perpendicular to the applied field.

\[ \tau_{\text{max}} = pE \sin 90^\circ = 3.4 \times 10^{-30} \times 3 \times 10^4 \ \text{Nm} \]

\[ \tau_{\text{max}} = 10.2 \times 10^{-26} \ \text{Nm} \]

Microwave oven works on the principle of torque acting on an electric dipole. The food we consume has water molecules which are permanent electric dipoles. Oven produces microwaves that are oscillating electromagnetic fields and produce torque on the water molecules. Due to this torque on each water molecule, the molecules rotate very fast and produce thermal energy. Thus, heat generated is used to heat the food.

Unit 1 Electrostatics
1.5 ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

Introduction
In mechanics, potential energy is defined for conservative forces. Since gravitational force is a conservative force, its gravitational potential energy is defined in XI standard physics (Unit 6). Since Coulomb force is an inverse-square-law force, it's also a conservative force like gravitational force. Therefore, we can define potential energy for charge configurations.

1.5.1 Electrostatic Potential energy and Electrostatic potential
Consider a positive charge q kept fixed at the origin which produces an electric field \( \vec{E} \) around it. A positive test charge \( q' \) is brought from point R to point P against the repulsive force between q and \( q' \) as shown in Figure 1.22. Work must be done to overcome this repulsion. This work done is stored as potential energy.

![Figure 1.22 Work done is equal to potential energy](image)

The test charge \( q' \) is brought from R to P with constant velocity which means that external force used to bring the test charge \( q' \) from R to P must be equal and opposite to the coulomb force \( \vec{F}_{\text{ext}} = -\vec{F}_{\text{coulomb}} \). The work done is

\[
W = \int_{R}^{P} \vec{F}_{\text{ext}} \cdot d\vec{r} \quad (1.25)
\]

Since coulomb force is conservative, work done is independent of the path and it depends only on the initial and final positions of the test charge. If potential energy associated with \( q' \) at P is \( U_P \) and that at R is \( U_R \), then difference in potential energy is defined as the work done to bring a test charge \( q' \) from point R to P and is given as

\[
U_P - U_R = W = \Delta U
\]

\[
\Delta U = \int_{R}^{P} \vec{F}_{\text{ext}} \cdot d\vec{r} \quad (1.26)
\]

Since \( \vec{F}_{\text{ext}} = -\vec{F}_{\text{coulomb}} = -q'\vec{E} \) \( (1.27) \)

\[
\Delta U = \int_{R}^{P} (q'\vec{E}) \cdot d\vec{r} = q' \int_{R}^{P} (-\vec{E}) \cdot d\vec{r} \quad (1.28)
\]

The potential energy difference per unit charge is given by

\[
\frac{\Delta U}{q'} = \frac{q' \int_{R}^{P} (-\vec{E}) \cdot d\vec{r}}{q'} = -\int_{R}^{P} \vec{E} \cdot d\vec{r} \quad (1.29)
\]

The above equation (1.29) is independent of \( q' \). The quantity \( \frac{\Delta U}{q'} = -\int_{R}^{P} \vec{E} \cdot d\vec{r} \) is called electric potential difference between P and R and is denoted as \( V_P - V_R = \Delta V \).

In other words, the electric potential difference is defined as the work done by an external force to bring unit positive charge from point R to point P.

\[
V_P - V_R = \Delta V = \int_{R}^{P} (-\vec{E}) \cdot d\vec{r} \quad (1.30)
\]
The electric potential energy difference can be written as \( \Delta U = q' \Delta V \). Physically potential difference between two points is a meaningful quantity. The value of the potential itself at one point is not meaningful. Therefore the point \( R \) is taken at infinity and its potential is considered as zero \( (V_\infty = 0) \).

Then the electric potential at a point \( P \) is equal to the work done by an external force to bring a unit positive charge with constant velocity from infinity to the point \( P \) in the region of the external electric field \( \vec{E} \). Mathematically this is written as

\[
V_p = -\int_{\infty}^{p} \vec{E} \cdot d\vec{r}
\]

(1.31)

**Important points**

1. Electric potential at point \( P \) depends only on the electric field which is due to the source charge \( q \) and not on the test charge \( q' \). Unit positive charge is brought from infinity to the point \( P \) with constant velocity because external agency should not impart any kinetic energy to the test charge.

2. From equation (1.29), the unit of electric potential is Joule per coulomb. The practical unit is volt (V) named after Alessandro Volta (1745-1827) who invented the electrical battery. The potential difference between two points is expressed in terms of voltage.

### 1.5.2 Electric potential due to a point charge

Consider a positive charge \( q \) kept fixed at the origin. Let \( P \) be a point at distance \( r \) from the charge \( q \). This is shown in Figure 1.23.

The electric potential at the point \( P \) is

\[
V = \int_{\infty}^{r} (-\vec{E}) \cdot d\vec{r} = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}
\]

(1.32)

Electric field due to positive point charge \( q \) is

\[
\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}
\]

and using \( \hat{r} \cdot \hat{r} = 1 \), we have

\[
V = -\frac{1}{4\pi\varepsilon_0} \int_{\infty}^{r} \frac{q}{r^2} \hat{r} \cdot d\hat{r} = -\frac{1}{4\pi\varepsilon_0} \int_{\infty}^{r} \frac{q}{r^2} dr
\]

After the integration,

\[
V = -\frac{1}{4\pi\varepsilon_0} q \frac{1}{r}\left(1 - \frac{1}{r}\right) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}
\]

Hence the electric potential due to a point charge \( q \) at a distance \( r \) is

\[
V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}
\]

(1.33)

**Important points**

(i) If the source charge \( q \) is positive, \( V > 0 \). If \( q \) is negative, then \( V \) is negative and equal to \( V = -\frac{1}{4\pi\varepsilon_0} \frac{q}{r} \).

(ii) The description of motion of objects using the concept of potential or potential energy is simpler than that using the concept of field.

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(iii) From expression (1.33), it is clear that the potential due to positive charge decreases as the distance increases, but for a negative charge the potential increases as the distance is increased. At infinity \((r = \infty)\) electrostatic potential is zero \((V = 0)\).

In the case of gravitational force, mass moves from a point of higher gravitational potential to a point of lower gravitational potential. Similarly a positive charge moves from a point of higher electrostatic potential to lower electrostatic potential. However a negative charge moves from lower electrostatic potential to higher electrostatic potential. This comparison is shown in Figure 1.24.

(iv) The electric potential at a point \(P\) due to a collection of charges \(q_1, q_2, q_3, \ldots, q_n\) is equal to sum of the electric potentials due to individual charges.

\[
V_{\text{tot}} = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3} + \ldots + \frac{kq_n}{r_n} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i} \tag{1.34}
\]

where \(r_1, r_2, r_3, \ldots, r_n\) are the distances of \(q_1, q_2, q_3, \ldots, q_n\) respectively from \(P\) (Figure 1.25).

**EXAMPLE 1.12**

(a) Calculate the electric potential at points \(P\) and \(Q\) as shown in the figure below.

(b) Suppose the charge \(+9 \mu C\) is replaced by \(-9 \mu C\) find the electrostatic potentials at points \(P\) and \(Q\)

(c) Calculate the work done to bring a test charge \(+2 \mu C\) from infinity to the point \(P\). Assume the charge \(+9 \mu C\) is held fixed at origin and \(+2 \mu C\) is brought from infinity to \(P\).
EXAMPLE 1.13

Consider a point charge +q placed at the origin and another point charge -2q placed at a distance of 9 m from the charge +q. Determine the point between the two charges at which electric potential is zero.

Solution

According to the superposition principle, the total electric potential at a point is equal to the sum of the potentials due to each charge at that point.

Consider the point at which the total potential zero is located at a distance x from the charge +q as shown in the figure.

The total electric potential at P is zero.

\[ V_{\text{tot}} = \frac{1}{4\pi \varepsilon_0} \left( \frac{q}{x} - \frac{2q}{9 - x} \right) = 0 \]

which gives

\[ \frac{q}{x} = \frac{2q}{9 - x} \]

or

\[ \frac{1}{x} = \frac{2}{9 - x} \]

Hence, \( x = 3 \text{ m} \)

1.5.3 Electrostatic potential at a point due to an electric dipole

Consider two equal and opposite charges separated by a small distance 2a as shown in Figure 1.26. The point P is located at a distance r from the midpoint of the dipole. Let \( \theta \) be the angle between the line OP and dipole axis AB.

**Solution**

(a) Electric potential at point P is given by

\[ V_p = \frac{1}{4\pi \varepsilon_0} \frac{q}{r} = \frac{9 \times 10^6 \times 9 \times 10^{-6}}{10} = 8.1 \times 10^3 \text{ V} \]

Electric potential at point Q is given by

\[ V_q = \frac{1}{4\pi \varepsilon_0} \frac{q}{r_q} = \frac{9 \times 10^6 \times 9 \times 10^{-6}}{16} = 5.06 \times 10^3 \text{ V} \]

Note that the electric potential at point Q is less than the electric potential at point P. If we put a positive charge at P, it moves from P to Q. However if we place a negative charge at P it will move towards the charge +9µC.

The potential difference between the points P and Q is given by

\[ \Delta V = V_p - V_q = 3.04 \times 10^3 \text{ V} \]

(b) Suppose we replace the charge +9µC by -9µC, then the corresponding potentials at the points P and Q are,

\[ V_p = -8.1 \times 10^3 \text{ V}, \quad V_q = -5.06 \times 10^3 \text{ V} \]

Note that in this case electric potential at the point Q is higher than at point P.

The potential difference or voltage between the points P and Q is given by

\[ \Delta V = V_p - V_q = -3.04 \times 10^3 \text{ V} \]

(c) The electric potential V at a point P due to some charge is defined as the work done by an external force to bring a unit positive charge from infinity to P. So to bring the q amount of charge from infinity to the point P, work done is given as follows.

\[ W = qV \]

\[ W_q = 2 \times 10^{-6} \times 5.06 \times 10^3 = 10.12 \times 10^{-3} \text{ J} \]
Let \( r_1 \) be the distance of point P from +q and \( r_2 \) be the distance of point P from -q.

Potential at P due to charge +q
\[
V = \frac{1}{4\pi\varepsilon} q \left( \frac{1}{r_1} - \frac{1}{r_2} \right)
\]

(1.35)

Suppose if the point P is far away from the dipole, such that \( r >> a \), then equation (1.35) can be expressed in terms of r.

By the cosine law for triangle BOP,
\[
r_1^2 = r^2 + a^2 - 2ra \cos \theta
\]
\[
r_1^2 = r^2 \left[ 1 + \frac{a^2}{r^2} - \frac{2a}{r} \cos \theta \right]
\]

Since the point P is very far from dipole, then \( r >> a \). As a result the term \( \frac{a^2}{r^2} \) is very small and can be neglected. Therefore
\[
r_1^2 = r^2 \left( 1 - \frac{2a}{r} \cos \theta \right)
\]

(1.36)

Similarly applying the cosine law for triangle AOP,
\[
r_2^2 = r^2 + a^2 - 2ra \cos (180 - \theta)
\]
\[
since \cos (180 - \theta) = -\cos \theta \quad we \ get\]
\[
r_2^2 = r^2 + a^2 + 2ra \cos \theta
\]

Neglecting the term \( \frac{a^2}{r^2} \) (because \( r >> a \))
\[
r_2^2 = r^2 \left( 1 + \frac{2a \cos \theta}{r} \right)
\]
\[
r_2 = r \left( 1 + \frac{2a \cos \theta}{r} \right)^\frac{1}{2}
\]

Using Binomial theorem, we get
\[
\frac{1}{r} = \frac{1}{r} \left( 1 - \frac{2a}{r} \cos \theta \right)^\frac{1}{2}
\]

(1.37)

Substituting equation (1.37) and (1.36) in equation (1.35),
\[
V = \frac{1}{4\pi\varepsilon} q \left( \frac{1}{r} \left( \frac{1 + a \cos \theta}{r} \right) - \frac{1}{r} \left( \frac{1 - a \cos \theta}{r} \right) \right)
\]
\[
V = \frac{q}{4\pi\varepsilon} \left( \frac{1}{r} \left( \frac{1 + a \cos \theta}{r} \right) - 1 + \frac{a \cos \theta}{r} \right)
\]
\[
V = \frac{1}{4\pi\varepsilon} \frac{2aq}{r^2} \cos \theta
\]
But the electric dipole moment \( p = 2qa \) and we get,

\[
V = \frac{1}{4\pi\epsilon_0} \left( \frac{p\cos\theta}{r^2} \right)
\]

Now we can write \( p\cos\theta = \vec{p} \cdot \hat{r} \), where \( \hat{r} \) is the unit vector from the point O to point P. Hence the electric potential at a point P due to an electric dipole is given by

\[
V = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2} \quad (r>>a) \quad (1.38)
\]

Equation (1.38) is valid for distances very large compared to the size of the dipole. But for a point dipole, the equation (1.38) is valid for any distance.

**Special cases**

**Case (i)** If the point P lies on the axial line of the dipole on the side of +q, then \( \theta = 0 \). Then the electric potential becomes

\[
V = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2} \quad (1.39)
\]

**Case (ii)** If the point P lies on the axial line of the dipole on the side of –q, then \( \theta = 180^\circ \), then

\[
V = -\frac{1}{4\pi\epsilon_0} \frac{p}{r^2} \quad (1.40)
\]

**Case (iii)** If the point P lies on the equatorial line of the dipole, then \( \theta = 90^\circ \). Hence

\[
V = 0 \quad (1.41)
\]

**Important points**

(i) The potential due to an electric dipole falls as \( \frac{1}{r^2} \) and the potential due to a single point charge falls as \( \frac{1}{r} \). Thus the potential due to the dipole falls faster than that due to a monopole (point charge). As the distance increases from electric dipole, the effects of positive and negative charges nullify each other.

(ii) The potential due to a point charge is spherically symmetric since it depends only on the distance \( r \). But the potential due to a dipole is not spherically symmetric because the potential depends on the angle between \( \vec{p} \) and position vector \( \hat{r} \) of the point. However the dipole potential is axially symmetric. If the position vector \( \vec{r} \) is rotated about \( \vec{p} \) by keeping \( \theta \) fixed, then all points on the cone at the same distance \( r \) will have the same potential as shown in Figure 1.27. In this figure, all the points located on the blue curve will have the same potential.

**Figure 1.27** Dipole potential is axially symmetric

**1.5.4 Equi-potential Surface**

Consider a point charge \( q \) located at some point in space and an imaginary sphere of radius \( r \) is chosen by keeping the charge \( q \) at its center (Figure 1.28(a)). The electric potential at all points on the surface of the given sphere is the same. Such a surface is called an equipotential surface.

An equipotential surface is a surface on which all the points are at the same potential. For a point charge the equipotential surfaces are concentric spherical surfaces as shown in Figure 1.28(b). Each spherical surface is an equipotential surface but the value of the
potential is different for different spherical surfaces.

For a uniform electric field, the equipotential surfaces form a set of planes normal to the electric field $\vec{E}$. This is shown in the Figure 1.29.

**Properties of equipotential surfaces**

(i) The work done to move a charge $q$ between any two points A and B, $W = q (V_B - V_A)$. If the points A and B lie on the same equipotential surface, work done is zero because $V_A = V_B$.

(ii) The electric field is normal to an equipotential surface. If it is not normal, then there is a component of the field parallel to the surface. Then work must be done to move a charge between two points on the same surface. This is a contradiction. Therefore the electric field must always be normal to equipotential surface.

### 1.5.5 Relation between electric field and potential

Consider a positive charge $q$ kept fixed at the origin. To move a unit positive charge by a small distance $dx$ in the electric field $\vec{E}$,
the work done is given by $dW = -E \, dx$. The minus sign implies that work is done against the electric field. This work done is equal to electric potential difference. Therefore,

$$dW = dV.$$  \hspace{1cm} \text{(or)} \hspace{1cm} dV = -E \, dx \hspace{1cm} \text{(1.42)}$$

Hence $E = -\frac{dV}{dx}$ \hspace{1cm} \text{(1.43)}$

The electric field is the negative gradient of the electric potential. In general,

$$E = -\left( \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right) \hspace{1cm} \text{(1.44)}$$

**EXAMPLE 1.14**

The following figure represents the electric potential as a function of the $x$-coordinate. Plot the corresponding electric field as a function of $x$.

![Electric Potential vs. x](image)

**Solution**

In the given problem, since the potential depends only on $x$, we can use $E = -\frac{dV}{dx}$ (the other two terms $\frac{\partial V}{\partial y}$ and $\frac{\partial V}{\partial z}$ are zero)

From 0 to 1 cm, the slope is constant and so $\frac{dV}{dx} = 25 \, V \, cm^{-1}$. So $E = -25 \, V \, cm^{-1} \hat{i}$

From 1 to 4 cm, the potential is constant, $V = 25 \, V$. It implies that $\frac{dV}{dx} = 0$. So $E = 0$

From 4 to 5 cm, the slope $\frac{dV}{dx} = -25 \, V \, cm^{-1}$. So $E = +25 \, V \, cm^{-1} \hat{i}$

The plot of electric field for the various points along the $x$ axis is given below.

![Electric Field](image)

### 1.5.6 Electrostatic potential energy for collection of point charges

The electric potential at a point at a distance $r$ from point charge $q_i$ is given by

$$V = \frac{1}{4\pi \varepsilon_0} \frac{q_i}{r}$$

This potential $V$ is the work done to bring a unit positive charge from infinity to the point. Now if the charge $q_2$ is brought from infinity to that point at a distance $r$ from $q_1$, the work done is the product of $q_2$ and the electric potential at that point. Thus we have

$$W = q_2 V$$
This work done is stored as the electrostatic potential energy $U$ of a system of charges $q_1$ and $q_2$ separated by a distance $r$. Thus we have

$$U = q_1V = \frac{1}{4\pi\varepsilon_0} \frac{q_1q_2}{r} \quad (1.45)$$

The electrostatic potential energy depends only on the distance between the two point charges. In fact, the expression (1.45) is derived by assuming that $q_1$ is fixed and $q_2$ is brought from infinity. The equation (1.45) holds true when $q_1$ is fixed and $q_2$ is brought from infinity or both $q_1$ and $q_2$ are simultaneously brought from infinity to a distance $r$ between them.

Three charges are arranged in the following configuration as shown in Figure 1.30.

![Figure 1.30](image)

**Figure 1.30** Electrostatic potential energy for Collection of point charges

To calculate the total electrostatic potential energy, we use the following procedure. We bring all the charges one by one and arrange them according to the configuration as shown in Figure 1.30.

(i) Bringing a charge $q_1$ from infinity to the point A requires no work, because there are no other charges already present in the vicinity of charge $q_1$.

(ii) To bring the second charge $q_2$ to the point B, work must be done against the electric field created by the charge $q_1$.

So the work done on the charge $q_2$ is $W = q_2V_{1B}$. Here $V_{1B}$ is the electrostatic potential due to the charge $q_1$ at point $B$.

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1q_2}{r_{12}} \quad (1.46)$$

Note that the expression is same when $q_2$ is brought first and then $q_1$ later.

(iii) Similarly to bring the charge $q_3$ to the point C, work has to be done against the total electric field due to both charges $q_1$ and $q_2$. So the work done to bring the charge $q_3$ is $W = q_3(V_{1C} + V_{2C})$. Here $V_{1C}$ is the electrostatic potential due to charge $q_1$ at point C and $V_{2C}$ is the electrostatic potential due to charge $q_2$ at point C.

The electrostatic potential is

$$U = \frac{1}{4\pi\varepsilon_0} \left( \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}} \right) \quad (1.47)$$

(iv) Adding equations (1.46) and (1.47), the total electrostatic potential energy for the system of three charges $q_1$, $q_2$, and $q_3$ is

$$U = \frac{1}{4\pi\varepsilon_0} \left( \frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}} \right) \quad (1.48)$$

Note that this stored potential energy $U$ is equal to the total external work done to assemble the three charges at the given locations. The expression (1.48) is same if the charges are brought to their positions in any other order. Since the Coulomb force is a conservative force, the electrostatic potential energy is independent of the manner in which the configuration of charges is arrived at.
EXAMPLE 1.15

Four charges are arranged at the corners of the square PQRS of side \( a \) as shown in the figure. (a) Find the work required to assemble these charges in the given configuration. (b) Suppose a charge \( q' \) is brought to the center of the square, by keeping the four charges fixed at the corners, how much extra work is required for this?

Solution

(a) The work done to arrange the charges in the corners of the square is independent of the way they are arranged. We can follow any order.

(i) First, the charge +q is brought to the corner P. This requires no work since no charge is already present, \( W_P = 0 \)

(ii) Work required to bring the charge \(-q\) to the corner Q = \( (-q) \times \text{potential at a point Q due to +q located at a point P} \).

\[
W_Q = -q \times \frac{1}{4\pi\varepsilon_0} \left( -\frac{q}{a} \right) = -\frac{1}{4\pi\varepsilon_0} \frac{q^2}{a}
\]

(iii) Work required to bring the charge +q to the corner R = \( q \times \text{potential at the point R due to charges at the point P and Q} \).

\[
W_R = q \times \frac{1}{4\pi\varepsilon_0} \left( -\frac{q}{a} + \frac{q}{\sqrt{2}a} \right) = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{a} \left( 1 + \frac{1}{\sqrt{2}} \right)
\]

(iv) Work required to bring the fourth charge \(-q\) at the position \( S = q \times \text{potential at the point S due to all the three charges at the point P, Q and R} \).

\[
W_S = -q \times \frac{1}{4\pi\varepsilon_0} \left( \frac{q}{a} + \frac{q}{a} - \frac{q}{\sqrt{2}a} \right) = -\frac{1}{4\pi\varepsilon_0} \frac{q^2}{a} \left( 2 - \frac{1}{\sqrt{2}} \right)
\]

(b) Work required to bring the charge \( q' \) to the center of the square = \( q' \times \text{potential at the center point O due to all the four charges in the four corners} \).

The potential created by the two \(+q\) charges are canceled by the potential created by the \(-q\) charges which are located in the opposite corners. Therefore the net electric potential at the center O due to all the charges in the corners is zero.

Hence no work is required to bring any charge to the point O. Physically this implies that if any charge \( q' \) when brought close to O, then it moves to the point O without any external force.

1.5.7 Electrostatic potential energy of a dipole in a uniform electric field

Consider a dipole placed in the uniform electric field \( \vec{E} \) as shown in the Figure 1.31. A dipole experiences a torque when kept in an uniform electric field \( \vec{E} \). This torque rotates the dipole to align it with the...
direction of the electric field. To rotate the dipole (at constant angular velocity) from its initial angle \( \theta \) to another angle \( \theta' \) against the torque exerted by the electric field, an equal and opposite external torque must be applied on the dipole.

The work done by the external torque to rotate the dipole from angle \( \theta \) to \( \theta' \) at constant angular velocity is

\[
W_{\text{ext}} = \int_{\theta}^{\theta'} \tau_{\text{E}} \, d\theta = \frac{1}{2} \frac{\tau_{\text{E}}}{\mu_{\text{p}}} (\cos \theta' - \cos \theta)
\]  

Since \( \tau_{\text{E}} = \mu_{\text{p}} E \), we have

\[
\tau_{\text{E}} = \mu_{\text{p}} E
\]

Substituting equation (1.50) in equation (1.49), we get

\[
W_{\text{p}} = \int_{\theta}^{\theta'} \tau_{\text{E}} \, d\theta
\]

where \( W_{\text{p}} \) is the potential energy difference between the angular positions \( \theta \) and \( \theta' \).

This work done is equal to the potential energy difference between the angular positions \( \theta \) and \( \theta' \).

If the initial angle is \( \theta' = 90^\circ \) and is taken as reference point, then \( U(\theta') = \mu_{\text{p}} E \cos 90^\circ = 0 \).

Thus, the potential energy is

\[
U(\theta) = -\mu_{\text{p}} E \cos \theta
\]

For one water molecule

\[
W = -\mu_{\text{p}} E \cos \theta = -\mu_{\text{p}} E \cos 90^\circ
\]

The work done to rotate one water molecule from \( \theta = 0^\circ \) to \( \theta' = 90^\circ \) is equal to the potential energy difference between these two configurations.

From the equation (1.51), we write

\[
U = U(\theta) - U(\theta') = \mu_{\text{p}} E (\cos \theta - \cos 90^\circ)
\]

When the water molecules are aligned in the direction of the electric field, it has minimum potential energy. The work done to rotate the dipole from \( \theta = 0^\circ \) to \( 90^\circ \) is equal to the potential energy difference between these two configurations.

EXAMPE 1.16

A water molecule has an electric dipole moment of \( 6.3 \times 10^{-30} \) Cm. A sample contains \( 10^{22} \) water molecules, with all the dipole moments aligned parallel to the external electric field of magnitude \( 3 \times 10^8 \) N C\(^{-1}\). How much work is required to rotate all the water molecules from \( \theta = 0^\circ \) to \( 90^\circ \)?

Solution

In addition to \( \mu_{\text{p}} \) and \( E \), the potential energy also depends on the orientation \( \theta \) of the dipole with respect to the external electric field.

The potential energy stored in the system of dipole kept in the uniform electric field is

\[
U = -\mu_{\text{p}} E \cos \theta
\]

Therefore, the potential energy difference between these two configurations is

\[
\Delta U = U(\theta) - U(\theta') = \mu_{\text{p}} E (\cos \theta - \cos 90^\circ)
\]

The work done to rotate the dipole from \( \theta = 0^\circ \) to \( 90^\circ \) is equal to the potential energy difference between these two configurations.

\[
W = \int_{\theta}^{\theta'} \tau_{\text{E}} \, d\theta
\]

The potential energy difference between these two configurations is

\[
\Delta U = \mu_{\text{p}} E (\cos \theta - \cos 90^\circ)
\]

The work done to rotate the dipole from \( \theta = 0^\circ \) to \( 90^\circ \) is equal to the potential energy difference between these two configurations.

\[
W = \int_{\theta}^{\theta'} \tau_{\text{E}} \, d\theta
\]

The work done to rotate the dipole from \( \theta = 0^\circ \) to \( 90^\circ \) is equal to the potential energy difference between these two configurations.

\[
W = \int_{\theta}^{\theta'} \tau_{\text{E}} \, d\theta
\]

The work done to rotate the dipole from \( \theta = 0^\circ \) to \( 90^\circ \) is equal to the potential energy difference between these two configurations.

\[
W = \int_{\theta}^{\theta'} \tau_{\text{E}} \, d\theta
\]
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these elements have the same area, the number of electric field lines crossing the element in region A is more than that crossing the element in region B. Therefore the electric flux in region A is more than that in region B. The electric field strength for a point charge decreases as the distance increases, then for a point charge electric flux also decreases as the distance increases. The above discussion gives a qualitative idea of electric flux. However a precise definition of electric flux is needed.

Electric flux for uniform Electric field

Consider a uniform electric field in a region of space. Let us choose an area A normal to the electric field lines as shown in Figure 1.33 (a). The electric flux for this case is

\[ \Phi_E = EA \]  

(1.52)

Suppose the same area A is kept parallel to the uniform electric field, then no electric field lines pierce through the area A, as shown in Figure 1.33(b). The electric flux for this case is zero.

\[ \Phi_E = 0 \]  

(1.53)

If the area is inclined at an angle \( \theta \) with the field, then the component of the electric field perpendicular to the area alone contributes to the electric flux. The electric field component parallel to the surface area will not contribute to the electric flux. This is shown in Figure 1.33 (c). For this case, the electric flux

\[ \Phi_E = (E \cos \theta) A \]  

(1.54)

Further, \( \theta \) is also the angle between the electric field and the direction normal to the area. Hence in general, for uniform electric field, the electric flux is defined as
Solution

The electric flux

\[ \Phi = \vec{E} \cdot \vec{A} = EA \cos \theta \] (1.55)

Here, note that \( \vec{A} \) is the area vector \( \vec{A} = A \hat{n} \). Its magnitude is simply the area \( A \) and the direction is along the unit vector \( \hat{n} \) perpendicular to the area as shown in Figure 1.33. Using this definition for flux, \( \Phi = \vec{E} \cdot \vec{A} \), equations (1.53) and (1.54) can be obtained as special cases.

In Figure 1.33 (a), \( \theta = 0^\circ \) so \( \Phi = \vec{E} \cdot \vec{A} = EA \)

In Figure 1.33 (b), \( \theta = 90^\circ \) so \( \Phi = \vec{E} \cdot \vec{A} = 0 \)

**Example 1.17**

Calculate the electric flux through the rectangle of sides 5 cm and 10 cm kept in the region of a uniform electric field 100 NC\(^{-1}\). The angle \( \theta \) is 60\(^\circ\). Suppose \( \theta \) becomes zero, what is the electric flux?

**Electric flux in a non uniform electric field and an arbitrarily shaped area**

Suppose the electric field is not uniform and the area \( A \) is not flat (Figure 1.34), then the entire area is divided into \( n \) small area segments \( \Delta \vec{A}_1, \Delta \vec{A}_2, \Delta \vec{A}_3, \ldots, \Delta \vec{A}_n \) such that each area element is almost flat and the electric field over each area element is considered to be uniform.

The electric flux for the entire area \( A \) is approximately written as

\[ \Phi = \sum_{i=1}^{n} \vec{E}_i \cdot \Delta \vec{A}_i \] (1.56)
By taking the limit $\Delta \vec{A}_i \to 0$ (for all $i$) the summation in equation (1.56) becomes integration. The total electric flux for the entire area is given by

$$\Phi_e = \int \vec{E} \cdot d\vec{A}$$  \hspace{1cm} (1.57)

From Equation (1.57), it is clear that the electric flux for a given surface depends on both the electric field pattern on the surface area and orientation of the surface with respect to the electric field.

### 1.6.2 Electric flux for closed surfaces

In the previous section, the electric flux for any arbitrary curved surface is discussed. Suppose a closed surface is present in the region of the non-uniform electric field as shown in Figure 1.35 (a).

The total electric flux over this closed surface is written as

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$  \hspace{1cm} (1.58)

Note the difference between equations (1.57) and (1.58). The integration in equation (1.58) is a closed surface integration and for each areal element, the outward normal is the direction of $d\vec{A}$ as shown in the Figure 1.35(b).

The total electric flux over a closed surface can be negative, positive or zero. In the Figure 1.35(b), it is shown that in one area element, the angle between $d\vec{A}$ and $\vec{E}$ is less than 90°, then the electric flux is positive and in another areal element, the angle between $d\vec{A}$ and $\vec{E}$ is greater than 90°, then the electric flux is negative.

In general, the electric flux is negative if the electric field lines enter the closed surface and positive if the electric field lines leave the closed surface.

### 1.6.3 Gauss law

A positive point charge $Q$ is surrounded by an imaginary sphere of radius $r$ as shown in Figure 1.36. We can calculate the total...
The electric flux through the closed surface of the sphere using the equation (1.58).

\[ \Phi_E = \oint E \cdot dA = \oint E \, dA \cos \theta \]

The electric field of the point charge is directed radially outward at all points on the surface of the sphere. Therefore, the direction of the area element \( dA \) is along the electric field \( \vec{E} \) and \( \theta = 0^\circ \).

The remarkable point about this result is that the equation (1.61) is equally true for any arbitrary shaped surface which encloses the charge \( Q \) and as shown in the Figure 1.37. It is seen that the total electric flux is the same for closed surfaces \( A_1, A_2 \) and \( A_3 \) as shown in the Figure 1.37.

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = Q_{\text{encl}} \]

Here \( Q_{\text{encl}} \) denotes the charges inside the closed surface.

**Discussion of Gauss law**

(i) The total electric flux through the closed surface depends only on the
charges enclosed by the surface and the charges present outside the surface will not contribute to the flux and the shape of the closed surface which can be chosen arbitrarily.

(ii) The total electric flux is independent of the location of the charges inside the closed surface.

(iii) To arrive at equation (1.62), we have chosen a spherical surface. This imaginary surface is called a Gaussian surface. The shape of the Gaussian surface to be chosen depends on the type of charge configuration and the kind of symmetry existing in that charge configuration. The electric field is spherically symmetric for a point charge, therefore spherical Gaussian surface is chosen. Cylindrical and planar Gaussian surfaces can be chosen for other kinds of charge configurations.

(iv) In the LHS of equation (1.62), the electric field \( \vec{E} \) is due to charges present inside and outside the Gaussian surface but the charge \( Q_{\text{enc}} \) denotes the charges which lie only inside the Gaussian surface.

(v) The Gaussian surface cannot pass through any discrete charge but it can pass through continuous charge distributions. It is because, very close to the discrete charges, the electric field is not well defined.

(vi) Gauss law is another form of Coulomb’s law and it is also applicable to the charges in motion. Because of this reason, Gauss law is treated as much more general law than Coulomb’s law.

**Example 1.18**

(i) In figure (a), calculate the electric flux through the closed areas \( A_1 \) and \( A_2 \).

(ii) In figure (b), calculate the electric flux through the cube.

**Solution**

(i) In figure (a), the area \( A_1 \) encloses the charge \( Q \). So electric flux through this closed surface \( A_1 \) is \( \frac{Q}{\varepsilon_0} \). But the closed surface \( A_2 \) contains no charges inside, so electric flux through \( A_2 \) is zero.

(ii) In figure (b), the net charge inside the cube is 3q and the total electric flux in the cube is therefore \( \Phi = \frac{3q}{\varepsilon_0} \).

Note that the charge -10 q lies outside the cube and it will not contribute the total flux through the surface of the cube.
1.6.4 Applications of Gauss law

Electric field due to any arbitrary charge configuration can be calculated using Coulomb’s law or Gauss law. If the charge configuration possesses some kind of symmetry, then Gauss law is a very efficient way to calculate the electric field. It is illustrated in the following cases.

(i) Electric field due to an infinitely long charged wire

Consider an infinitely long straight wire having uniform linear charge density \( \lambda \). Let \( P \) be a point located at a perpendicular distance \( r \) from the wire (Figure 1.38(a)). The electric field at the point \( P \) can be found using Gauss law. We choose two small charge elements \( A_1 \) and \( A_2 \) on the wire which are at equal distances from the point \( P \). The resultant electric field due to these two charge elements points radially away from the charged wire and the magnitude of electric field is same at all points on the circle of radius \( r \). This is shown in the Figure 1.38(b). From this property, we can infer that the charged wire possesses a cylindrical symmetry.

Let us choose a cylindrical Gaussian surface of radius \( r \) and length \( L \) as shown in the Figure 1.39.

The total electric flux in this closed surface is calculated as follows.

\[
\Phi_E = \oint \vec{E} \cdot d\vec{A} = \int_{\text{Curved surface}} \vec{E} \cdot d\vec{A} + \int_{\text{top surface}} \vec{E} \cdot d\vec{A} + \int_{\text{bottom surface}} \vec{E} \cdot d\vec{A} \quad (1.63)
\]

It is seen from Figure (1.39) that for the curved surface, \( \vec{E} \) is parallel to \( \vec{A} \) and \( \vec{E} \cdot d\vec{A} = E dA \). For the top and bottom surfaces, \( \vec{E} \) is perpendicular to \( \vec{A} \) and \( \vec{E} \cdot d\vec{A} = 0 \)

Substituting these values in the equation (1.63) and applying Gauss law to the cylindrical surface, we have

\[
\Phi_E = \int_{\text{Curved surface}} E dA = \frac{Q_{\text{enc}}}{\varepsilon_0} \quad (1.64)
\]

Figure 1.38 Electric field due to infinite long charged wire
Since the magnitude of the electric field for the entire curved surface is constant, \( E \) is taken out of the integration and \( Q_{\text{encl}} \) is given by
\[
Q_{\text{encl}} = \lambda L.
\]
Equation (1.65) indicates that the electric field is always along the perpendicular direction \( \hat{r} \) to wire. In fact, if \( \lambda > 0 \) then \( \vec{E} \) points perpendicular outward \( \hat{r} \) from the wire and if \( \lambda < 0 \), then \( \vec{E} \) points perpendicular inward \( -\hat{r} \).

The equation (1.67) is true only for an infinitely long charged wire. For a charged wire of finite length, the electric field need not be radial at all points. However, equation (1.67) for such a wire is taken approximately true around the mid-point of the wire and far away from the both ends of the wire.

(ii) Electric field due to charged infinite plane sheet

Consider an infinite plane sheet of charges with uniform surface charge density \( \sigma \). Let \( P \) be a point at a distance of \( r \) from the sheet as shown in the Figure 1.40.

Since the plane is infinitely large, the electric field should be same at all points equidistant from the plane and radially directed at all points. A cylindrical shaped Gaussian surface of length \( 2r \) and area \( A \) of the flat surfaces is chosen such that the infinite plane sheet passes perpendicularly through the middle part of the Gaussian surface.

Applying Gauss law for this cylindrical surface,
In vector form, \( \vec{E} = \frac{\sigma}{2\varepsilon_0} \hat{n} \) (1.71)

Here \( \hat{n} \) is the outward unit vector normal to the plane. Note that the electric field due to an infinite plane sheet of charge depends on the surface charge density and is independent of the distance \( r \).

The electric field will be the same at any point farther away from the charged plane. Equation (1.71) implies that if \( \sigma > 0 \) the electric field at any point \( P \) is outward perpendicular \( \hat{n} \) to the plane and if \( \sigma < 0 \) the electric field points inward perpendicularly \( (\hat{n}) \) to the plane.

For a finite charged plane sheet, equation (1.71) is approximately true only in the middle region of the plane and at points far away from both ends.

(iii) Electric field due to two parallel charged infinite sheets

Consider two infinitely large charged plane sheets with equal and opposite charge densities \( +\sigma \) and \( -\sigma \) which are placed parallel to each other as shown in the Figure 1.41.

![Figure 1.41 Electric field due to two parallel charged sheets](image)
The electric field between the plates and outside the plates is found using Gauss law. The magnitude of the electric field due to an infinite charged plane sheet is \( \frac{\sigma}{2\varepsilon_0} \) and it points perpendicularly outward if \( \sigma > 0 \) and points inward if \( \sigma < 0 \).

At the points \( P_2 \) and \( P_3 \), the electric field due to both plates are equal in magnitude and opposite in direction (Figure 1.41). As a result, electric field at a point outside the plates is zero. But inside the plate, electric fields are in same direction i.e., towards the right, the total electric field at a point \( P_1 \)

\[
E_{\text{inside}} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} \quad (1.72)
\]

The direction of the electric field inside the plates is directed from positively charged plate to negatively charged plate and is uniform everywhere inside the plate.

(iv) Electric field due to a uniformly charged spherical shell

Consider a uniformly charged spherical shell of radius \( R \) and total charge \( Q \) as shown in Figure 1.42. The electric field at points outside and inside the sphere is found using Gauss law.

Case (a) At a point outside the shell \( (r > R) \)

Let us choose a point \( P \) outside the shell at a distance \( r \) from the center as shown in Figure 1.42 (a). The charge is uniformly distributed on the surface of the sphere (spherical symmetry). Hence the electric field must point radially outward if \( Q > 0 \) and points radially inward if \( Q < 0 \). So we choose a spherical Gaussian surface of radius \( r \) is chosen and the total charge enclosed by this Gaussian surface is \( Q \). Applying Gauss law

\[
\int \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0} \quad (1.73)
\]

Figure 1.42 The electric field due to a charged spherical shell
The electric field $\vec{E}$ and $d\vec{A}$ point in the same direction (outward normal) at all the points on the Gaussian surface. The magnitude of $\vec{E}$ is also the same at all points due to the spherical symmetry of the charge distribution.

Hence $E \oint_{\text{Gaussian surface}} dA = \frac{Q}{\varepsilon_0}$ \hspace{1cm} (1.74)

But $\oint_{\text{Gaussian surface}} dA = \text{total area of Gaussian surface} = 4\pi r^2$. Substituting this value in equation (1.74)

$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$ (or) $E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2}$

In vector form $\vec{E} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \hat{r}$ \hspace{1cm} (1.75)

The electric field is radially outward if $Q > 0$ and radially inward if $Q < 0$. From equation (1.75), we infer that the electric field at a point outside the shell will be same as if the entire charge $Q$ is concentrated at the center of the spherical shell. (A similar result is observed in gravitation, for gravitational force due to a spherical shell with mass $M$)

Case (b): At a point on the surface of the spherical shell ($r = R$)

The electrical field at points on the spherical shell ($r = R$) is given by

$\vec{E} = \frac{Q}{4\pi \varepsilon_0 R^2} \hat{r}$ \hspace{1cm} (1.76)

Case (c) At a point inside the spherical shell ($r < R$)

Consider a point $P$ inside the shell at a distance $r$ from the center. A Gaussian sphere of radius $r$ is constructed as shown in the Figure 1.42 (b). Applying Gauss law

Since Gaussian surface encloses no charge, So $Q = 0$. The equation (1.77) becomes

$E = 0 \hspace{1cm} (r < R)$ \hspace{1cm} (1.78)

The electric field due to the uniformly charged spherical shell is zero at all points inside the shell.

A graph is plotted between the electric field and radial distance. This is shown in Figure 1.43.

**Figure 1.43** Electric field versus distance for a spherical shell of radius $R$

**Note** Gauss law is a powerful technique whenever a given charge configuration possesses spherical, cylindrical or planar symmetry, then the electric field due to such a charge configuration can be easily found. If there is no such symmetry, the direct method (Coulomb's law and calculus) can be used. For example, it is difficult to use Gauss law to find the electric field for a dipole since it has no spherical, cylindrical or planar symmetry.
1.7
ELECTROSTATICS OF CONDUCTORS AND DIELECTRICS

1.7.1 Conductors at electrostatic equilibrium

An electrical conductor has a large number of mobile charges which are free to move in the material. In a metallic conductor, these mobile charges are free electrons which are not bound to any atom and therefore are free to move on the surface of the conductor. When there is no external electric field, the free electrons are in continuous random motion in all directions. As a result, there is no net motion of electrons along any particular direction which implies that the conductor is in electrostatic equilibrium. Thus at electrostatic equilibrium, there is no net current in the conductor. A conductor at electrostatic equilibrium has the following properties.

(i) The electric field is zero everywhere inside the conductor. This is true regardless of whether the conductor is solid or hollow.

This is an experimental fact. Suppose the electric field is not zero inside the metal, then there will be a force on the mobile charge carriers due to this electric field. As a result, there will be a net motion of the mobile charges, which contradicts the conductors being in electrostatic equilibrium. Thus the electric field is zero everywhere inside the conductor. We can also understand this fact by applying an external uniform electric field on the conductor. This is shown in Figure 1.44.

Before applying the external electric field, the free electrons in the conductor are uniformly distributed in the conductor. When an electric field is applied, the free electrons accelerate to the left causing the left plate to be negatively charged and the right plate to be positively charged as shown in Figure 1.44.

Due to this realignment of free electrons, there will be an internal electric field created inside the conductor which increases until it nullifies the external electric field. Once the external electric field is nullified the conductor is said to be in electrostatic equilibrium. The time taken by a conductor to reach electrostatic equilibrium is in the order of $10^{-16}$s, which can be taken as almost instantaneous.

(ii) There is no net charge inside the conductors. The charges must reside only on the surface of the conductors.

We can prove this property using Gauss law. Consider an arbitrarily shaped conductor as shown in Figure 1.45. A Gaussian surface is drawn inside the conductor such that it is very close to the surface of the conductor. Since the electric field is zero everywhere inside
We now prove that the electric field has magnitude \( \frac{\sigma}{\varepsilon_0} \) just outside the conductor's surface. Consider a small cylindrical Gaussian surface, as shown in the Figure 1.47. One half of this cylinder is embedded inside the conductor.

(iii) **The electric field outside the conductor is perpendicular to the surface of the conductor and has a magnitude of** \( \frac{\sigma}{\varepsilon_0} \). where \( \sigma \) is the surface charge density at that point.

If the electric field has components parallel to the surface of the conductor, then free electrons on the surface of the conductor would experience acceleration (Figure 1.46(a)). This means that the conductor is not in equilibrium. Therefore at electrostatic equilibrium, the electric field must be perpendicular to the surface of the conductor. This is shown in Figure 1.46 (b).
Since electric field is normal to the surface of the conductor, the curved part of the cylinder has zero electric flux. Also inside the conductor, the electric field is zero. Hence the bottom flat part of the Gaussian surface has no electric flux. Therefore the top flat surface alone contributes to the electric flux. The electric field is parallel to the area vector and the total charge inside the surface is $\sigma A$. By applying Gauß’s law,

$$EA = \frac{\sigma A}{\varepsilon_0}$$

In vector form,

$$\vec{E} = \frac{\sigma}{\varepsilon_0} \hat{n}$$  \hspace{1cm} (1.79)

Here $\hat{n}$ represents the unit vector outward normal to the surface of the conductor. Suppose $\sigma < 0$, then electric field points inward perpendicular to the surface.

(iv) The electrostatic potential has the same value on the surface and inside of the conductor.

We know that the conductor has no parallel electric component on the surface which means that charges can be moved on the surface without doing any work. This is possible only if the electrostatic potential is constant at all points on the surface and there is no potential difference between any two points on the surface.

Since the electric field is zero inside the conductor, the potential is the same as the surface of the conductor. Thus at electrostatic equilibrium, the conductor is always at equipotential.

1.7.2 Electrostatic shielding

Using Gauss law, we proved that the electric field inside the charged spherical shell is zero. Further, we showed that the electric field inside both hollow and solid conductors is zero. It is a very interesting property which has an important consequence.

Consider a cavity inside the conductor as shown in Figure 1.48 (a). Whatever the charges at the surfaces and whatever the electrical disturbances outside, the electric field inside the cavity is zero. A sensitive electrical instrument which is to be protected from external electrical disturbance is kept inside this cavity. This is called electrostatic shielding.

Faraday cage is an instrument used to demonstrate this effect. It is made up of metal bars configured as shown in Figure 1.48 (b). If an artificial lightning jolt is created outside, the person inside is not affected.

Figure 1.48  (a) Electric field inside the cavity  (b) Faraday cage
During lightning accompanied by a thunderstorm, it is always safer to sit inside a bus than in open ground or under a tree. The metal body of the bus provides electrostatic shielding, since the electric field inside is zero. During lightning, the charges flow through the body of the conductor to the ground with no effect on the person inside that bus.

1.7.3 Electrostatic induction

In section 1.1, we have learnt that an object can be charged by rubbing using an appropriate material. Whenever a charged rod is touched by another conductor, charges start to flow from charged rod to the conductor. Is it possible to charge a conductor without any contact? The answer is yes. This type of charging without actual contact is called electrostatic induction.

(i) Consider an uncharged (neutral) conducting sphere at rest on an insulating stand. Suppose a negatively charged rod is brought near the conductor without touching it, as shown in Figure 1.49(a).

(ii) Now the conducting sphere is connected to the ground through a conducting wire. This is called grounding. Since the ground can always receive any amount of electrons, grounding removes the electron from the conducting sphere. Note that positive charges will not flow to the ground because they are attracted by the negative charges of the rod (Figure 1.49(b)).

(iii) When the grounding wire is removed from the conductor, the positive charges remain near the charged rod (Figure 1.49(c)).

(iv) Now the charged rod is taken away from the conductor. As soon as the charged rod is removed, the positive charge gets distributed uniformly on the surface of the conductor (Figure 1.49(d)). By this process, the neutral conducting sphere becomes positively charged.

For an arbitrary shaped conductor, the intermediate steps and conclusion are the same except the final step. The distribution of positive charges is not uniform for arbitrarily-shaped conductors. Why is it not uniform? The reason for it is discussed in the section 1.9.
It is important here to note that the acceleration depends on the mass of the object. Galileo's conclusion that all objects fall at the same rate towards the Earth is true only in a uniform gravitational field. When a uniform electric field is included, the acceleration of a charged object depends on both mass and charge.

\[ \mathbf{a} = \frac{g + \frac{qE}{m}}{m} \mathbf{j} \]

But still the acceleration \( \mathbf{a} \) is constant throughout the motion. Hence we use kinematic equations to calculate the range, maximum height and time of flight.

In fact we can simply replace \( g \) by \( g + \frac{qE}{m} \) in the usual expressions of range, maximum height and time of flight of a projectile.

<table>
<thead>
<tr>
<th>Without charge</th>
<th>With the charge +q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time of flight T</td>
<td>( \frac{2v \sin \theta}{g} )</td>
</tr>
<tr>
<td>Maximum height ( h_{\text{max}} )</td>
<td>( \frac{v^2 \sin^2 \theta}{2g} )</td>
</tr>
<tr>
<td>Range R</td>
<td>( \frac{v^2 \sin 2\theta}{g} )</td>
</tr>
</tbody>
</table>

Note that the time of flight, maximum height, range are all inversely proportional to the acceleration of the object. Since \( \left(g + \frac{qE}{m}\right) > g \) for charge +q, the quantities T, \( h_{\text{max}} \) and R will decrease when compared to the motion of an object of mass m and zero net charge. Suppose the charge is –q, then \( \left(g - \frac{qE}{m}\right) < g \), and the quantities T, \( h_{\text{max}} \) and

**EXAMPLE 1.19**

A small ball of conducting material having a charge +q and mass m is thrown upward at an angle \( \theta \) to horizontal surface with an initial speed \( v_0 \) as shown in the figure. There exists an uniform electric field \( E \) downward along with the gravitational field \( g \). Calculate the range, maximum height and time of flight in the motion of this charged ball. Neglect the effect of air and treat the ball as a point mass.

**Solution**

If the conductor has no net charge, then its motion is the same as usual projectile motion of a mass m which we studied in Kinematics (unit 2, vol-1 XI physics). Here, in this problem, in addition to downward gravitational force, the charge also will experience a downward uniform electrostatic force.

The acceleration of the charged ball due to gravity = \( -g \mathbf{j} \)

The acceleration of the charged ball due to uniform electric field = \( -\frac{qE}{m} \mathbf{j} \)

The total acceleration of charged ball in downward direction \( \mathbf{a} = -\left(g + \frac{qE}{m}\right) \mathbf{j} \)
R will increase. Interestingly the trajectory is still parabolic as shown in the figure.

\[ + - + - \]

\[ \rightarrow \]

\[ E \rightarrow \]

Center of negative charge coincides with center of positive charge

\[ (a) \quad (b) \]

\[ \text{Figure 1.50} \quad \text{(a) Non polar molecules without external field (b) With the external field} \]

\[ 1.7.4 \text{ Dielectrics or insulators} \]

A dielectric is a non-conducting material and has no free electrons. The electrons in a dielectric are bound within the atoms. Ebonite, glass and mica are some examples of dielectrics. When an external electric field is applied, the electrons are not free to move anywhere but they are realigned in a specific way. A dielectric is made up of either polar molecules or non-polar molecules.

Non-polar molecules

A non-polar molecule is one in which centers of positive and negative charges coincide. As a result, it has no permanent dipole moment. Examples of non-polar molecules are hydrogen \((H_2)\), oxygen \((O_2)\), and carbon dioxide \((CO_2)\) etc. When an external electric field is applied, the centers of positive and negative charges are separated by a small distance which induces dipole moment in the direction of the external electric field. Then the dielectric is said to be polarized by an external electric field. This is shown in Figure 1.50.

\[ \text{Figure 1.50} \quad \text{(a) Non polar molecules without external field (b) With the external field} \]

Polar molecules

In polar molecules, the centers of the positive and negative charges are separated even in the absence of an external electric field. They have a permanent dipole moment. Due to thermal motion, the direction of each dipole moment is oriented randomly (Figure 1.51(a)). Hence the net dipole moment is zero in the absence of an external electric field. Examples of polar molecules are \(H_2O, N_2O, HCl, NH_3\).

When an external electric field is applied, the dipoles inside the polar molecule tend to align in the direction of the electric field. Hence a net dipole moment is induced in it. Then the dielectric is said to be polarized by an external electric field (Figure 1.51(b)).

\[ \text{Figure 1.51} \quad \text{(a) Randomly oriented polar molecules (b) Align with the external electric field} \]
Polarisation

In the presence of an external electric field, the dipole moment is induced in the dielectric material. **Polarisation** $\vec{p}$ is defined as the total dipole moment per unit volume of the dielectric. For most dielectrics (linear isotropic), the Polarisation is directly proportional to the strength of the external electric field. This is written as

$$\vec{p} = \chi_e \vec{E}_{\text{ext}}$$

(1.80)

where $\chi_e$ is a constant called the electric susceptibility which is a characteristic of each dielectric.

### 1.7.5 Induced Electric field inside the dielectric

When an external electric field is applied on a conductor, the charges are aligned in such a way that an internal electric field is created which cancels the external electric field. But in the case of a dielectric, which has no free electrons, the external electric field only realigns the charges so that an internal electric field is produced. The magnitude of the internal electric field is smaller than that of external electric field. Therefore the net electric field inside the dielectric is not zero but is parallel to an external electric field with magnitude less than that of the external electric field. For example, let us consider a rectangular dielectric slab placed between two oppositely charged plates (capacitor) as shown in the Figure 1.52(b).

The uniform electric field between the plates acts as an external electric field $\vec{E}_{\text{ext}}$ which polarizes the dielectric placed between plates. The positive charges are induced on one side surface and negative charges are induced on the other side of surface. But inside the dielectric, the net charge is zero even in a small volume. So the dielectric in the external field is equivalent to two oppositely charged sheets with the surface charge densities $+\sigma_b$ and $-\sigma_b$. These charges are called bound charges. They are not free to move like free electrons in conductors. This is shown in the Figure 1.52(b).

**Figure 1.52** Induced electric field lines inside the dielectric
CAPACITORS AND CAPACITANCE

1.8.1 Capacitors

Capacitor is a device used to store electric charge and electrical energy. It consists of two conducting objects (usually plates or sheets) separated by some distance. Capacitors are widely used in many electronic circuits and have applications in many areas of science and technology.

A simple capacitor consists of two parallel metal plates separated by a small distance as shown in Figure 1.54 (a).

When a capacitor is connected to a battery of potential difference $V$, the electrons are transferred from one plate to the other plate by battery so that one plate becomes negatively charged with a charge of $-Q$ and the other plate positively charged with $+Q$. The potential difference between the plates is equivalent to the battery’s terminal voltage. This is shown in Figure 1.54(b). If the battery voltage is increased, the amount of charges stored in the plates also increase. In general, the charge stored in the capacitor

For example, the charged balloon after rubbing sticks onto a wall. The reason is that the negatively charged balloon is brought near the wall, it polarizes opposite charges on the surface of the wall, which attracts the balloon. This is shown in Figure 1.53.

1.7.6 Dielectric strength

When the external electric field applied to a dielectric is very large, it tears the atoms apart so that the bound charges become free charges. Then the dielectric starts to conduct electricity. This is called dielectric breakdown. The maximum electric field the dielectric can withstand before it breakdowns is called dielectric strength. For example, the dielectric strength of air is $3 \times 10^6$ V m$^{-1}$. If the applied electric field increases beyond this, a spark is produced in the air. The dielectric strengths of some dielectrics are given in the Table 1.1.

<table>
<thead>
<tr>
<th>Substance</th>
<th>Dielectric strength (Vm$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mica</td>
<td>$100 \times 10^6$</td>
</tr>
<tr>
<td>Teflon</td>
<td>$60 \times 10^6$</td>
</tr>
<tr>
<td>Paper</td>
<td>$16 \times 10^6$</td>
</tr>
<tr>
<td>Air</td>
<td>$3 \times 10^6$</td>
</tr>
<tr>
<td>Pyrex glass</td>
<td>$14 \times 10^6$</td>
</tr>
</tbody>
</table>
is proportional to the potential difference between the plates.

\[ Q \propto V \]

so that \[ Q = CV \]

where the \( C \) is the proportionality constant called capacitance. The capacitance \( C \) of a capacitor is defined as the ratio of the magnitude of charge on either of the conductor plates to the potential difference existing between the conductors.

\[ C = \frac{Q}{V} \quad (1.81) \]

The SI unit of capacitance is *coulomb per volt* or *farad* (F) in honor of Michael Faraday. Farad is a very large unit of capacitance. In practice, capacitors are available in the range of microfarad \((1\mu F = 10^{-6} \text{ F})\) to picofarad \((1\text{pf} = 10^{-12} \text{ F})\). A capacitor is represented by the symbol \( \pm \) or \( \pm \). Note that the total charge stored in the capacitor is zero \((Q - Q = 0)\). When we say the capacitor stores charges, it means the amount of charge that can be stored in any one of the plates.

Nowadays there are capacitors available in various shapes (cylindrical, disk) and types (tantalum, ceramic and electrolytic), as shown in Figure 1.55. These capacitors are extensively used in various kinds of electronic circuits.

**Capacitance of a parallel plate capacitor**

Consider a capacitor with two parallel plates each of cross-sectional area \( A \) and separated by a distance \( d \) as shown in Figure 1.56.

The electric field between two infinite parallel plates is uniform and is given by \( E = \frac{\sigma}{\varepsilon_0} \), where \( \sigma \) is the surface charge density on the plates \( \left[\sigma = \frac{Q}{A}\right] \). If the separation distance \( d \) is very much smaller than the size of the plate \((d^2 \ll A)\), then the above result is used even for finite-sized parallel plate capacitor.

The electric field between the plates is

\[ E = \frac{Q}{A\varepsilon_0} \quad (1.82) \]

Since the electric field is uniform, the electric potential between the plates having separation \( d \) is given by
(i) If the area of cross-section of the capacitor plates is increased, more charges can be distributed for the same potential difference. As a result, the capacitance is increased.

(ii) If the distance $d$ between the two plates is reduced, the potential difference between the plates ($V = Ed$) decreases with $E$ constant. As a result, voltage difference between the terminals of the battery increases which in turn leads to an additional flow of charge to the plates from the battery, till the voltage on the capacitor equals to the battery’s terminal voltage. Suppose the distance is increased, the capacitor voltage increases and becomes greater than the battery voltage. Then, the charges flow from capacitor plates to battery till both voltages becomes equal.

**EXAMPLE 1.20**

A parallel plate capacitor has square plates of side 5 cm and separated by a distance of 1 mm. (a) Calculate the capacitance of this capacitor. (b) If a 10 V battery is connected to the capacitor, what is the charge stored in any one of the plates? (The value of $\varepsilon_0 = 8.85 \times 10^{-12}$ Nm$^2$ C$^{-2}$)

**Solution**

(a) The capacitance of the capacitor is

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 25 \times 10^{-4}}{1 \times 10^{-3}}$$

$$C = 221.2 \times 10^{-13} F$$

(b) The charge stored in any one of the plates is $Q = CV$, then

$$Q = 221.2 \times 10^{-13} \times 10 = 221.2 \times 10^{-12} C = 221.2 \mu C$$
Sometimes we notice that the ceiling fan does not start rotating as soon as it is switched on. But when we rotate the blades, it starts to rotate as usual. Why is it so? We know that to rotate any object, there must be a torque applied on the object. For the ceiling fan, the initial torque is given by the capacitor widely known as a condenser. If the condenser is faulty, it will not give sufficient initial torque to rotate the blades when the fan is switched on.

### 1.8.2 Energy stored in the capacitor

Capacitor not only stores the charge but also it stores energy. When a battery is connected to the capacitor, electrons of total charge \(-Q\) are transferred from one plate to the other plate. To transfer the charge, work is done by the battery. This work done is stored as electrostatic potential energy in the capacitor.

To transfer an infinitesimal charge \(dQ\) for a potential difference \(V\), the work done is given by

\[
dW = V\,dQ
\]

where \(V = \frac{Q}{C}\)  

The total work done to charge a capacitor is

\[
W = \int_0^Q \frac{Q}{C}\,dQ = \frac{Q^2}{2C}
\]

This work done is stored as electrostatic potential energy \(U_E\) in the capacitor.

\[
U_E = \frac{Q^2}{2C} = \frac{1}{2}CV^2 \quad (\because Q = CV) \quad (1.87)
\]

where \(Q = CV\) is used. This stored energy is thus directly proportional to the capacitance of the capacitor and the square of the voltage between the plates of the capacitor. But where is this energy stored in the capacitor? To understand this question, the equation (1.87) is rewritten as follows using the results \(C = \frac{\epsilon_0 A}{d}\) and \(V = Ed\)

\[
U_E = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\epsilon_0 (Ad)E^2
\]

where \(Ad = \text{volume of the space between the capacitor plates. The energy stored per unit volume of space is defined as energy density } u_E = \frac{U}{\text{Volume}}\). From equation (1.88), we get

\[
u_E = \frac{1}{2}\epsilon_0 E^2
\]

From equation (1.89), we infer that the energy is stored in the electric field existing between the plates of the capacitor. Once the capacitor is allowed to discharge, the energy is retrieved.

It is important to note that the energy density depends only on the electric field and not on the size of the plates of the capacitor. In fact, expression (1.89) is true for the electric field due to any type of charge configuration.

### 1.8.3 Applications of capacitors

Capacitors are used in various electronics circuits. A few of the applications.

\begin{center}
\textbf{UNIT 1 Electrostatics}
\end{center}
1.8.4 Effect of dielectrics in capacitors

In earlier discussions, we assumed that the space between the parallel plates of a capacitor is either empty or filled with air. Suppose dielectrics like mica, glass or paper are introduced between the plates, then the capacitance of the capacitor is altered. The dielectric can be inserted into the plates in two different ways. (i) when the capacitor is disconnected from the battery. (ii) when the capacitor is connected to the battery.

(i) when the capacitor is disconnected from the battery

Consider a capacitor with two parallel plates each of cross-sectional area $A$ and are separated by a distance $d$. The capacitor is charged by a battery of voltage $V_0$ and the charge stored is $Q_0$. The capacitance of the capacitor without the dielectric is

$$C_0 = \frac{Q_0}{V_0} \quad (1.90)$$

The battery is then disconnected from the capacitor and the dielectric is inserted between the plates. This is shown in Figure 1.58.

(a) Most people are now familiar with the digital camera. The flash which comes from the camera when we take photographs is due to the energy released from the capacitor, called a flash capacitor (Figure 1.57 (a))

(b) During cardiac arrest, a device called heart defibrillator is used to give a sudden surge of a large amount of electrical energy to the patient’s chest to retrieve the normal heart function. This defibrillator uses a capacitor of $175 \mu F$ charged to a high voltage of around $2000 \text{ V}$. This is shown in Figure 1.57(b).

(c) Capacitors are used in the ignition system of automobile engines to eliminate sparking

(d) Capacitors are used to reduce power fluctuations in power supplies and to increase the efficiency of power transmission.

However, capacitors have disadvantage as well. Even after the battery or power supply is removed, the capacitor stores charges and energy for some time. For example if the TV is switched off, it is always advisable to not touch the back side of the TV panel.
Since $\varepsilon_r > 1$, we have $C > C_0$. Thus insertion of the dielectric constant $\varepsilon_r$ increases the capacitance.

Using equation (1.84),

$$C = \frac{Q}{V} = \varepsilon_r \frac{Q_0}{V_0} = \varepsilon_r C_0 \quad (1.93)$$

where $\varepsilon = \varepsilon_r \varepsilon_0$ is the permittivity of the dielectric medium.

The energy stored in the capacitor before the insertion of a dielectric is given by

$$U = \frac{1}{2} \frac{Q_0^2}{C_0} \quad (1.95)$$

After the dielectric is inserted, the charge $Q_0$ remains constant but the capacitance is increased. As a result, the stored energy is decreased.

$$U = \frac{1}{2} \frac{Q_0^2}{C} = \frac{1}{2} \frac{Q_0^2}{\varepsilon_r C_0} = \frac{U_0}{\varepsilon_r} \quad (1.96)$$

Since $\varepsilon_r > 1$ we get $U < U_0$. There is a decrease in energy because, when the dielectric is inserted, the capacitor spends some energy in pulling the dielectric inside.

(ii) When the battery remains connected to the capacitor

Let us now consider what happens when the battery of voltage $V_0$ remains connected to the capacitor when the dielectric is inserted into the capacitor. This is shown in Figure 1.59.

The potential difference $V_0$ across the plates remains constant. But it is found experimentally (first shown by Faraday) that when dielectric is inserted, the charge stored in the capacitor is increased by a factor $\varepsilon_r$. 

$$V = Ed = \frac{E_0}{\varepsilon_r} d = \frac{V_0}{\varepsilon_r} \quad (1.92)$$

We know that capacitance is inversely proportional to the potential difference. Therefore as $V$ decreases, $C$ increases. Thus new capacitance in the presence of a dielectric is

$$C = \frac{Q}{V} = \varepsilon_r \frac{Q_0}{V_0} = \varepsilon_r C_0 \quad (1.93)$$
After the dielectric is inserted, the capacitance is increased; hence the stored energy is also increased.

\[ U = \frac{1}{2} C V_o^2 = \frac{1}{2} \varepsilon_r C_0 V_0^2 = \varepsilon_c U_o \]  

(1.101)

Since \( \varepsilon_r > 1 \) we have \( U > U_o \).

It may be noted here that since voltage between the capacitor \( V_0 \) is constant, the electric field between the plates also remains constant.

The energy density is given by

\[ u = \frac{1}{2} \varepsilon E_0^2 \]  

(1.102)

where \( \varepsilon \) is the permittivity of the given dielectric material.

The results of the above discussions are summarised in the following Table 1.2

**Q = \varepsilon_r Q_o**  

(1.97)

Due to this increased charge, the capacitance is also increased. The new capacitance is

\[ C = \frac{Q}{V_0} = \frac{Q_o}{V_0} = \frac{\varepsilon_c}{\varepsilon} C_0 \]  

(1.98)

However the reason for the increase in capacitance in this case when the battery remains connected is different from the case when the battery is disconnected before introducing the dielectric.

\[ \text{Now, } C_0 = \frac{\varepsilon \varepsilon_o A}{d} \]

and

\[ C = \frac{\varepsilon A}{d} \]  

(1.99)

The energy stored in the capacitor before the insertion of a dielectric is given by

\[ U_o = \frac{1}{2} C_0 V_o^2 \]  

(1.100)

Note that here we have not used the expression \( U_o = \frac{1}{2} \frac{Q_o^2}{C_0} \) because here, both charge and capacitance are changed, whereas in equation (1.100), \( V_o \) remains constant.

---

**Figure 1.59** (a) Capacitor is charged through a battery (b) Dielectric is inserted when the battery is connected.

**Computer keyboard keys are constructed using capacitors with a dielectric as shown in the figure.**

When the key is pressed, the separation between the plates decreases leading to an increase in the capacitance. This in turn triggers the electronic circuits in the computer to identify which key is pressed.
increases. As a result, the capacitance is decreased. New capacitance is
\[ C = \frac{C}{\epsilon_r} \]
\[ = \frac{44.25 \times 10^{-9}}{5} \]
\[ = 8.85 \times 10^{-9} F = 8.85 \mu F \]
The stored charge remains same and 442.5 nC. Hence newly stored energy is
\[ U = \frac{Q^2}{2C} = \frac{Q^2 \epsilon_r}{2} = \epsilon_f U \]
\[ = 5 \times 2.21 \mu J = 11.05 \mu J \]
The increased energy is
\[ \Delta U = 11.05 \mu J - 2.21 \mu J = 8.84 \mu J \]
When the dielectric is removed, it experiences an inward pulling force due to the plates. To remove the dielectric, an external agency has to do work on the dielectric which is stored as additional energy. This is the source for the extra energy 8.84 \mu J.

### 1.8.5 Capacitor in series and parallel

#### (i) Capacitor in series

Consider three capacitors of capacitance \( C_1 \), \( C_2 \) and \( C_3 \) connected in series with a battery of voltage \( V \) as shown in the Figure 1.60 (a).

As soon as the battery is connected to the capacitors in series, the electrons of charge
If three capacitors in series are considered to form an equivalent single capacitor $C_s$ shown in Figure 1.60(b), then we have $V = \frac{Q}{C_s}$. Substituting this expression into equation (1.104), we get

$$\frac{Q}{C_s} = Q \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)$$

Thus, the inverse of the equivalent capacitance $C_s$ of three capacitors connected in series is equal to the sum of the inverses of each capacitance. This equivalent capacitance $C_s$ is always less than the smallest individual capacitance in the series.

(ii) Capacitance in parallel

Consider three capacitors of capacitance $C_1$, $C_2$ and $C_3$ connected in parallel with a battery of voltage $V$ as shown in Figure 1.61 (a).
The equivalent capacitance \( C_p \) in a parallel connection is always greater than the largest individual capacitance. In a parallel connection, it is equivalent as area of each capacitance adds to give more effective area such that total capacitance increases.

**Example 1.22**

Find the equivalent capacitance between P and Q for the configuration shown below in the figure (a).

The equivalent capacitance \( C_p \) in a parallel connection is always greater than the largest individual capacitance. In a parallel connection, it is equivalent as area of each capacitance adds to give more effective area such that total capacitance increases.

**Solution**

The capacitors 1 \( \mu F \) and 3\( \mu F \) are connected in parallel and 6\( \mu F \) and 2 \( \mu F \) are also separately connected in parallel. So these parallel combinations reduced to equivalent single capacitances in their respective positions, as shown in the figure (b).

**Figure 1.61** (a) capacitors in parallel (b) equivalent capacitance with the same total charge

Since corresponding sides of the capacitors are connected to the same positive and negative terminals of the battery, the voltage across each capacitor is equal to the battery’s voltage. Since capacitance of the capacitors is different, the charge stored in each capacitor is not the same. Let the charge stored in the three capacitors be \( Q_1 \), \( Q_2 \), and \( Q_3 \) respectively. According to the law of conservation of total charge, the sum of these three charges is equal to the charge \( Q \) transferred by the battery,

\[
Q = Q_1 + Q_2 + Q_3 \quad (1.106)
\]

Now, since \( Q = CV \), we have

\[
Q = C_1V + C_2V + C_3V \quad (1.107)
\]

If these three capacitors are considered to form a single capacitance \( C_p \) which stores the total charge \( Q \) as shown in the Figure 1.61(b), then we can write \( Q = C_pV \). Substituting this in equation (1.107), we get

\[
C_pV = C_1V + C_2V + C_3V
\]

\[
C_p = C_1 + C_2 + C_3 \quad (1.108)
\]

Thus, the equivalent capacitance of capacitors connected in parallel is equal to the sum of the individual capacitances.
\[ C_{eq} = 1\mu F + 3\mu F = 4\mu F \]
\[ C_{eq} = 6\mu F + 2\mu F = 8\mu F \]

From the figure (b), we infer that the two 4 \mu F capacitors are connected in series and the two 8 \mu F capacitors are connected in series. By using formula for the series, we can reduce to their equivalent capacitances as shown in figure (c).

\[ \frac{1}{C_{eq}} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \quad \Rightarrow \quad C_{eq} = 2 \mu F \]

and

\[ \frac{1}{C_{eq}} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4} \quad \Rightarrow \quad C_{eq} = 4 \mu F \]

From the figure (c), we infer that 2\mu F and 4\mu F are connected in parallel. So the equivalent capacitance is given in the figure (d).

\[ C_{eq} = 2\mu F + 4\mu F = 6\mu F \]

Thus the combination of capacitances in figure (a) can be replaced by a single capacitance 6 \mu F.

### 1.9 DISTRIBUTION OF CHARGES IN A CONDUCTOR AND ACTION AT POINTS

#### 1.9.1 Distribution of charges in a conductor

Consider two conducting spheres A and B of radii \( r_1 \) and \( r_2 \) respectively connected to each other by a thin conducting wire as shown in the Figure 1.62. The distance between the spheres is much greater than the radii of either spheres.

If a charge Q is introduced into any one of the spheres, this charge Q is redistributed into both the spheres such that the electrostatic potential is same in both the spheres. They are now uniformly charged and attain electrostatic equilibrium. Let \( q_1 \) be the charge residing on the surface of sphere A and \( q_2 \) is the charge residing on the surface of sphere B such that \( Q = q_1 + q_2 \). The charges are distributed only on the surface and there is no net charge inside the conductor.

The electrostatic potential at the surface of the sphere A is given by

\[ V_A = \frac{1}{4\pi \varepsilon_0} \frac{q_1}{r_1} \quad (1.110) \]

The electrostatic potential at the surface of the sphere B is given by

\[ V_B = \frac{1}{4\pi \varepsilon_0} \frac{q_2}{r_2} \quad (1.111) \]

The surface of the conductor is an equipotential. Since the spheres are connected by the conducting wire, the surfaces of both the spheres together form an equipotential surface. This implies that

\[ V_A = V_B \]

or

\[ \frac{q_1}{r_1} = \frac{q_2}{r_2} \quad (1.112) \]

Let us take the charge density on the surface of sphere A is \( \sigma_i \) and charge density on the surface of sphere B is \( \sigma_2 \). This implies that \( q_1 = 4\pi r_1^2 \sigma_i \) and
q_2 = 4\pi r_2^2 \sigma_2. Substituting these values into equation (1.112), we get

\[ \sigma_1 r_1 = \sigma_2 r_2 \]  
(1.113)

from which we conclude that

\[ \sigma r = \text{constant} \]  
(1.114)

Thus the surface charge density \( \sigma \) is inversely proportional to the radius of the sphere. For a smaller radius, the charge density will be larger and vice versa.

**Example 1.23**

Two conducting spheres of radius \( r_1 = 8 \text{ cm} \) and \( r_2 = 2 \text{ cm} \) are separated by a distance much larger than 8 cm and are connected by a thin conducting wire as shown in the figure. A total charge of \( Q = +100 \text{ nC} \) is placed on one of the spheres. After a fraction of a second, the charge \( Q \) is redistributed and both the spheres attain electrostatic equilibrium.

(a) Calculate the charge and surface charge density on each sphere.

(b) Calculate the potential at the surface of each sphere.

**Solution**

(a) The electrostatic potential on the surface of the sphere \( A \) is

\[ V_A = \frac{1}{4\pi \varepsilon_0} \frac{q_1}{r_1} \]

The electrostatic potential on the surface of the sphere \( B \) is

\[ V_B = \frac{1}{4\pi \varepsilon_0} \frac{q_2}{r_2} \]

Since \( V_A = V_B \), we have

\[ \frac{q_1}{r_1} = \frac{q_2}{r_2} \Rightarrow q_1 = \left( \frac{r_1}{r_2} \right) q_2 \]
But from the conservation of total charge, \( Q = q_1 + q_2 \), we get \( q_1 = Q - q_2 \). By substituting this in the above equation,

\[ Q - q_2 = \left( \frac{r_1}{r_2} \right) q_2 \]

so that

\[ q_2 = Q \left( \frac{r_2}{r_1 + r_2} \right) \]

Therefore,

\[ q_2 = 100 \times 10^{-9} \times \left( \frac{2}{10} \right) = 20 \text{ nC} \]

and \( q_1 = Q - q_2 = 80 \text{ nC} \)

The electric charge density for sphere \( A \) is

\[ \sigma_1 = \frac{q_1}{4\pi r_1^2} \]

Therefore,

\[ \sigma_1 = \frac{80 \times 10^{-9}}{4 \times 64 \times 10^{-4}} = 0.99 \times 10^{-6} \text{ C m}^{-2} \]

and

\[ \sigma_2 = \frac{20 \times 10^{-9}}{4\pi \times 4 \times 10^{-4}} = 3.9 \times 10^{-6} \text{ C m}^{-2} \]

Note that the surface charge density is greater on the smaller sphere compared to the larger sphere (\( \sigma_2 \approx 4\sigma_1 \)) which confirms the result \( \frac{\sigma_2}{\sigma_1} = \frac{r_2}{r_1} \).

The potential on both spheres is the same. So we can calculate the potential on any one of the spheres.

\[ V_A = \frac{1}{4\pi \varepsilon_0} \frac{q_1}{r_1} = \frac{9 \times 10^7 \times 80 \times 10^{-9}}{8 \times 10^{-2}} = 9 \text{ kV} \]
### 1.9.2 Action at points or Corona discharge

Consider a charged conductor of irregular shape as shown in Figure 1.63 (a).

![Figure 1.63](image)

**Figure 1.63** Action at a points or corona discharge

We know that smaller the radius of curvature, the larger is the charge density. The end of the conductor which has larger curvature (smaller radius) has a large charge accumulation as shown in Figure 1.63 (b). As a result, the electric field near this edge is very high and it ionizes the surrounding air. The positive ions are repelled at the sharp edge and negative ions are attracted towards the sharper edge. This reduces the total charge of the conductor near the sharp edge. This is called action at points or corona discharge.

### 1.9.3 Lightning arrester or lightning conductor

This is a device used to protect tall buildings from lightning strikes. It works on the principle of action at points or corona discharge. This device consists of a long thick copper rod passing from top of the building to the ground. The upper end of the rod has a sharp spike or a sharp needle as shown in Figure 1.64 (a) and (b).

![Figure 1.64](image)

**Figure 1.64** (a) Schematic diagram of a lightning arrester. (b) A house with a lightning arrester

The lower end of the rod is connected to the copper plate which is buried deep into the ground. When a negatively charged cloud is passing above the building, it induces
a positive charge on the spike. Since the induced charge density on thin sharp spike is large, it results in a corona discharge. This positive charge ionizes the surrounding air which in turn neutralizes the negative charge in the cloud. The negative charge pushed to the spikes passes through the copper rod and is safely diverted to the Earth. The lightning arrester does not stop the lightning; rather it diverts the lightning to the ground safely.

1.9.4 Van de Graaff Generator

In the year 1929, Robert Van de Graaff designed a machine which produces a large amount of electrostatic potential difference, up to several million volts (10^7 V). This Van de Graaff generator works on the principle of electrostatic induction and action at points.

A large hollow spherical conductor is fixed on the insulating stand as shown in Figure 1.65. A pulley B is mounted at the center of the hollow sphere and another pulley C is fixed at the bottom. A belt made up of insulating materials like silk or rubber runs over both pulleys. The pulley C is driven continuously by the electric motor. Two comb shaped metallic conductors E and D are fixed near the pulleys.

The comb D is maintained at a positive potential of 10^4 V by a power supply. The upper comb E is connected to the inner side of the hollow metal sphere.

Due to the high electric field near comb D, air between the belt and comb D gets ionized. The positive charges are pushed towards the belt and negative charges are attracted towards the comb D. The positive charges stick to the belt and move up. When the positive charges reach the comb E, a large amount of negative and positive charges are induced on either side of comb E due to electrostatic induction. As a result, the positive charges are pushed away from the comb E and they reach the outer surface of the sphere. Since the sphere is a conductor, the positive charges are distributed uniformly on the outer surface of the hollow sphere. At the same time, the negative charges nullify the positive charges in the belt due to corona discharge before it passes over the pulley.

When the belt descends, it has almost no net charge. At the bottom, it again gains a large positive charge. The belt goes up and delivers the positive charges to the
The electric field on the surface of the sphere (by Gauss law) is given by

\[ E = \frac{1}{4\pi \varepsilon} \frac{Q}{R^2} \]

The potential on the surface of the hollow metallic sphere is given by

\[ V = \frac{1}{4\pi \varepsilon} \frac{Q}{R} = ER \]

with \( V_{\text{max}} = E_{\text{max}} R \)

Here \( E_{\text{max}} = 3 \times 10^6 \frac{V}{m} \). So the maximum potential difference created is given by

\[ V_{\text{max}} = 3 \times 10^6 \times 0.5 \]

\[ = 1.5 \times 10^6 \text{ V (or) 1.5 million volt} \]

outer surface of the sphere. This process continues until the outer surface produces the potential difference of the order of \( 10^7 \) which is the limiting value. We cannot store charges beyond this limit since the extra charge starts leaking to the surroundings due to ionization of air. The leakage of charges can be reduced by enclosing the machine in a gas filled steel chamber at very high pressure.

The high voltage produced in this Van de Graaff generator is used to accelerate positive ions (protons and deuterons) for nuclear disintegrations and other applications.

**Example 1.24**

Dielectric strength of air is \( 3 \times 10^6 \text{ V m}^{-1} \). Suppose the radius of a hollow sphere in the Van de Graaff generator is \( R = 0.5 \text{ m} \), calculate the maximum potential difference created by this Van de Graaff generator.
Like charges repel and unlike charges attract
The total charge in the universe is conserved
Charge is quantized. Total charge in an object \( q = ne \) where \( n = 0,1,2,3\ldots \) and \( e \) is electron charge.

Coulomb's law in vector form:
\[
\vec{F} = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r} \quad (\hat{r} \text{ is unit vector along joining } q_1, q_2)
\]

For continuous charge distributions, integration methods can be used.

Electrostatic force obeys the superposition principle.

Electric field at a distance \( r \) from a point charge:
\[
\vec{E} = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} \hat{r}
\]

Electric field lines starts at a positive charge and end at a negative charge or at infinity

Electric field due to electric dipole at points on the axial line:
\[
\vec{E}_{\text{tot}} = \frac{1}{4\pi \varepsilon_0} \left( \frac{2\hat{p}}{r^3} \right)
\]

Electric field due to electric dipole at points on the equatorial line:
\[
\vec{E}_{\text{tot}} = -\frac{1}{4\pi \varepsilon_0} \left( \frac{\vec{p}}{r^3} \right)
\]

Torque experienced by a dipole in a uniform electric field:
\[
\vec{\tau} = \vec{p} \times \vec{E}
\]

Electrostatic potential at a distance \( r \) from the point charge:
\[
V = \frac{1}{4\pi \varepsilon_0} \frac{q}{r}
\]

Electrostatic potential due to an electric dipole:
\[
V = \frac{1}{4\pi \varepsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^3}
\]

The electrostatic potential is the same at all points on an equipotential surface.

The relation between electric field and electrostatic potential:
\[
\vec{E} = -\left( \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right)
\]

Electrostatic potential energy for system of charges is equal to the work done to arrange the charges in the given configuration.

Electrostatic potential energy stored in a dipole system in a uniform electric field:
\[
U = -\vec{p} \cdot \vec{E}
\]

The total electric flux through a closed surface: \( \Phi_E = \frac{Q}{\varepsilon_0} \) where \( Q \) is the net charge enclosed by the surface

Electric field due to a charged infinite wire:
\[
\vec{E} = \frac{1}{2\pi \varepsilon_0} \frac{\lambda}{r} \hat{r}
\]

Electric field due to a charged infinite plane:
\[
\vec{E} = \frac{\sigma}{\varepsilon_0} \hat{n} \quad (\hat{n} \text{ is normal to the plane})
\]

Electric field inside a charged spherical shell is zero. For points outside:
\[
\vec{E} = \frac{Q}{4\pi \varepsilon_0 r^2} \hat{r}
\]
Electric field inside a conductor is zero. The electric field at the surface of the conductor is normal to the surface and has magnitude \( E = \frac{\sigma}{\varepsilon_0} \).

The surface of the conductor has the same potential, at all points on the surface.

Conductor can be charged using the process of induction.

A dielectric or insulator has no free electrons. When an electric field is applied, the dielectric is polarised.

Capacitance is given by \( C = \frac{Q}{V} \).

Capacitance of a parallel plate capacitor: \( C = \frac{\varepsilon_0 A}{d} \)

Electrostatic energy stored in a capacitor: \( U = \frac{1}{2} CV^2 \)

The equivalent capacitance for parallel combination is equal to the sum of individual capacitance of capacitors.

For a series combination: The inverse of equivalent capacitance is equal to sum of inverse of individual capacitances of capacitors.

The distribution of charges in the conductors depends on the shape of conductor. For sharper edge, the surface charge density is greater. This principle is used in the lightning arrestor.

To create a large potential difference, a Van de Graaff generator is used.
**I Multiple choice questions**

1. Two identical point charges of magnitude \(-q\) are fixed as shown in the figure below. A third charge \(+q\) is placed midway between the two charges at the point P. Suppose this charge \(+q\) is displaced a small distance from the point P in the directions indicated by the arrows, in which direction(s) will \(+q\) be stable with respect to the displacement?

   - (a) A₁ and A₂
   - (b) B₁ and B₂
   - (c) both directions
   - (d) No stable

2. Which charge configuration produces a uniform electric field?
   - (a) point Charge
   - (b) infinite uniform line charge
   - (c) uniformly charged infinite plane
   - (d) uniformly charged spherical shell

3. What is the ratio of the charges \(\frac{q_1}{q_2}\) for the following electric field line pattern?

   - (a) \(\frac{1}{5}\)
   - (b) \(\frac{25}{11}\)
   - (c) 5
   - (d) \(\frac{11}{25}\)

4. An electric dipole is placed at an alignment angle of 30° with an electric field of \(2 \times 10^5 \text{ N C}^{-1}\). It experiences a torque equal to 8 N m. The charge on the dipole if the dipole length is 1 cm is

   - (a) 4 mC
   - (b) 8 mC
   - (c) 5 mC
   - (d) 7 mC

5. Four Gaussian surfaces are given below with charges inside each Gaussian surface. Rank the electric flux through each Gaussian surface in increasing order.

   - (a) D < C < B < A
   - (b) A < B = C < D
   - (c) C < A = B < D
   - (d) D > C > B > A

6. The total electric flux for the following closed surface which is kept inside water...
7. Two identical conducting balls having positive charges \( q_1 \) and \( q_2 \) are separated by a center to center distance \( r \). If they are made to touch each other and then separated to the same distance, the force between them will be (NSEP 04-05)
(a) less than before
(b) same as before
(c) more than before
(d) zero

8. Rank the electrostatic potential energies for the given system of charges in increasing order.

(a) \( \frac{80q}{\varepsilon_0} \)   (b) \( \frac{q}{40\varepsilon_0} \)
(c) \( \frac{q}{80\varepsilon_0} \)   (d) \( \frac{q}{160\varepsilon_0} \)

9. An electric field \( \vec{E} = 10x\hat{i} \) exists in a certain region of space. Then the potential difference \( V = V_o - V_A \), where \( V_o \) is the potential at the origin and \( V_A \) is the potential at \( x = 2 \text{ m} \) is:
(a) 10 J   (b) -20 J
(c) +20 J   (d) -10 J

10. A thin conducting spherical shell of radius \( R \) has a charge \( Q \) which is uniformly distributed on its surface. The correct plot for electrostatic potential due to this spherical shell is

11. Two points A and B are maintained at a potential of 7 V and -4 V respectively. The work done in moving 50 electrons from A to B is
(a) \( 8.80 \times 10^{-17} \text{ J} \)
(b) \( -8.80 \times 10^{-17} \text{ J} \)
(c) \( 4.40 \times 10^{-17} \text{ J} \)
(d) \( 5.80 \times 10^{-17} \text{ J} \)

12. If voltage applied on a capacitor is increased from \( V \) to \( 2V \), choose the correct conclusion.
(a) \( Q \) remains the same, \( C \) is doubled
(b) \( Q \) is doubled, \( C \) doubled
(c) \( C \) remains same, \( Q \) doubled
(d) Both \( Q \) and \( C \) remain same
13. A parallel plate capacitor stores a charge Q at a voltage V. Suppose the area of the parallel plate capacitor and the distance between the plates are each doubled then which is the quantity that will change?
   (a) Capacitance  
   (b) Charge  
   (c) Voltage  
   (d) Energy density

14. Three capacitors are connected in triangle as shown in the figure. The equivalent capacitance between the points A and C is

![Diagram of capacitors connected in triangle]

(a) 1µF  
(b) 2 µF  
(c) 3 µF  
(d) \( \frac{1}{4} \) µF

15. Two metallic spheres of radii 1 cm and 3 cm are given charges of -1 \( \times 10^{-2} \) C and 5 \( \times 10^{-2} \) C respectively. If these are connected by a conducting wire, the final charge on the bigger sphere is (AIIPMT -2012)

(a) 3 \( \times 10^{-2} \) C  
(b) 4 \( \times 10^{-2} \) C  
(c) 1 \( \times 10^{-2} \) C  
(d) 2 \( \times 10^{-2} \) C

**Answers**

1) b  
2) c  
3) d  
4) b  
5) a

6) b  
7) c  
8) a  
9) b  
10) b

11) a  
12) c  
13) d  
14) b  
15) a

**II Short Answer Questions**

1. What is meant by quantisation of charges?
2. Write down Coulomb’s law in vector form and mention what each term represents.
3. What are the differences between Coulomb force and gravitational force?
4. Write a short note on superposition principle.
5. Define ‘Electric field’.
6. What is mean by ‘Electric field lines’?
7. The electric field lines never intersect. Justify.
8. Define ‘Electric dipole’
9. What is the general definition of electric dipole moment?
10. Define ‘electrostatic potential”.
11. What is an equipotential surface?
12. What are the properties of an equipotential surface?
13. Give the relation between electric field and electric potential.
14. Define ‘electrostatic potential energy’. 
15. Define ‘electric flux’
16. What is meant by electrostatic energy density?
17. Write a short note on ‘electrostatic shielding’.
18. What is Polarisation?
19. What is dielectric strength?
20. Define 'capacitance'. Give its unit.
21. What is corona discharge?

III Long Answer questions
1. Discuss the basic properties of electric charges.
2. Explain in detail Coulomb's law and its various aspects.
3. Define 'Electric field' and discuss its various aspects.
4. How do we determine the electric field due to a continuous charge distribution? Explain.
5. Calculate the electric field due to a dipole on its axial line and equatorial plane.
6. Derive an expression for the torque experienced by a dipole due to a uniform electric field.
7. Derive an expression for electrostatic potential due to a point charge.
8. Derive an expression for electrostatic potential due to an electric dipole.
9. Obtain an expression for potential energy due to a collection of three point charges which are separated by finite distances.
10. Derive an expression for electrostatic potential energy of the dipole in a uniform electric field.
11. Obtain Gauss law from Coulomb's law.
12. Obtain the expression for electric field due to an infinitely long charged wire.
13. Obtain the expression for electric field due to an charged infinite plane sheet.
14. Obtain the expression for electric field due to an uniformly charged spherical shell.
15. Discuss the various properties of conductors in electrostatic equilibrium.
16. Explain the process of electrostatic induction.
17. Explain dielectrics in detail and how an electric field is induced inside a dielectric.
18. Obtain the expression for capacitance for a parallel plate capacitor.
19. Obtain the expression for energy stored in the parallel plate capacitor.
20. Explain in detail the effect of a dielectric placed in a parallel plate capacitor.
21. Derive the expression for resultant capacitance, when capacitors are connected in series and in parallel.
22. Explain in detail how charges are distributed in a conductor, and the principle behind the lightning conductor.
23. Explain in detail the construction and working of a Van de Graaff generator.

Exercises
1. When two objects are rubbed with each other, approximately a charge of 50 nC can be produced in each object. Calculate the number of electrons that must be transferred to produce this charge.
   Ans: $31.25 \times 10^{10}$ electrons
2. The total number of electrons in the human body is typically in the order of $10^{28}$. Suppose, due to some reason, you and your friend lost 1% of this number
of electrons. Calculate the electrostatic force between you and your friend separated at a distance of 1m. Compare this with your weight. Assume mass of each person is 60 kg and use point charge approximation.

Ans: \( F_e = 9 \times 10^6 \text{ N}, W = 588 \text{ N} \)

3. Five identical charges \( Q \) are placed equidistant on a semicircle as shown in the figure. Another point charge \( q \) is kept at the center of the circle of radius \( R \). Calculate the electrostatic force experienced by the charge \( q \).

\[
\overrightarrow{F} = \frac{1}{4\pi \varepsilon_0} \frac{qQ}{R^2}(1+\sqrt{2}) \overrightarrow{N}\hat{i}
\]

4. Suppose a charge \(+q\) on Earth’s surface and another \(+q\) charge is placed on the surface of the Moon. (a) Calculate the value of \( q \) required to balance the gravitational attraction between Earth and Moon (b) Suppose the distance between the Moon and Earth is halved, would the charge \( q \) change?

(Take \( m_e = 5.9 \times 10^{24} \text{ kg}, m_M = 7.9 \times 10^{22} \text{ kg} \))

Ans: (a) \( q \approx +5.64 \times 10^{15} \text{ C} \), (b) no change

5. Draw the free body diagram for the following charges as shown in the figure (a), (b) and (c).

6. Consider an electron travelling with a speed \( v_0 \) and entering into a uniform electric field \( \vec{E} \) which is perpendicular to \( \vec{v}_0 \) as shown in the Figure. Ignoring gravity, obtain the electron’s acceleration, velocity and position as functions of time.

\[
\vec{a} = -\frac{eE}{m} \hat{j}, \vec{v} = v_0 \hat{i} - \frac{eE}{m} t \hat{j}, \vec{r} = v_0 t \hat{i} - \frac{1}{2} \frac{eE}{m} t^2 \hat{j}
\]

7. A closed triangular box is kept in an electric field of magnitude \( E = 2 \times 10^3 \) N C\(^{-1}\) as shown in the figure.
Calculate the electric flux through the (a) vertical rectangular surface (b) slanted surface and (c) entire surface. Ans: (a) 15 Nm² C⁻¹ (b) 15 Nm² C⁻¹ (c) zero

8. The electrostatic potential is given as a function of x in figure (a) and (b). Calculate the corresponding electric fields in regions A, B, C and D. Plot the electric field as a function of x for the figure (b).

Ans: (a) $E_x = 15 \text{ Vm}^{-1}$ (region A), $E_x = -10 \text{ Vm}^{-1}$ (region C)
$E_x = 0$ (region B), $E_x = 30 \text{ Vm}^{-1}$ (region D)

9. A spark plug in a bike or a car is used to ignite the air-fuel mixture in the engine. It consists of two electrodes separated by a gap of around 0.6 mm gap as shown in the figure.

To create the spark, an electric field of magnitude $3 \times 10^6 \text{ Vm}^{-1}$ is required. (a) What potential difference must be applied to produce the spark? (b) If the gap is increased, does the potential difference increase, decrease or remains the same? (c) Find the potential difference if the gap is 1 mm.
Ans: (a) 1800 V, (b) increases (c) 3000 V

10. A point charge of $+10 \mu\text{C}$ is placed at a distance of 20 cm from another identical point charge of $+10 \mu\text{C}$. A point charge of $-2 \mu\text{C}$ is moved from point a to b as shown in the figure. Calculate the change in potential energy of the system? Interpret your result.

Ans: $\Delta U = -3.246 \text{ J}$, negative sign implies that to move the charge -2µC no external work is required. System spends its stored
energy to move the charge from point a to point b.

11. Calculate the resultant capacitances for each of the following combinations of capacitors.

(a) \( C_0 \)

(b) \( C_0 \)

(c) \( C_0 \)

(d) \( C_0 \)

(e) \( C_0 \)

Ans: (a) \( \frac{2}{3}C_0 \) (b) \( C_0 \) (c) \( 3C_0 \)

12. An electron and a proton are allowed to fall through the separation between the plates of a parallel plate capacitor of voltage 5 V and separation distance \( h = 1 \text{ mm} \) as shown in the figure.

(a) Calculate the time of flight for both electron and proton (b) Suppose if a neutron is allowed to fall, what is the time of flight? (c) Among the three, which one will reach the bottom first? (Take \( m_e = 1.6 \times 10^{-27} \text{ kg} \), \( m_p = 9.1 \times 10^{-31} \text{ kg} \) and \( g = 10 \text{ m s}^{-2} \))

Ans:

(a) \( t_e = \sqrt{\frac{2hm_e}{eE}} \approx 1.5\text{ns} \) (ignoring the gravity),

(b) \( t_p = \sqrt{\frac{2hm_p}{eE}} \approx 63\text{ns} \) (ignoring the gravity)

(b) \( t_n = \sqrt{\frac{2h}{g}} \approx 14.1\text{ms} \)

(c) electron will reach first

13. During a thunder storm, the movement of water molecules within the clouds creates friction, partially causing the bottom part of the clouds to become negatively charged. This implies that the bottom of the cloud and the ground act as a parallel plate capacitor. If the electric field between the cloud and ground exceeds the dielectric breakdown of the air \( (3 \times 10^6 \text{ V m}^{-1}) \), lightning will occur.
(a) If the bottom part of the cloud is 1000 m above the ground, determine the electric potential difference that exists between the cloud and ground.

(b) In a typical lightning phenomenon, around 25C of electrons are transferred from cloud to ground. How much electrostatic potential energy is transferred to the ground?

Ans: (a) \( V = 3 \times 10^9 \) V, (b) \( U = 75 \times 10^9 \) J

14. For the given capacitor configuration
(a) Find the charges on each capacitor
(b) potential difference across them
(c) energy stored in each capacitor

\[
\begin{align*}
Q_a &= 24 \ \mu\text{C}, & Q_b &= 18 \ \mu\text{C}, \\
Q_c &= 6 \ \mu\text{C}, & Q_d &= 24 \ \mu\text{C} \\
V_a &= 3 \text{V}, & V_b &= 3 \text{V}, \\
V_c &= 3 \text{V}, & V_d &= 3 \text{V}, \\
U_a &= 36 \ \mu\text{J}, & U_b &= 27 \ \mu\text{J}, \\
U_c &= 9 \ \mu\text{J}, & U_d &= 36 \ \mu\text{J}
\end{align*}
\]

15. Capacitors P and Q have identical cross sectional areas A and separation d. The space between the capacitors is filled with a dielectric of dielectric constant \( \varepsilon_r \) as shown in the figure. Calculate the capacitance of capacitors P and Q.

\[
C_p = \frac{\varepsilon_r A}{2d} \left( 1 + \varepsilon_r \right), \quad C_Q = \frac{2\varepsilon_r A}{d} \left( \frac{\varepsilon_r}{1 + \varepsilon_r} \right)
\]
In this activity you will be able to learn about capacitor and the factors affecting capacitance.

**STEPS:**

- Open the browser and type “phet.colorado.edu/en/simulation/legacy/capacitor-lab” in the address bar. Go to the tab ‘Dielectric’.
- Change the plate area, distance between the plate and dielectric. Identify what you would maximize or minimize to make a capacitor with the greatest capacitance.
- Explore the relationships between charge, voltage, and stored energy for a capacitor. Design a capacitor system to store the greatest energy.
- Charge the capacitor with 1.0 v using the battery. Disconnect the battery. Now insert a dielectric between the plates. Discuss how electric field changes in between the plates when dielectric is introduced.
- What is the effect of introducing a dielectric between plates? (Change dielectric materials)

**URL:**
https://phet.colorado.edu/en/simulation/legacy/capacitor-lab

* Pictures are indicative only.
* If browser requires, allow Flash Player or Java Script to load the page.
In this unit, the student is exposed to
- Flow of charges in a metallic conductor
- Ohm’s law, electrical resistance, V-I characteristics
- Carbon resistors and combination of resistors
- Kirchhoff’s laws - Wheatstone’s bridge and its applications
- Electric power and Electric energy
- Heating effect - Joule’s law – Experimental verification and applications
- Thermoelectric effects – Seebeck effect – Peltier effect – Thomson effect

In unit 1, we studied the properties of charges when it is at rest. In reality, the charges are always moving within the materials. For example, the electrons in a copper wire are never at rest and are continuously in random motion. Therefore it is important to analyse the behaviour of charges when it is at motion. The motion of charges is called ‘electric current’. Current electricity is the study of flow of electric charges. It owes its origin to Alessandro Volta (1745-1827), who invented the electric battery which produced the first steady flow of electric current. Modern world depends heavily on the use of electricity. It is used to operate machines, communication systems, electronic devices, home appliances etc., In this unit, we will study about the electric current, resistance and related phenomenon in materials.
2.1 ELECTRIC CURRENT

Matter is made up of atoms. Each atom consists of a positively charged nucleus with negatively charged electrons moving around the nucleus. Atoms in metals have one or more electrons which are loosely bound to the nucleus. These electrons are called free electrons and can be easily detached from the atoms. The substances which have an abundance of these free electrons are called conductors. These free electrons move at random throughout the conductor at a given temperature. In general due to this random motion, there is no net transfer of charges from one end of the conductor to other end and hence no current. When a potential difference is applied by the battery across the ends of the conductor, the free electrons drift towards the positive terminal of the battery, producing a net electric current. This is easily understandable from the analogy given in the Figure 2.1.

In the XI Volume 2, unit 6, we studied, that the mass move from higher gravitational potential to lower gravitational potential. Likewise, positive charge flows from higher electric potential to lower electric potential and negative charge flows from lower electric potential to higher electric potential. So battery or electric cell simply creates potential difference across the conductor.

![Figure 2.1 Water current and Electric current](image-url)
The electric current in a conductor is defined as the rate of flow of charges through a given cross-sectional area \( A \). It is shown in the Figure 2.2.

**Figure 2.2** Charges flow across the area \( A \)

If a net charge \( Q \) passes through any cross section of a conductor in time \( t \), then the current is defined as \( I = \frac{Q}{t} \). But charge flow is not always constant. Hence current can more generally be defined as

\[
I_{\text{avg}} = \frac{\Delta Q}{\Delta t}
\]

(2.1)

Where \( \Delta Q \) is the amount of charge that passes through the conductor at any cross section during the time interval \( \Delta t \). If the rate at which charge flows changes in time, the current also changes. The instantaneous current \( I \) is defined as the limit of the average current, as \( \Delta t \to 0 \)

\[
I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}
\]

(2.2)

The SI unit of current is the **ampere** (A)

\[
1 \text{A} = \frac{1 \text{C}}{1 \text{s}}
\]

That is, 1A of current is equivalent to 1 Coulomb of charge passing through a perpendicular cross section in 1 second. The electric current is a scalar quantity.

### Example 2.1

Compute the current in the wire if a charge of 120 C is flowing through a copper wire in 1 minute.

**Solution**

The current (rate of flow of charge) in the wire is

\[
I = \frac{Q}{t} = \frac{120}{60} = 2 \text{A}
\]

### 2.1.1 Conventional Current

In an electric circuit, arrow heads are used to indicate the direction of flow of current. By convention, this flow in the circuit should be from the positive terminal of the battery to the negative terminal. This current is called the conventional current or simply current and is in the direction in which a positive test charge would move. In typical circuits the charges that flow are actually electrons, from the negative terminal of the battery to the positive. As a result, the flow of electrons and the direction of conventional current points in opposite direction as shown in Figure 2.3. Mathematically, a transfer of positive charge...
unit 2 current electricity

is the same as a transfer of negative charge in the opposite direction.

Electric current is not only produced by batteries. In nature, lightning bolt produces enormous electric current in a short time. During lightning, very high potential difference is created between the clouds and ground so charges flow between the clouds and ground.

2.1.2 Drift velocity

In a conductor the charge carriers are free electrons. These electrons move freely through the conductor and collide repeatedly with the positive ions. If there is no electric field, the electrons move in random directions, so the directions of their velocities are also completely random direction. On an average, the number of electrons travelling in any direction will be equal to the number of electrons travelling in the opposite direction. As a result, there is no net flow of electrons in any direction and hence there will not be any current.

Suppose a potential difference is set across the conductor by connecting a battery, an electric field $\vec{E}$ is created in the conductor. This electric field exerts a force on the electrons, producing a current. The electric field accelerates the electrons, while ions scatter the electrons and change the direction of motion. Thus, we have zigzag paths of electrons. In addition to the zigzag motion due to the collisions, the electrons move slowly along the conductor in a direction opposite to that of $\vec{E}$ as shown in the Figure 2.4.

Ions

Any material is made up of neutral atoms with equal number of electrons and protons. If the outermost electrons leave the atoms, they become free electrons and are responsible for electric current. The atoms after losing their outer most electrons will have more positive charges and hence are called positive ions. These ions will not move freely within the material like the free electrons. Hence the positive ions will not give rise to current.

This velocity is called drift velocity $\vec{v}_d$. The drift velocity is the average velocity acquired by the electrons inside the conductor when
EXAMPLE 2.2

If an electric field of magnitude 570 N C\(^{-1}\), is applied in the copper wire, find the acceleration experienced by the electron.

**Solution:**

\[ E = 570 \text{ N C}\(^{-1}\), \quad e = 1.6 \times 10^{-19} \text{ C}, \quad m = 9.11 \times 10^{-31} \text{ kg and } a = ? \]

\[ F = ma = eE \]

\[ a = \frac{eE}{m} = \frac{570 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}} \]

\[ = 912 \times 10^{-19} \times 10^{31} \]

\[ = 1.001 \times 10^{14} \text{ m s}\(^{-2}\) \]

**Misconception**

(i) There is a common misconception that the battery is the source of electrons. It is not true. When a battery is connected across the given wire, the electrons in the closed circuit resulting the current. Battery sets the potential difference (electrical energy) due to which these electrons in the conducting wire flow in a particular direction. The resulting electrical energy is used by electric bulb, electric fan etc. Similarly the electricity board is supplying the electrical energy to our home.

(ii) We often use the phrases like 'charging the battery in my mobile' and 'my mobile phone battery has no charge' etc. These sentences are not correct.

The typical drift velocity of electrons in the wire is \(10^{-4} \text{ m s}\(^{-1}\). If an electron drifts with this speed, then the electrons leaving the battery will take hours to reach the light bulb. Then how electric bulbs glow as soon as we switch on the battery? When battery is switched on, the electrons begin to move away from the negative terminal of the battery and this electron exerts force on the nearby electrons. This process creates a propagating influence (electric field) that travels through the wire at the speed of light. In other words, the energy is transported from the battery to light bulb at the speed of light through propagating influence (electric field). Due to this reason, the light bulb glows as soon as the battery is switched on.
When we say ‘battery has no charge’, it means, that the battery has lost ability to provide energy or provide potential difference to the electrons in the circuit. When we say ‘mobile is charging’, it implies that the battery is receiving energy from AC power supply and not electrons.

### 2.1.3 Microscopic model of current

Consider a conductor with area of cross section \( A \) and an electric field \( \vec{E} \) applied from right to left. Suppose there are \( n \) electrons per unit volume in the conductor and assume that all the electrons move with the same drift velocity \( \vec{v}_d \) as shown in Figure 2.5.

The drift velocity of the electrons = \( \vec{v}_d \)

The electrons move through a distance \( dx \) within a small interval of \( dt \)

\[
\vec{v}_d = \frac{dx}{dt} \quad \text{and} \quad dx = \vec{v}_d \, dt \quad (2.7)
\]

Since \( A \) is the area of cross section of the conductor, the electrons available in the volume of length \( dx \) is

\[
= \text{volume} \times \text{number per unit volume}
\]

\[
= A \, dx \times n \quad (2.8)
\]

Substituting for \( dx \) from equation (2.7) in (2.8)

\[
= (A \, \vec{v}_d \, dt) \, n
\]

Total charge in volume element \( dQ = (\text{charge}) \times (\text{number of electrons in the volume element}) \)

\[
dQ = (e)(Av_d dt)n
\]

Hence the current \( I = \frac{dQ}{dt} = \frac{neAv_d dt}{dt} \)

\[
I = neAv_d. \quad (2.9)
\]

**Current density (J)**

The current density \( J \) is defined as the current per unit area of cross section of the conductor.

\[
J = \frac{I}{A}
\]

The S.I unit of current density is \( \frac{A}{m^2} \) (or) \( A \, m^{-2} \)

\[
J = \frac{neA v_d}{A} \quad (\text{from equation 2.9})
\]

\[
J = nev_d \quad (2.10)
\]

The above expression is valid only when the direction of the current is perpendicular to the area \( A \). In general, the current density is a vector quantity and it is given by

\[
\vec{J} = ne\vec{v}_d
\]

Substituting \( \vec{v}_d \) from equation (2.4)

\[
\vec{J} = -\frac{n \cdot e^2 \tau}{m} \vec{E} \quad (2.11)
\]

\[
\vec{J} = -\sigma \vec{E}
\]

But conventionally, we take the direction of (conventional) current density as the direction of electric field. So the above equation becomes

\[
\vec{J} = \sigma \vec{E} \quad (2.12)
\]

where \( \sigma = \frac{n e^2 \tau}{m} \) is called conductivity.

The equation 2.12 is called microscopic form of ohm’s law.
EXAMPLE 2.4

Determine the number of electrons flowing per second through a conductor, when a current of 32 A flows through it.

Solution

I = 32 A, t = 1 s

Charge of an electron, e = 1.6 \times 10^{-19} C

The number of electrons flowing per second, n =?

\[ n = \frac{It}{e} \]

\[ n = \frac{32 \times 1}{1.6 \times 10^{-19} C} \]

n = 2 \times 10^{20} electrons

2.2

OHM’S LAW

The ohm’s law can be derived from the equation \( J = \sigma E \). Consider a segment of wire of length \( l \) and cross sectional area \( A \) as shown in Figure 2.7.

EXAMPLE 2.3

A copper wire of cross-sectional area 0.5 mm² carries a current of 0.2 A. If the free electron density of copper is \( 8.4 \times 10^{28} \) m⁻³ then compute the drift velocity of free electrons.

Solution

The relation between drift velocity of electrons and current in a wire of cross-sectional area \( A \) is

\[ v_d = \frac{I}{neA} \]

\[ v_d = \frac{0.2}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 0.5 \times 10^{-6}} \]

\[ v_d = 0.03 \times 10^{-3} \text{ m s}^{-1} \]
When a potential difference \( V \) is applied across the wire, a net electric field is created in the wire which constitutes the current. For simplicity, we assume that the electric field is uniform in the entire length of the wire, the potential difference (voltage \( V \)) can be written as

\[
V = El
\]

As we know, the magnitude of current density

\[
J = \sigma E = \sigma \frac{V}{l}
\]  

(2.14)

But \( J = \frac{I}{A} \), so we write the equation (2.14) as

\[
\frac{I}{A} = \sigma \frac{V}{l}
\]

By rearranging the above equation, we get

\[
V = I \left( \frac{l}{\sigma A} \right)
\]  

(2.15)

The quantity \( \frac{l}{\sigma A} \) is called resistance of the conductor and it is denoted as \( R \). Note that the resistance is directly proportional to the length of the conductor and inversely proportional to area of cross section.

Therefore, the macroscopic form of ohm’s law can be stated as

\[
V = IR
\]  

(2.16)

From the above equation, the resistance is the ratio of potential difference across the given conductor to the current passing through the conductor.

\[
R = \frac{V}{I}
\]  

(2.17)

The SI unit of resistance is ohm (\( \Omega \)). From the equation (2.16), we infer that the graph between current versus voltage is straight line with a slope equal to the inverse of resistance \( R \) of the conductor. It is shown in the Figure 2.8 (a).

\[\text{Figure 2.8} \text{ Current against voltage for (a) a conductor which obey Ohm’s law and (b) for a non-ohmic device (Diode given in XII physics, unit 9 is an example of a non-ohmic device)}\]

Materials for which the current against voltage graph is a straight line through the origin, are said to obey Ohm’s law and their behaviour is said to be ohmic as shown in Figure 2.8(a). Materials or devices that do not follow Ohm’s law are said to be non-ohmic. These materials have more complex relationships between voltage and current. A plot of \( I \) against \( V \) for a non-ohmic material is non-linear and they do not have a constant resistance (Figure 2.8(b)).
A potential difference across 24 Ω resistor is 12 V. What is the current through the resistor?

**Solution**

\[ V = 12 \text{ V} \text{ and } R = 24 \Omega \]

Current, \( I = ? \)

From Ohm’s law, \[ I = \frac{V}{R} = \frac{12}{24} = 0.5 \text{ A} \]

2.2.1 Resistivity

In the previous section, we have seen that the resistance \( R \) of any conductor is given by

\[ R = \frac{l}{\sigma A} \quad (2.18) \]

where \( \sigma \) is called the conductivity of the material and it depends only on the type of the material used and not on its dimension.

The resistivity of a material is equal to the reciprocal of its conductivity.

\[ \rho = \frac{1}{\sigma} \quad (2.19) \]

Now we can rewrite equation (2.18) using equation (2.19)

\[ R = \rho \frac{l}{A} \quad (2.20) \]

The resistance of a material is directly proportional to the length of the conductor and inversely proportional to the area of cross section of the conductor. The proportionality constant \( \rho \) is called the resistivity of the material.

If \( l = 1 \text{ m} \) and \( A = 1 \text{ m}^2 \), then the resistance \( R = \rho \). In other words, the electrical resistivity of a material is defined as the resistance offered to current flow by a conductor of unit length having unit area of cross section. The SI unit of \( \rho \) is ohm-metre (Ω m). Based on the resistivity, materials are classified as conductors, insulators and semiconductors. The conductors have lowest resistivity, insulators have highest resistivity and semiconductors have resistivity greater than conductors but less than insulators. The typical resistivity values of some conductors, insulators and semiconductors are given in the Table 2.1

<table>
<thead>
<tr>
<th>Material</th>
<th>Resistivity, ( \rho ) (Ω m) at 20°C</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Insulators</strong></td>
<td></td>
</tr>
<tr>
<td>Pure Water</td>
<td>( 2.5 \times 10^5 )</td>
</tr>
<tr>
<td>Glass</td>
<td>( 10^{10} - 10^{14} )</td>
</tr>
<tr>
<td>Hard Rubber</td>
<td>( 10^{13} - 10^{16} )</td>
</tr>
<tr>
<td>NaCl</td>
<td>( - 10^{14} )</td>
</tr>
<tr>
<td>Fused Quartz</td>
<td>( - 10^{16} )</td>
</tr>
<tr>
<td><strong>Semiconductors</strong></td>
<td></td>
</tr>
<tr>
<td>Germanium</td>
<td>0.46</td>
</tr>
<tr>
<td>Silicon</td>
<td>640</td>
</tr>
<tr>
<td><strong>Conductors</strong></td>
<td></td>
</tr>
<tr>
<td>Silver</td>
<td>( 1.6 \times 10^{-8} )</td>
</tr>
<tr>
<td>Copper</td>
<td>( 17 \times 10^{-8} )</td>
</tr>
<tr>
<td>Aluminium</td>
<td>( 2.7 \times 10^{-8} )</td>
</tr>
<tr>
<td>Tungsten</td>
<td>( 5.6 \times 10^{-8} )</td>
</tr>
<tr>
<td>Iron</td>
<td>( 10 \times 10^{-8} )</td>
</tr>
</tbody>
</table>
EXAMPLE 2.6

The resistance of a wire is 20 Ω. What will be new resistance, if it is stretched uniformly 8 times its original length?

**Solution**

\[ R_1 = 20 \, \Omega, \quad R_2 = \, ? \]

Let the original length \( (l_1) \) be \( l \).

The new length, \( l_2 = 8l \) (i.e. \( l_2 = 8l \))

The original resistance, \( R_1 = \frac{\rho l}{A_1} \)

The new resistance \( R_2 = \frac{\rho l}{A_2} = \frac{\rho (8l)}{A_2} \)

Though the wire is stretched, its volume is unchanged.

Initial volume = Final volume

\[ A_1 l_1 = A_2 l_2 \quad \Rightarrow \quad A_1 l = A_2 (8l) \]

\[ \frac{A_1}{A_2} = 8 \]

By dividing equation \( R_2 \) by equation \( R_1 \), we get

\[ \frac{R_2}{R_1} = \frac{\rho l}{A_2} \times \frac{A_1}{\rho l} \]

\[ \frac{R_2}{R_1} = \frac{A_1}{A_2} \times 8 \]

Substituting the value of \( \frac{A_1}{A_2} \), we get

\[ \frac{R_2}{R_1} = 8 \times 8 = 64 \]

\[ R_2 = 64 \times 20 = 1280 \, \Omega \]

Hence, stretching the length of the wire has increased its resistance.

EXAMPLE 2.7

Consider a rectangular block of metal of height \( A \), width \( B \) and length \( C \) as shown in the figure.

If a potential difference of \( V \) is applied between the two faces A and B of the block (figure (a)), the current \( I_{AB} \) is observed. Find the current that flows if the same potential difference \( V \) is applied between the two faces B and C of the block (figure (b)). Give your answers in terms of \( I_{AB} \).

**Solution**

In the first case, the resistance of the block

\[ R_{AB} = \rho \frac{\text{length}}{\text{Area}} = \rho \frac{C}{AB} \]

The current \( I_{AB} = \frac{V}{R_{AB}} = \frac{V}{\rho} \frac{AB}{C} \)  \hspace{1cm} (1)

In the second case, the resistance of the block \( R_{BC} = \rho \frac{A}{BC} \)

The current \( I_{BC} = \frac{V}{R_{BC}} = \frac{V}{\rho} \frac{BC}{A} \) \hspace{1cm} (2)

To express \( I_{BC} \) in terms of \( I_{AB} \), we multiply and divide equation (2) by \( AC \), we get

\[ I_{BC} = \frac{V}{\rho} \frac{BC}{AC} \frac{AC}{AC} = \left( \frac{V}{\rho} \frac{AB}{A} \right) \frac{C^2}{A^2} \]

\[ I_{BC} = \frac{C^2}{A^2} \cdot I_{AB} \]

Since \( C > A \), the current \( I_{BC} > I_{AB} \)

The human body contains a large amount of water which has low resistance of around 200 Ω and the dry skin has high resistance of around 500 k Ω. But when the skin is wet, the resistance is reduced to around 1000 Ω. This is the reason, repairing the electrical connection with the wet skin is always dangerous.
2.2.2 Resistors in series and parallel

An electric circuit may contain a number of resistors which can be connected in different ways. For each type of circuit, we can calculate the equivalent resistance produced by a group of individual resistors.

**Resistors in series**

When two or more resistors are connected end to end, they are said to be in series. The resistors could be simple resistors or bulbs or heating elements or other devices. Figure 2.9 (a) shows three resistors \( R_1 \), \( R_2 \) and \( R_3 \) connected in series.

The amount of charge passing through resistor \( R_1 \) must also pass through resistors \( R_2 \) and \( R_3 \) since the charges cannot accumulate anywhere in the circuit. Due to this reason, the current \( I \) passing through all the three resistors is the same. According to Ohm’s law, if same current pass through different resistors of different values, then the potential difference across each resistor must be different. Let \( V_1 \), \( V_2 \) and \( V_3 \) be the potential difference (voltage) across each of the resistors \( R_1 \), \( R_2 \) and \( R_3 \) respectively, then we can write

\[
V_1 = IR_1, \quad V_2 = IR_2, \quad V_3 = IR_3.
\]

But the total voltage \( V \) is equal to the sum of voltages across each resistor.

\[
V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 \quad (2.21)
\]

\[
V = I (R_1 + R_2 + R_3) \quad (2.22)
\]

where \( R_3 \) is the equivalent resistance,

\[
R_3 = R_1 + R_2 + R_3 \quad (2.23)
\]

When several resistances are connected in series, the total or equivalent resistance is the sum of the individual resistances as shown in the Figure 2.9 (b).

**Note:** The value of equivalent resistance in series connection will be greater than each individual resistance.

**Example 2.8**

Calculate the equivalent resistance for the circuit which is connected to 24 V battery and also find the potential difference across 4 \( \Omega \) and 6 \( \Omega \) resistors in the circuit.
**Solution**

Since the resistors are connected in series, the effective resistance in the circuit
\[ R_{eq} = 4\,\Omega + 6\,\Omega = 10\,\Omega \]

The current in the circuit
\[ \frac{V}{R_{eq}} = \frac{24}{10} = 2.4\,A \]

Voltage across 4Ω resistor
\[ V_1 = IR_1 = 2.4\,A \times 4\,\Omega = 9.6\,V \]

Voltage across 6Ω resistor
\[ V_2 = IR_1 = 2.4\,A \times 6\,\Omega = 14.4\,V \]

**Resistors in parallel**

Resistors are in parallel when they are connected across the same potential difference as shown in Figure 2.10 (a).

In this case, the total current that leaves the battery is split into three separate paths. Let \( I_1, I_2, \) and \( I_3 \) be the current through the resistors \( R_1, R_2, \) and \( R_3 \) respectively. Due to the conservation of charge, total current in the circuit \( I \) is equal to sum of the currents through each of the three resistors.

\[ I = I_1 + I_2 + I_3 \]  
(2.24)

Since the voltage across each resistor is the same, applying Ohm’s law to each resistor, we have

\[ I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3} \]  
(2.25)

Substituting these values in equation (2.24), we get

\[ I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V \left[ \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right] \]

\[ I = \frac{V}{R_p} \]

\[ \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \]  
(2.26)

**Resistors in parallel**

Resistors are in parallel when they are connected across the same potential difference as shown in Figure 2.10 (a).

In this case, the total current that leaves the battery is split into three separate paths. Let \( I_1, I_2, \) and \( I_3 \) be the current through the resistors \( R_1, R_2, \) and \( R_3 \) respectively. Due to the conservation of charge, total current in the circuit \( I \) is equal to sum of the currents through each of the three resistors.

\[ I = I_1 + I_2 + I_3 \]  
(2.24)

Since the voltage across each resistor is the same, applying Ohm’s law to each resistor, we have

\[ I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3} \]  
(2.25)

Substituting these values in equation (2.24), we get

\[ I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V \left[ \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right] \]

\[ I = \frac{V}{R_p} \]

\[ \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \]  
(2.26)

**Figure 2.10** Resistors in parallel

Here \( R_p \) is the equivalent resistance of the parallel combination of the resistors. Thus, when a number of resistors are connected in parallel, the sum of the reciprocal of the values of resistance of the individual resistor is equal to the reciprocal of the effective resistance of the combination as shown in the Figure 2.10 (b)

Note: The value of equivalent resistance in parallel connection will be lesser than each individual resistance.

Household appliances are always connected in parallel so that even if one is switched off, the other devices could function properly.
EXAMPLE 2.9

Calculate the equivalent resistance in the following circuit and also find the current $I_1$, $I_1$, and $I_2$ in the given circuit.

\[
\begin{align*}
\text{Solution} \\
\text{Since the resistances are connected in parallel, therefore, the equivalent resistance in the circuit is} \\
\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4} + \frac{1}{6} \Rightarrow \frac{12}{R_p} = \frac{5}{12} \quad \text{or} \quad R_p = \frac{12}{5} \Omega \\
\text{The resistors are connected in parallel, the potential (voltage) across each resistor is the same.} \\
I_1 = \frac{V}{R_1} = \frac{24V}{6\Omega} = 6A \\
I_2 = \frac{V}{R_2} = \frac{24V}{6} = 4A \\
The current I is the total of the currents in the two branches. Then, \\
I = I_1 + I_2 = 6A + 4A = 10A
\end{align*}
\]

EXAMPLE 2.10

When two resistances connected in series and parallel their equivalent resistances are 15 $\Omega$ and $\frac{56}{15} \Omega$ respectively. Find the individual resistances.

\[
\begin{align*}
\text{Solution} \\
R_s = R_1 + R_2 = 15 \Omega \quad (1) \\
R_p = \frac{R_1R_2}{R_1 + R_2} = \frac{56}{15} \Omega \quad (2) \\
\text{From equation (1) substituting for } R_1 + R_2 \text{ in equation (2)} \\
\frac{R_1R_2}{15} = \frac{56}{15} \Omega \\
\therefore R_1R_2 = 56 \quad (3) \\
\text{Substituting for } R_2 \text{ in equation (1) from equation (3)} \\
R_1 + \frac{56}{R_1} = 15 \\
\text{Then, } \frac{R_1^2 + 56}{R_1} = 15 \\
R_1^2 + 56 = 15R_1 \\
R_1^2 - 15R_1 + 56 = 0 \\
The above equation can be solved using factorisation. \\
R_1^2 - 8R_1 - 8R_1 + 56 = 0 \\
R_1(R_1 - 8) - 7(R_1 - 8) = 0 \\
(R_1 - 8)(R_1 - 7) = 0 \\
\text{If } (R_1 = 8 \Omega) \\
\text{using in equation (1)} \\
8 + R_2 = 15 \\
R_2 = 15 - 8 = 7 \Omega, \\
R_2 = 7 \Omega \text{ i.e., (when } R_1 = 8 \Omega; R_2 = 7 \Omega) \\
\text{If } (R_1 = 7 \Omega) \\
\text{Substituting in equation (1)} \\
7 + R_2 = 15 \\
R_2 = 8 \Omega, \text{ i.e., (when } R_1 = 8 \Omega; R_2 = 7 \Omega)
\end{align*}
\]
**EXAMPLE 2.11**

Calculate the equivalent resistance between A and B in the given circuit.

![Circuit Diagram]

**Solution**

Parallel connection

Part 1

\[ \frac{1}{R_{p_1}} = \frac{1}{R_1} + \frac{1}{R_2} \]

\[ \frac{1}{R_{p_1}} = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} \]

\[ R_{p_1} = 1\Omega \]

Part II

\[ \frac{1}{R_{p_2}} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} \]

\[ \frac{1}{R_{p_2}} = \frac{1}{2} \]

\[ R_{p_2} = 2\Omega \]

Part III

\[ \frac{1}{R_{p_3}} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} \]

\[ \frac{1}{R_{p_3}} = \frac{1}{3} \]

\[ R_{p_3} = 3\Omega \]

\[ R = R_{p_1} + R_{p_2} + R_{p_3} \]

\[ R = 1 + 2 + 3 \]

\[ R = 6\Omega \]

The circuit became:

![Simplified Circuit Diagram]

**EXAMPLE 2.12**

Five resistors are connected in the configuration as shown in the figure. Calculate the equivalent resistance between the points a and b.

![Circuit Diagram]

**Solution**

Case (a)

To find the equivalent resistance between the points a and b, we assume that current is entering the junction a. Since all the resistances in the outside loop are the same (1Ω), the current in the branches ac and ad must be equal. So the electric potential at the point c and d is the same hence no current flows into 5 Ω resistance. It implies that the 5 Ω has no role in determining the equivalent resistance and it can be removed. So the circuit is simplified as shown in the figure.
shows the tolerance of the resistor at 10% or 5% as shown in the Figure 2.12. If there is no fourth ring, the tolerance is 20%.

For the resistor shown in Figure 2.12, the first digit = 5 (green), the second digit = 6 (blue), decimal multiplier = $10^3$ (orange) and tolerance = 5% (gold). The value of resistance = $56 \times 10^3 \ \Omega$ or 56 k$\Omega$ with the tolerance value 5%.

### 2.2.3 Color code for Carbon resistors

Carbon resistors consists of a ceramic core, on which a thin layer of crystalline carbon is deposited as shown in Figure 2.11. These resistors are inexpensive, stable and compact in size. Color rings are used to indicate the value of the resistance according to the rules given in the Table 2.2.

Three coloured rings are used to indicate the value of a resistor: the first two rings are significant figures of resistances, the third ring indicates the decimal multiplier after them. The fourth color, silver or gold, shows the tolerance of the resistor at 10% or 5% as shown in the Figure 2.12. If there is no fourth ring, the tolerance is 20%.

For the resistor shown in Figure 2.12, the first digit = 5 (green), the second digit = 6 (blue), decimal multiplier = $10^3$ (orange) and tolerance = 5% (gold). The value of resistance = $56 \times 10^3 \ \Omega$ or 56 k$\Omega$ with the tolerance value 5%.
increase in temperature according to the expression,

$$\rho_T = \rho_o [1 + \alpha (T - T_o)]$$  \hspace{1cm} (2.27)

where $\rho_T$ is the resistivity of a conductor at $T^\circ C$, $\rho_o$ is the resistivity of the conductor at some reference temperature $T_o$ (usually at 20°C) and $\alpha$ is the temperature coefficient of resistivity. It is defined as the ratio of increase in resistivity per degree rise in temperature to its resistivity at $T_o$.

From the equation (2.27), we can write

$$\rho_T - \rho_o = \alpha \rho_o (T - T_o)$$

\[\therefore \alpha = \frac{\rho_T - \rho_o}{\rho_o (T - T_o)} = \frac{\Delta \rho}{\rho_o \Delta T}\]

where $\Delta \rho = \rho_T - \rho_o$ is change in resistivity for a change in temperature $\Delta T = T - T_o$. Its unit is per °C.

$\alpha$ of conductors

For conductors $\alpha$ is positive. If the temperature of a conductor increases, the average kinetic energy of electrons in the conductor increases. This results in more frequent collisions and hence the resistivity increases. The graph of the equation (2.27) is shown in Figure 2.13

Even though, the resistivity of conductors like metals varies linearly for wide range of temperatures, there also exists a non-linear region at very low temperatures. The resistivity approaches some finite value as the temperature approaches absolute zero as shown in Figure 2.13(b).

As the resistance is directly proportional to resistivity of the material, we can also write the resistance of a conductor at temperature T °C as

$$R_T = R_o [1 + \alpha (T - T_o)]$$  \hspace{1cm} (2.28)
Figure 2.13 (a) Temperature dependence of resistivity for a conductor (b) Non linear region at low temperature

The temperature coefficient can be also be obtained from the equation (2.28),

\[
R_T - R_0 = \alpha R_0 (T - T_0)
\]

\[\therefore \alpha = \frac{R_T - R_0}{R_0 (T - T_0)} = \frac{1}{R_0} \frac{\Delta R}{\Delta T}
\]

\[\alpha = \frac{1}{R_0} \frac{\Delta R}{\Delta T} \quad (2.29)
\]

where \(\Delta R = R_T - R_0\) is change in resistance during the change in temperature \(\Delta T = T - T_0\).

\(\alpha\) of semiconductors

For semiconductors, the resistivity decreases with increase in temperature. As the temperature increases, more electrons will be liberated from their atoms (Refer unit 9 for conduction in semiconductors). Hence the current increases and therefore the resistivity decreases as shown in Figure 2.14. A semiconductor with a negative temperature coefficient of resistance is called a thermistor.

The typical values of temperature coefficients of various materials are given in table 2.3.

<table>
<thead>
<tr>
<th>Color</th>
<th>Temperature Coefficient (\alpha) [(\degree C^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>3.8 \times 10^{-3}</td>
</tr>
<tr>
<td>Copper</td>
<td>3.9 \times 10^{-3}</td>
</tr>
<tr>
<td>Gold</td>
<td>3.4 \times 10^{-3}</td>
</tr>
<tr>
<td>Aluminum</td>
<td>3.9 \times 10^{-3}</td>
</tr>
<tr>
<td>Tungsten</td>
<td>4.5 \times 10^{-3}</td>
</tr>
<tr>
<td>Iron</td>
<td>5.0 \times 10^{-3}</td>
</tr>
<tr>
<td>Platinum</td>
<td>3.92 \times 10^{-3}</td>
</tr>
<tr>
<td>Lead</td>
<td>3.9 \times 10^{-3}</td>
</tr>
<tr>
<td>Nichrome</td>
<td>0.4 \times 10^{-3}</td>
</tr>
<tr>
<td>Carbon</td>
<td>-0.5 \times 10^{-3}</td>
</tr>
<tr>
<td>Germanium</td>
<td>-48 \times 10^{-3}</td>
</tr>
<tr>
<td>Silicon</td>
<td>-75 \times 10^{-3}</td>
</tr>
</tbody>
</table>

We can understand the temperature dependence of resistivity in the following way. In section 2.1.3, we have shown that the electrical conductivity, \(\sigma = \frac{ne^2}{m}\). As the resistivity is inverse of \(\sigma\), it can be written as,

\[\rho = \frac{m}{ne^2\tau} \quad (2.30)
\]
The resistivity of materials is
i) inversely proportional to the number density \((n)\) of the electrons
ii) inversely proportional to the average time between the collisions \((\tau)\).

In metals, if the temperature increases, the average time between the collision \((\tau)\) decreases and \(n\) is independent of temperature. In semiconductors when temperature increases, \(n\) increases and \(\tau\) decreases, but increase in \(n\) is dominant than decreasing \(\tau\), so that overall resistivity decreases.

**EXAMPLE 2.14**

Resistance of a material at 10°C and 40°C are 45 Ω and 85 Ω respectively. Find its temperature co-efficient of resistance.

**Solution**

\(T_0 = 10\, ^\circ\text{C},\ T = 40\, ^\circ\text{C},\ R_0 = 45\, \Omega,\ R = 85\, \Omega\)

\[
\alpha = \frac{1}{R} \frac{\Delta R}{\Delta T} = \frac{1}{45} \frac{85 - 45}{40 - 10} = \frac{1}{45} \left( \frac{40}{30} \right) = 0.0296\ \text{per}^\circ\text{C}
\]

**ENERGY AND POWER IN ELECTRICAL CIRCUITS**

When a battery is connected between the ends of a conductor, a current is established. The battery is transporting energy to the device which is connected in the circuit. Consider a circuit in which a battery of voltage \(V\) is connected to the resistor as shown in Figure 2.15.

**EXAMPLE 2.13**

If the resistance of coil is 3 Ω at 20 °C and \(\alpha = 0.004/\text{oC}\) then determine its resistance at 100 °C.

**Solution**

\(R_0 = 3\, \Omega,\ T = 100^\circ\text{C},\ T_0 = 20^\circ\text{C}\)

\(\alpha = 0.004/\text{oC},\ R_T = ?\)

\[
R_T = R_0(1 + \alpha(T - T_0))
\]

\[
R_{100} = 3(1 + 0.004 \times 80)
\]

\[
R_{100} = 3(1 + 0.32)
\]

\[
R_{100} = 3(1.32)
\]

\[
R_{100} = 3.96\, \Omega
\]

**UNIT 2 CURRENT ELECTRICITY**
Usually these voltage rating refers AC RMS voltages. For a given bulb, if the voltage drop across the bulb is greater than voltage rating, the bulb will fuse.

Using Ohm’s law, power delivered to the resistance $R$ is expressed in other forms

$$P = IV = I(\frac{V}{R}) = F R$$ (2.33)

$$P = IV = \frac{V}{R}V = \frac{V^2}{R}$$ (2.34)

The electrical power produced (dissipated) by a resistor is $FR$. It depends on the square of the current. Hence, if current is doubled, the power will increase by four times. Similar explanation holds true for voltage also.

The total energy used by any device is obtained by multiplying the power and duration of the time when it is ON. If the power is in watts and the time is in seconds, the energy will be in joules. In practice, electrical energy is measured in kilowatt hour (kWh). 1 kWh is known as 1 unit of electrical energy.

$$(1 \text{ kWh} = 1000 \text{ Wh} = (1000 \text{ W})(3600\text{ s}) = 3.6 \times 10^6 \text{ J})$$

The Tamilnadu Electricity Board is charging for the amount of energy you use and not for the power. A current of 1A flowing through a potential difference of 1V produces a power of 1W.
EXAMPLE 2.15
A battery of voltage V is connected to 30 W bulb and 60 W bulb as shown in the figure. (a) Identify brightest bulb (b) which bulb has greater resistance? (c) Suppose the two bulbs are connected in series, which bulb will glow brighter?

Solution
(a) The power delivered by the battery P = VI. Since the bulbs are connected in parallel, the voltage drop across each bulb is the same. If the voltage is kept fixed, then the power is directly proportional to current (P ∝ I). So 60 W bulb draws twice as much as current as 30 W and it will glow brighter then others.

(b) To calculate the resistance of the bulbs, we use the relation \( P = \frac{V^2}{R} \). In both the bulbs, the voltage drop is the same, so the power is inversely proportional to the resistance or resistance is inversely proportional to the power \( R \propto \frac{1}{P} \). It implies that, the 30W has twice as much as resistance as 60 W bulb.

(c) When these two bulbs are connected in series, the current passing through each bulb is the same. It is equivalent to two resistors connected in series. The bulb which has higher resistance has higher voltage drop. So 30W bulb will glow brighter than 60W bulb. So the higher power rating does not always imply more brightness and it depends whether bulbs are connected in series or parallel.

EXAMPLE 2.16
Two electric bulbs marked 20 W – 220 V and 100 W – 220 V are connected in series to 440 V supply. Which bulb will be fused?

Solution
To check which bulb will be fused, the voltage drop across each bulb has to be calculated.

The resistance of a bulb, \( R = \frac{V^2}{P} = \frac{(\text{Rated voltage})^2}{\text{Rated power}} \)

For 20W-220V bulb,
\[ R_1 = \frac{(220)^2}{20} \Omega = 2420 \Omega \]

For 100W-220V bulb,
\[ R_2 = \frac{(220)^2}{100} \Omega = 484 \Omega \]

Both the bulbs are connected in series. So the current which passes through both the bulbs are same. The current that passes through the circuit, \( I = \frac{V}{R_{\text{total}}} \).
is connected to a circuit, electrons flow from the negative terminal to the positive terminal through the circuit. By using chemical reactions, a battery produces potential difference across its terminals. This potential difference provides the energy to move the electrons through the circuit. Commercially available electric cells and batteries are shown in Figure 2.18.

If we connect copper and zinc rod in a lemon, it acts as an electric cell. The citric acid in the lemon acts as an electrolyte. The potential can be measured using a multimeter.

Several electric cells connected together form a battery. When a cell or battery

\[
R_{tot} = (R_1 + R_2)
\]

\[
R_{tot} = (484 + 2420) \Omega = 2904 \Omega
\]

\[
I = \frac{440V}{2904 \Omega} \approx 0.151 A
\]

The voltage drop across the 20W bulb is

\[
V_1 = IR_1 = \frac{440}{2904} \times 2420 \approx 366.6 V
\]

The voltage drop across the 100W bulb is

\[
V_2 = IR_2 = \frac{440}{2904} \times 484 \approx 73.3 V
\]

The 20 W bulb will be fused because its voltage rating is only 220 V and 366.6 V is dropped across it.

2.4 Electric Cells and Batteries

An electric cell converts chemical energy into electrical energy to produce electricity. It contains two electrodes immersed in an electrolyte as shown in Figure 2.17.

Several electric cells connected together form a battery. When a cell or battery
2.4.1 Electromotive force and internal resistance

A battery or cell is called a source of electromotive force (emf). The term ‘electromotive force’ is a misnomer since it does not really refer to a force but describes a potential difference in volts. The emf of a battery or cell is the voltage provided by the battery when no current flows in the external circuit. It is shown in Figure 2.19.

![Figure 2.19 Measuring the emf of a cell](image)

Electromotive force determines the amount of work a battery or cell does to move a certain amount of charge around the circuit. It is denoted by the symbol $\xi$ and to be pronounced as ‘xi’. An ideal battery has zero internal resistance and the potential difference (terminal voltage) across the battery equals to its emf. But a real battery is made of electrodes and electrolyte, there is resistance to the flow of charges within the battery. This resistance is called internal resistance $r$. For a real battery, the terminal voltage is not equal to the emf of the battery. A freshly prepared cell has low internal resistance and it increases with ageing.

2.4.2 Determination of internal resistance

The circuit connections are made as shown in Figure 2.20.

![Figure 2.20 Internal resistance of the cell](image)

The emf of cell $\xi$ is measured by connecting a high resistance voltmeter across it without connecting the external resistance $R$ as shown in Figure 2.20(a). Since the voltmeter draws very little current for deflection, the circuit may be considered as open. Hence the voltmeter reading gives the emf of the cell. Then, external resistance $R$ is included in the circuit and current $I$ is established in the circuit. The potential difference across $R$ is equal to the potential difference across the cell ($V$) as shown in Figure 2.20(b).

The potential drop across the resistor $R$ is

$$V = IR \quad (2.35)$$

Due to internal resistance $r$ of the cell, the voltmeter reads a value $V$, which is less than the emf of cell $\xi$. It is because, certain
amount of voltage (Ir) has dropped across the internal resistance \( r \).

Then \( V = \xi - Ir \)

\[
Ir = \xi - V
\]  \hspace{1cm} (2.36)

Dividing equation (2.36) by equation (2.35), we get

\[
\frac{Ir}{IR} = \frac{\xi - V}{V}
\]

\[
r = \left[ \frac{\xi - V}{V} \right] R
\]  \hspace{1cm} (2.37)

Since \( \xi \), \( V \) and \( R \) are known, internal resistance \( r \) can be determined. We can also find the total current that flows in the circuit.

Due to this internal resistance, the power delivered to the circuit is not equal to power rating mentioned in the battery. For a battery of emf \( \xi \), with an internal resistance \( r \), the power delivered to the circuit of resistance \( R \) is given by

\[
P = I\xi = I(V + Ir) \text{ (from equation 2.36)}
\]

Here \( V \) is the voltage drop across the resistance \( R \) and it is equal to IR.

Therefore, \( P = I (IR + Ir) \)

\[
P = F R + F r
\]  \hspace{1cm} (2.38)

Here \( F \) \( r \) is the power delivered to the internal resistance and \( F \) \( R \) is the power delivered to the electrical device (here it is the resistance \( R \)). For a good battery, the internal resistance \( r \) is very small, then \( F \) \( r \) \(<\) \( F \) \( R \) and almost entire power is delivered to the resistance.

**Example 2.17**

A battery has an emf of 12 V and connected to a resistor of 3 \( \Omega \). The current in the circuit is 3.93 A. Calculate (a) terminal voltage and the internal resistance of the battery (b) power delivered by the battery and power delivered to the resistor

**Solution**

The given values \( I = 3.93 \) A, \( \xi = 12 \) V, \( R = 3 \) \( \Omega \)

(a) The terminal voltage of the battery is equal to voltage drop across the resistor

\[
V = IR = 3.93 \times 3 = 11.79 \text{ V}
\]

The internal resistance of the battery,

\[
r = \left[ \frac{\xi - V}{V} \right] R = \left[ \frac{12 - 11.79}{11.79} \right] \times 3 = 0.05 \text{ \( \Omega \)}
\]

(b) The power delivered by the battery \( P = I\xi = 3.93 \times 12 = 47.1 \text{ W} \)

The power delivered to the resistor = \( I^2 \) \( R \) = 46.3 \text{ W}

The remaining power = \( (47.1 - 46.3) \) \( P = 0.772 \) W is delivered to the internal resistance and cannot be used to do useful work. (it is equal to \( F^2 r \)).

### 2.4.3 Cells in series

Several cells can be connected to form a battery. In series connection, the negative terminal of one cell is connected to the positive terminal of the second cell, the negative terminal of second cell is connected to the positive terminal of the third cell and so on. The free positive terminal of the first cell and the free negative terminal of the last cell become the terminals of the battery.

Suppose \( n \) cells, each of emf \( \xi \) volts and internal resistance \( r \) ohms are connected in series with an external resistance \( R \) as shown in Figure 2.21
Thus series connection of cells is advantageous only when the effective internal resistance of the cells is negligibly small compared with $R$.

**Activity**

Construct lemon cells in series and observe the potential of this combination.

**EXAMPLE 2.18**

From the given circuit,

$$I = \frac{\text{total emf}}{\text{total resistance}} = \frac{n\xi}{nr + R} \quad (2.39)$$

Case (a) If $r << R$, then,

$$I = \frac{n\xi}{R} \approx nI_1 \quad (2.40)$$

where, $I_1$ is the current due to a single cell

$$I_1 = \frac{\xi}{R}$$

Thus, if $r$ is negligible when compared to $R$ the current supplied by the battery is $n$ times that supplied by a single cell.

Case (b) If $r >> R$, $I = \frac{n\xi}{nr} \approx \frac{\xi}{r} \quad (2.41)$

It is the current due to a single cell. That is, current due to the whole battery is the same as that due to a single cell and hence there is no advantage in connecting several cells.
ii) Equivalent internal resistance \( r_{eq} = nr \)
\( = 4 \times 0.1 = 0.4 \, \Omega \)

iii) Total current \( I = \frac{n \xi}{R + nr} \)
\( = \frac{4 \times 9}{10 + (4 \times 0.1)} \)
\( = \frac{4 \times 9}{10 + 0.4} = \frac{36}{10.4} \)
\( I = 3.46 \, \text{A} \)

iv) Potential difference across external resistance \( V = IR = 3.46 \times 10 = 34.6 \, \text{V} \). The remaining 1.4 V is dropped across the internal resistance of cells.

v) Potential difference across each cell
\( V = \frac{34.6}{4} = 8.65 \, \text{V} \)

2.4.4 Cells in parallel

In parallel connection all the positive terminals of the cells are connected to one point and all the negative terminals to a second point. These two points form the positive and negative terminals of the battery.

Let \( n \) cells be connected in parallel between the points A and B and a resistance \( R \) is connected between the points A and B as shown in Figure 2.22. Let \( \xi \) be the emf and \( r \) the internal resistance of each cell.

The equivalent internal resistance of the battery is
\[ \frac{1}{r_{eq}} = \frac{1}{r} + \frac{1}{r} + \ldots + \frac{1}{r} \text{ (terms) } = \frac{n}{r}. \]
So
\[ r_{eq} = \frac{r}{n} \]
and the total resistance in the circuit
\( = R + \frac{r}{n}. \) The total emf is the potential difference between the points A and B, which is equal to \( \xi. \) The current in the circuit is given by
\[ I = \frac{\xi}{r_{eq} + R} \]

**Figure 2.22 Cells in parallel**

\[ I = \frac{n \xi}{r + nR} \quad (2.42) \]

**Case (a)** If \( r \gg R, \) \( I = \frac{n \xi}{r} = nI_i \quad (2.43) \)

where \( I_i \) is the current due to a single cell and is equal to \( \frac{\xi}{r} \) when \( R \) is negligible. Thus, the current through the external resistance due to the whole battery is \( n \) times the current due to a single cell.

**Case (b)** If \( r \ll R, \) \( I = \frac{\xi}{R} \quad (2.44) \)

When the car engine is started with headlights turned on, they sometimes become dim. This is due to the internal resistance of the car battery.
Ohm’s law is useful only for simple circuits. For more complex circuits, Kirchhoff’s rules can be used to find current and voltage. There are two generalized rules: i) Kirchhoff’s current rule ii) Kirchhoff’s voltage rule.

### 2.5.1 Kirchhoff’s first rule
(Current rule or Junction rule)

It states that the algebraic sum of the currents at any junction of a circuit is zero. It is a statement of conservation of electric charge. All charges that enter a given junction in a circuit must leave that junction since charge cannot build up or disappear at a junction. Current entering the junction is taken as positive and current leaving the junction is taken as negative.

From Figure 2.23 Kirchhoff’s current rule

Applying this law to the junction A in Figure 2.23
The product of current and resistance is taken as positive when the direction of the current is followed. Suppose if the direction of current is opposite to the direction of the loop, then product of current and voltage across the resistor is negative. It is shown in Figure 2.24 (a) and (b). The emf is considered positive when proceeding from the negative to the positive terminal of the cell. It is shown in Figure 2.24 (c) and (d).

\[
\begin{align*}
I_1 + I_2 - I_3 - I_4 - I_5 &= 0 \\
(I_1 + I_2) &= (I_3 + I_4 + I_5)
\end{align*}
\]

**Example 2.20**

From the given circuit find the value of I.

**Solution**

Applying Kirchhoff’s rule to the point P in the circuit,

The arrows pointing towards P are positive and away from P are negative.

Therefore, \(0.2A - 0.4A + 0.6A - 0.5A + 0.7A - I = 0\)
\(1.5A - 0.9A - I = 0\)
\(0.6A - I = 0\)
\(I = 0.6 \text{ A}\)

**2.5.2 Kirchhoff’s Second rule (Voltage rule or Loop rule)**

It states that in a closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf included in the circuit. This rule follows from the law of conservation of energy for an isolated system (The energy supplied by the emf sources is equal to the sum of the energy delivered to all resistors).

Kirchhoff voltage rule has to be applied only when all currents in the circuit reach a steady state condition (the current in various branches are constant).

**Example 2.21**

The following figure shows a complex network of conductors which can be divided into two closed loops like ACE and ABC. Apply Kirchhoff’s voltage rule.
We can denote the current that flows from 9V battery as $I_1$ and it splits into $I_2$ and $I_1 - I_2$ in the junction according Kirchoff’s current rule (KCR). It is shown below.

Now consider the loop EFCBE and apply KVR, we get

$$1I_2 + 3I_1 + 2I_1 = 9$$

$$5I_1 + I_2 = 9 \quad (1)$$

Applying KVR to the loop EADFE, we get

$$3(I_1 - I_2) - 1I_2 = 6$$

$$3I_1 - 4I_2 = 6 \quad (2)$$

Solving equation (1) and (2), we get

$$I_1 = 1.83 \text{ A} \text{ and } I_2 = -0.13 \text{ A}$$

It implies that the current in the 1 ohm resistor flows from F to E.

### Example 2.22

Calculate the current that flows in the 1 Ω resistor in the following circuit.

**Solution**

Thus applying Kirchoff’s second law to the closed loop EACE

$$I_1R_1 + I_2R_2 + I_3R_3 = \xi$$

and for the closed loop ABCA

$$I_4R_4 + I_5R_5 - I_2R_2 = 0$$

**2.5.3 Wheatstone’s bridge**

An important application of Kirchhoff’s rules is the Wheatstone’s bridge. It is used to compare resistances and also helps in determining the unknown resistance in electrical network. The bridge consists of four resistances P, Q, R and S connected as shown in Figure 2.25. A galvanometer G is connected between the points B and D. The battery is connected between the points B and D. The current through the galvanometer is $I_G$ and its resistance is G.

Applying Kirchhoff’s current rule to junction B

$$I_1 - I_G - I_3 = 0 \quad (2.45)$$

Applying Kirchhoff’s current rule to junction D,

$$I_2 + I_G - I_4 = 0 \quad (2.46)$$
Dividing equation (2.52) by equation (2.51), we get
\[
\frac{P+Q}{P} = \frac{R+S}{R} \\
1 + \frac{Q}{P} = 1 + \frac{S}{R} \\
\frac{Q}{P} = \frac{S}{R} \\
\frac{P}{Q} = \frac{R}{S}
\]

This is the bridge balance condition. Only under this condition, galvanometer shows null deflection. Suppose we know the values of two adjacent resistances, the other two resistances can be compared. If three of the resistances are known, the value of unknown resistance (fourth one) can be determined.

A galvanometer is an instrument used for detecting and measuring even very small electric currents. It is extensively useful to compare the potential difference between various parts of the circuit.

**EXAMPLE 2.23**

In a Wheatstone's bridge \( P = 100 \, \Omega \), \( Q = 1000 \, \Omega \) and \( R = 40 \, \Omega \). If the galvanometer shows zero deflection, determine the value of \( S \).

**UNIT 2 CURRENT ELECTRICITY**
**2.5.4 Meter bridge**

The meter bridge is another form of Wheatstone's bridge. It consists of a uniform manganin wire AB of one meter length. This wire is stretched along a meter scale on a wooden board between two copper strips C and D. Between these two copper strips another copper strip E is mounted to enclose two gaps $G_1$ and $G_2$ as shown in Figure 2.26. An unknown resistance $P$ is connected in $G_1$ and a standard resistance $Q$ is connected in $G_2$. A jockey (conducting wire) is connected to the terminal E on the central copper strip through a galvanometer (G) and a high resistance (HR). The exact position of jockey on the wire can be read on the scale. A Lechlanche cell and a key (K) are connected across the ends of the bridge wire.

![Figure 2.26 Meter bridge](image)

**Solution**

\[
\frac{P}{Q} = \frac{R}{S}
\]

\[
P = 500 \Omega, Q = 800 \Omega, R = x + 400, S = 1000 \Omega
\]

**Solution**

\[
\frac{P}{Q} = \frac{R}{S}
\]

\[
\frac{500}{800} = \frac{x + 400}{1000}
\]

\[
x + 400 = \frac{500 \times 1000}{800}
\]

\[
x + 400 = 625 \times 1000
\]

\[
x + 400 = 625
\]

\[
x = 625 - 400
\]

\[
x = 225 \Omega
\]
where $R'$ is the resistance per unit length of wire

\[
\frac{P}{Q} = \frac{AJ}{JB} = \frac{l_1}{l_2} \tag{2.55}
\]

\[
P = Q \frac{l_1}{l_2} \tag{2.56}
\]

The bridge wire is soldered at the ends of the copper strips. Due to imperfect contact, some resistance might be introduced at the contact. These are called end resistances. This error can be eliminated, if another set of readings are taken with $P$ and $Q$ interchanged and the average value of $P$ is found.

To find the specific resistance of the material of the wire in the coil $P$, the radius $r$ and length $l$ of the wire is measured. The specific resistance or resistivity $\rho$ can be calculated using the relation

\[
\text{Resistance} = \rho \frac{l}{A}
\]

By rearranging the above equation, we get

\[
\rho = \text{Resistance} \times \frac{A}{l} \tag{2.57}
\]

If $P$ is the unknown resistance equation (2.57) becomes,

\[
\rho = \frac{P\pi r^2}{l}
\]

**EXAMPLE 2.25**

In a meter bridge with a standard resistance of 15 $\Omega$ in the right gap, the ratio of balancing length is 3:2. Find the value of the other resistance.

**Solution**

\[
Q = 15 \Omega, \quad \frac{l_1}{l_2} = 3:2
\]

\[
\frac{l_1}{l_2} = \frac{3}{2}
\]

\[
P = \frac{Q}{A} \frac{l_1}{l_2}
\]

\[
P = 15 \times \frac{3}{2} = 22.5 \Omega
\]

**2.5.5 Potentiometer**

Potentiometer is used for the accurate measurement of potential differences, current and resistances. It consists of ten meter long uniform wire of manganin or constantan stretched in parallel rows each of 1 meter length, on a wooden board. The two free ends A and B are brought to the same side and fixed to copper strips with binding screws. A meter scale is fixed parallel to the wire. A jockey is provided for making contact.

The principle of the potentiometer is illustrated in Figure 2.27. A steady current is maintained across the wire CD by a battery.
**UNIT 2 CURRENT ELECTRICITY**

The battery, key and the potentiometer wire are connected in series forms the primary circuit. The positive terminal of a primary cell of emf $\xi$ is connected to the point C and negative terminal is connected to the jockey through a galvanometer G and a high resistance HR. This forms the secondary circuit.

![Figure 2.27 Potentiometer](image)

Let contact be made at any point J on the wire by jockey. If the potential difference across CJ is equal to the emf of the cell $\xi$ then no current will flow through the galvanometer and it will show zero deflection. CJ is the balancing length $l$. The potential difference across CJ is equal to $Irl$ where $I$ is the current flowing through the wire and $r$ is the resistance per unit length of the wire.

Hence $\xi = Irl$ (2.58)

Since $I$ and $r$ are constants, $\xi \propto l$. The emf of the cell is directly proportional to the balancing length.

### 2.5.6 Comparison of emf of two cells with a potentiometer

To compare the emf of two cells, the circuit connections are made as shown in Figure 2.28. Potentiometer wire CD is connected to a battery $Bt$ and a key K in series. This is the primary circuit. The end C of the wire is connected to the terminal M of a DPDT (Double Pole Double Throw) switch and the other terminal N is connected to a jockey through a galvanometer G and a high resistance HR. The cells whose emf $\xi_1$ and $\xi_2$ to be compared are connected to the terminals $M_1, N_1$ and $M_2, N_2$ of the DPDT switch. The positive terminals of $Bt$, $\xi_1$ and $\xi_2$ should be connected to the same end C.

![Figure 2.28 Comparison of emf of two cells](image)

The DPDT switch is pressed towards $M_1$, $N_1$ so that cell $\xi_1$ is included in the secondary circuit and the balancing length $l_1$ is found by adjusting the jockey for zero deflection. Then the second cell $\xi_2$ is included in the circuit and the balancing length $l_2$ is determined. Let $r$ be the resistance per unit length of the potentiometer wire and $I$ be the current flowing through the wire.

we have $\xi_1 = Irl_1$ (2.59)  
$\xi_2 = Irl_2$ (2.60)

By dividing equation (2.59) by (2.60)

$$\frac{\xi_1}{\xi_2} = \frac{l_1}{l_2}$$ (2.61)
By including a rheostat (Rh) in the primary circuit, the experiment can be repeated several times by changing the current flowing through it.

### 2.5.7 Measurement of internal resistance of a cell by potentiometer

To measure the internal resistance of a cell, the circuit connections are made as shown in Figure 2.29. The end C of the potentiometer wire is connected to the positive terminal of the battery Bt and the negative terminal of the battery is connected to the end D through a key K₁. This forms the primary circuit.

The positive terminal of the cell ξ whose internal resistance is to be determined is also connected to the end C of the wire. The negative terminal of the cell ξ is connected to a jockey through a galvanometer and a high resistance. A resistance box R and key K₂ are connected across the cell ξ. With K₂ open, the balancing point J is obtained and the balancing length CJ = l₁ is measured. Since the cell is in open circuit, its emf is

\[ ξ \propto l₁ \]  \hspace{1cm} (2.62)

A suitable resistance (say, 10 Ω) is included in the resistance box and key K₂ is closed. Let r be the internal resistance of the cell. The current passing through the cell and the resistance R is given by

\[ I = \frac{ξ}{R + r} \]

The potential difference across R is

\[ V = \frac{ξR}{R + r} \]

When this potential difference is balanced on the potentiometer wire, let l₂ be the balancing length.

Then

\[ \frac{ξR}{R + r} \propto l₂ \]  \hspace{1cm} (2.63)

From equations (2.62) and (2.63)

\[ \frac{R + r}{R} = \frac{l₁}{l₂} \]  \hspace{1cm} (2.64)

\[ 1 + \frac{r}{R} = \frac{l₁}{l₂} \]

\[ r = R \left( \frac{l₂ - l₁}{l₁} \right) \]

\[ \therefore r = R \left( \frac{l₁ - l₂}{l₁} \right) \]  \hspace{1cm} (2.65)

Substituting the values of the R, l₁ and l₂, the internal resistance of the cell is determined. The experiment can be repeated for different values of R. It is found that the internal resistance of the cell is not constant but increases with increase of external resistance connected across its terminals.

### 2.6 Heating effect of electric current

When current flows through a resistor, some of the electrical energy delivered to the resistor is converted into heat energy and it is dissipated. This heating effect of
current is known as Joule’s heating effect. Just as current produces thermal energy, thermal energy may also be suitably used to produce an electromotive force. This is known as thermoelectric effect.

### 2.6.1 Joule’s law

If a current \( I \) flows through a conductor kept across a potential difference \( V \) for a time \( t \), the work done or the electric potential energy spent is

\[
W = VIt \quad (2.66)
\]

In the absence of any other external effect, this energy is spent in heating the conductor. The amount of heat \( H \) produced is

\[
H = VIt \quad (2.67)
\]

For a resistance \( R \),

\[
H = I^2Rt \quad (2.68)
\]

This relation was experimentally verified by Joule and is known as Joule’s law of heating. It states that the heat developed in an electrical circuit due to the flow of current varies directly as

(i) the square of the current
(ii) the resistance of the circuit and
(iii) the time of flow.

**EXAMPLE 2.27**

Find the heat energy produced in a resistance of 10 \( \Omega \) when 5 A current flows through it for 5 minutes.

**Solution**

\[
R = 10 \, \Omega, \quad I = 5 \, A, \quad t = 5 \, \text{minutes} = 5 \times 60 \, \text{s}
\]

\[
H = I^2Rt
\]

\[
= 5^2 \times 10 \times 5 \times 60
\]

\[
= 25 \times 10 \times 300
\]

\[
= 25 \times 3000
\]

\[
= 75000 \, \text{J (or) 75 kJ}
\]

---

### 2.6.2 Application of Joule’s heating effect

**1. Electric heaters**

Electric iron, electric heater, electric toaster shown in Figure 2.30 are some of the home appliances that utilize the heating effect of current. In these appliances, the heating elements are made of nichrome, an alloy of nickel and chromium. Nichrome has a high specific resistance and can be heated to very high temperatures without oxidation.

![Electric Iron box, electric heater, electric Toaster](image)

**Figure 2.30** (a) Electric Iron box, (b) electric heater (c) electric Toaster

**EXAMPLE 2.28**

An electric heater of resistance 10 \( \Omega \) connected to 220 V power supply is immersed in the water of 1 kg. How long the electrical heater has to be switched on to increase its temperature from 30°C to 60°C. (The specific heat of water is \( s = 4200 \, \text{J kg}^{-1} \))
Whenever there is an excessive current produced due to faulty wire connection, the circuit breaker switch opens. After repairing the faulty connection, we can close the circuit breaker switch. It is shown in the Figure 2.32.

The amount of energy to increase the temperature of 1 kg water from 30°C to 60°C is

\[ Q = ms \Delta T \] (Refer XI physics vol 2, unit 8)

Here \( m = 1 \text{ kg,} \)
\[ s = 4200 \text{ J kg}^{-1}, \]
\[ \Delta T = 30, \]
so \( Q = 1 \times 4200 \times 30 = 126 \text{ kJ} \)

The time required to produce this heat energy
\[ t = \frac{Q}{I^2 R} = \frac{126 \times 10^3}{4840} \approx 26.03 \text{ s} \]

2. Electric fuses

Fuses as shown in Figure 2.31, are connected in series in a circuit to protect the electric devices from the heat developed by the passage of excessive current. It is a short length of a wire made of a low melting point material. It melts and breaks the circuit if current exceeds a certain value. Lead and copper wire melts and burns out when the current increases above 5 A and 35 A respectively.

The only disadvantage with the above fuses is that once fuse wire is burnt due to excessive current, they need to be replaced. Nowadays in houses, circuit breakers (trippers) are also used instead of fuses.

Figure 2.31 Electric Fuse

Whenever there is an excessive current produced due to faulty wire connection, the circuit breaker switch opens. After repairing the faulty connection, we can close the circuit breaker switch. It is shown in the Figure 2.32.

Figure 2.32 circuit breakers

3. Electric furnace

Furnaces as shown in Figure 2.33 are used to manufacture a large number of technologically important materials such as steel, silicon carbide, quartz, gallium arsenide, etc. To produce temperatures up to 1500°C, molybdenum-nichrome wire wound on a silica tube is used. Carbon arc furnaces produce temperatures up to 3000 °C.

Solution

According to Joule’s heating law \( H = I^2 R t \)
The current passed through the electrical heater \( = \frac{220 \text{ V}}{10 \Omega} = 22 \text{ A} \)
The heat produced in one second by the electrical heater \( H = I^2 R \)
The heat produced in one second \( H = (22)^2 \times 10 = 4840 \text{ J} = 4.84 \text{ kJ} \). In fact the power rating of this electrical heater is 4.84 kW.

The time required to produce this heat energy
\[ t = \frac{Q}{I^2 R} = \frac{126 \times 10^3}{4840} \approx 26.03 \text{ s} \]
Seebeck effect

Seebeck discovered that in a closed circuit consisting of two dissimilar metals, when the junctions are maintained at different temperatures an emf (potential difference) is developed. The current that flows due to the emf developed is called thermoelectric current. The two dissimilar metals connected to form two junctions is known as thermocouple (Figure 2.35).

Applications of Seebeck effect

1. Seebeck effect is used in thermoelectric generators (Seebeck generators). These thermoelectric generators are used in power plants to convert waste heat into electricity.

4. Electrical lamp

It consists of a tungsten filament (melting point 3380 °C) kept inside a glass bulb and heated to incandescence by current. In incandescent electric lamps only about 5% of electrical energy is converted into light and the rest is wasted as heat. Electric discharge lamps, electric welding and electric arc also utilize the heating effect of current as shown in Figure 2.34.

2.7 THERMOELECTRIC EFFECT

Conversion of temperature differences into electrical voltage and vice versa is known as thermoelectric effect. A thermoelectric device generates voltage when there is a temperature difference on each side. If a voltage is applied, it generates a temperature difference.

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2. This effect is utilized in automobiles as automotive thermoelectric generators for increasing fuel efficiency.
3. Seebeck effect is used in thermocouples and thermopiles to measure the temperature difference between the two objects.

### 2.7.2 Peltier effect

In 1834, Peltier discovered that when an electric current is passed through a circuit of a thermocouple, heat is evolved at one junction and absorbed at the other junction. This is known as Peltier effect.

![Figure 2.36 Peltier effect: Cu – Fe thermocouple](image)

In the Cu-Fe thermocouple the junctions A and B are maintained at the same temperature. Let a current from a battery flow through the thermocouple (Figure 2.36(a)). At the junction A, where the current flows from Cu to Fe, heat is absorbed and the junction A becomes cold. At the junction B, where the current flows from Fe to Cu heat is liberated and it becomes hot. When the direction of current is reversed, junction A gets heated and junction B gets cooled as shown in the Figure 2.36(b). Hence Peltier effect is reversible.

### 2.7.3 Thomson effect

Thomson showed that if two points in a conductor are at different temperatures, the density of electrons at these points will differ and as a result the potential difference is created between these points. Thomson effect is also reversible.

![Figure 2.37 (a) Positive Thomson effect (b) Negative Thomson effect](image)

If current is passed through a copper bar AB which is heated at the middle point C, the point C will be at higher potential. This indicates that the heat is absorbed along AC and evolved along CB of the conductor as shown in Figure 2.37(a). Thus heat is transferred due to the current flow in the direction of the current. It is called positive Thomson effect. Similar effect is observed in metals like silver, zinc, and cadmium.

When the copper bar is replaced by an iron bar, heat is evolved along CA and absorbed along BC. Thus heat is transferred due to the current flow in the direction opposite to the direction of current. It is called negative Thomson effect as shown in the Figure 2.37(b). Similar effect is observed in metals like platinum, nickel, cobalt, and mercury.
The current, I flowing in a conductor \( I = \frac{dQ}{dt} \), where \( dQ \) is the charge that flows through a cross-section in a time interval \( dt \). SI unit of current is ampere (A).

\[ 1 \text{A} = 1 \text{C s}^{-1}. \]

- The current density \( J \) in a conductor is the current flowing per unit area. \( J = \frac{I}{A} \)
- Current is a scalar but current density is a vector.
- The general form of Ohm's law \( \vec{J} = \sigma \vec{E} \)
- Practical form of Ohm's law states that \( V \propto I \), or \( V = IR \) where I is the current.
- The resistance \( R \) of a conductor is \( R = \frac{V}{I} \). SI unit of resistance is ohm (Ω) and \( 1 \Omega = \frac{1}{1A} \)
- The resistance of a material \( R = \frac{\rho l}{A} \) where \( l \) is length of the material and \( A \) is the area of cross section.
- The resistivity of a material determines how much resistance it offers to the flow of current.
- The equivalent resistance \( R_s \) of several resistances (\( R_1, R_2, R_3, \ldots \)) connected in series combination is \( R_s = (R_1 + R_2 + R_3 + \ldots) \)
- The equivalent resistance \( R_p \) of several resistances (\( R_1, R_2, R_3, \ldots \)) connected in parallel combination is \( \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \)
- Kirchhoff's first rule (Current rule or junction rule): The algebraic sum of the currents at any junction is zero.
- Kirchhoff's second rule (Voltage rule or loop rule): In a closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf included in the circuit.
- Electric power is the rate at which energy is transformed.
- If a current \( I \) flows across a potential difference \( V \), the power delivered to the circuit is \( P = IV \).
- In a resistor \( R \), the electrical power converted to heat is \( P = I^2R = \frac{V^2}{R} \)
- The energy equivalent of one kilowatt-hour (kWh) is \( 1 \text{kWh} = 3.6 \times 10^6 \text{J} \).
- Metre bridge is one form of Wheatstone's bridge.
- Potentiometer is used to compare potential differences.
- Joule's law of heating is \( H = VI \text{t} \) (or) \( H = I^2R \text{t} \).
- Thermoelectric effect: Conversion of temperature differences into electrical voltage and vice versa.

**SUMMARY**

- The current, I flowing in a conductor \( I = \frac{dQ}{dt} \), where \( dQ \) is the charge that flows through a cross-section in a time interval \( dt \). SI unit of current is ampere (A).
- \( 1 \text{A} = 1 \text{C s}^{-1} \).
- The current density \( J \) in a conductor is the current flowing per unit area. \( J = \frac{I}{A} \)
- Current is a scalar but current density is a vector.
- The general form of Ohm's law \( \vec{J} = \sigma \vec{E} \)
- Practical form of Ohm's law states that \( V \propto I \), or \( V = IR \) where I is the current.
- The resistance \( R \) of a conductor is \( R = \frac{V}{I} \). SI unit of resistance is ohm (Ω) and \( 1 \Omega = \frac{1}{1A} \)
- The resistance of a material \( R = \frac{\rho l}{A} \) where \( l \) is length of the material and \( A \) is the area of cross section.
- The resistivity of a material determines how much resistance it offers to the flow of current.
- The equivalent resistance \( R_s \) of several resistances (\( R_1, R_2, R_3, \ldots \)) connected in series combination is \( R_s = (R_1 + R_2 + R_3 + \ldots) \)
- The equivalent resistance \( R_p \) of several resistances (\( R_1, R_2, R_3, \ldots \)) connected in parallel combination is \( \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \)
- Kirchhoff's first rule (Current rule or junction rule): The algebraic sum of the currents at any junction is zero.
- Kirchhoff's second rule (Voltage rule or loop rule): In a closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf included in the circuit.
- Electric power is the rate at which energy is transformed.
- If a current \( I \) flows across a potential difference \( V \), the power delivered to the circuit is \( P = IV \).
- In a resistor \( R \), the electrical power converted to heat is \( P = I^2R = \frac{V^2}{R} \)
- The energy equivalent of one kilowatt-hour (kWh) is \( 1 \text{kWh} = 3.6 \times 10^6 \text{J} \).
- Metre bridge is one form of Wheatstone's bridge.
- Potentiometer is used to compare potential differences.
- Joule's law of heating is \( H = VI \text{t} \) (or) \( H = I^2R \text{t} \).
- Thermoelectric effect: Conversion of temperature differences into electrical voltage and vice versa.
I Multiple Choice Questions

1. The following graph shows current versus voltage values of some unknown conductor. What is the resistance of this conductor?

<table>
<thead>
<tr>
<th>V (V)</th>
<th>I (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

(a) 2 ohm  
(b) 4 ohm  
(c) 8 ohm  
(d) 1 ohm

2. A wire of resistance 2 ohms per meter is bent to form a circle of radius 1m. The equivalent resistance between any two diametrically opposite points, A and B as shown in the figure is

(a) $\pi \Omega$  
(b) $\frac{\pi}{2} \Omega$  
(c) $2\pi \Omega$  
(d) $\frac{\pi}{4} \Omega$

3. A toaster operating at 240 V has a resistance of 120 $\Omega$. The power is

(a) 400 W  
(b) 2 W  
(c) 480 W  
(d) 240 W

4. A carbon resistor of $(47 \pm 4.7) \text{ k}\Omega$ to be marked with rings of different colours for its identification. The colour code sequence will be

a) Yellow – Green – Violet – Gold  
b) Yellow – Violet – Orange – Silver  
c) Violet – Yellow – Orange – Silver  
d) Green – Orange – Violet – Gold

5. What is the value of resistance of the following resistor?

(a) 100 k$\Omega$  
(b) 10 k$\Omega$  
(c) 1 k$\Omega$  
(d) 1000 k$\Omega$

6. Two wires of A and B with circular cross section made up of the same material with equal lengths. Suppose $R_A = 3 R_B$, then what is the ratio of radius of wire A to that of B?

(a) $3$  
(b) $\sqrt{3}$  
(c) $\frac{1}{\sqrt{3}}$  
(d) $\frac{1}{3}$

7. A wire connected to a power supply of 230 V has power dissipation $P_1$. Suppose the wire is cut into two equal pieces and connected parallel to the same power supply. In this case power dissipation is $P_2$. The ratio $\frac{P_2}{P_1}$ is

(a) $1$  
(b) $2$  
(c) $3$  
(d) $4$
8. In India electricity is supplied for domestic use at 220 V. It is supplied at 110 V in USA. If the resistance of a 60W bulb for use in India is \( R \), the resistance of a 60W bulb for use in USA will be

(a) \( R \)  
(b) \( 2R \)  
(c) \( \frac{R}{4} \)  
(d) \( \frac{R}{2} \)

9. In a large building, there are 15 bulbs of 40W, 5 bulbs of 100W, 5 fans of 80W and 1 heater of 1kW are connected. The voltage of electric mains is 220V. The minimum capacity of the main fuse of the building will be (IIT-JEE 2014)

(a) 14 A  
(b) 8 A  
(c) 10 A  
(d) 12 A

10. There is a current of 1.0 A in the circuit shown below. What is the resistance of \( P \)?

\[ \begin{align*}
5V & \quad 15\Omega & \quad 15\Omega & \quad 15\Omega \\
& \quad P & \quad 3\Omega & \quad 2.5\Omega \\
\end{align*} \]

(a) 1.5 \( \Omega \)  
(b) 2.5 \( \Omega \)  
(c) 3.5 \( \Omega \)  
(d) 4.5 \( \Omega \)

11. What is the current out of the battery?

\[ \begin{align*}
5V & \quad 15\Omega & \quad 15\Omega & \quad 15\Omega \\
& \quad P & \quad 15\Omega & \quad 15\Omega & \quad 15\Omega \\
\end{align*} \]

(a) 1A  
(b) 2A  
(c) 3A  
(d) 4A

12. The temperature coefficient of resistance of a wire is 0.00125 per °C. At 300 K, its resistance is 1 \( \Omega \). The resistance of the wire will be 2 \( \Omega \) at

(a) 1154 K  
(b) 1100 K  
(c) 1400 K  
(d) 1127 K

13. The internal resistance of a 2.1 V cell which gives a current of 0.2 A through a resistance of 10 \( \Omega \) is

(a) 0.2 \( \Omega \)  
(b) 0.5 \( \Omega \)  
(c) 0.8 \( \Omega \)  
(d) 1.0 \( \Omega \)

14. A piece of copper and another of germanium are cooled from room temperature to 80 K. The resistance of

(a) each of them increases  
(b) each of them decreases  
(c) copper increases and germanium decreases  
(d) copper decreases and germanium increases

15. In Joule's heating law, when \( I \) and \( t \) are constant, if the \( H \) is taken along the \( y \) axis and \( P \) along the \( x \) axis, the graph is

(a) straight line  
(b) parabola  
(c) circle  
(d) ellipse

**Answers**

1) a  
2) b  
3) c  
4) b  
5) a  
6) c  
7) d  
8) c  
9) d  
10) c  
11) a  
12) d  
13) b  
14) d  
15) a

**II Short Answer Questions**

1. Why current is a scalar?
2. Distinguish between drift velocity and mobility.
3. State microscopic form of Ohm's law.
4. State macroscopic form of Ohm's law.
5. What are ohmic and non ohmic devices?
6. Define electrical resistivity.
7. Define temperature coefficient of resistance.
8. What is superconductivity?
9. What is electric power and electric energy?
10. Define current density.
11. Derive the expression for power P=VI in electrical circuit.
12. Write down the various forms of expression for power in electrical circuit.
13. State Kirchhoff’s current rule.
15. State the principle of potentiometer.
16. What do you mean by internal resistance of a cell?
17. State Joule’s law of heating.
18. What is Seebeck effect?
19. What is Thomson effect?
20. What is Peltier effect?
21. State the applications of Seebeck effect.

III Long Answer Questions
1. Describe the microscopic model of current and obtain general form of Ohm’s law
2. Obtain the macroscopic form of Ohm’s law from its microscopic form and discuss its limitation.
3. Explain the equivalent resistance of a series and parallel resistor network
4. Explain the determination of the internal resistance of a cell using voltmeter.

IV Numerical problems
1. The following graphs represent the current versus voltage and voltage versus current for the six conductors A,B,C,D,E and F. Which conductor has least resistance and which has maximum resistance?

Ans: Least: RF = 0.4 Ω, maximum RC = 2.5 Ω
2. Lightning is very good example of natural current. In typical lightning, there is $10^9$ J energy transfer across the potential difference of $5 \times 10^7$ V during a time interval of 0.2 s.
Using this information, estimate the following quantities (a) total amount of charge transferred between cloud and ground (b) the current in the lightning bolt (c) the power delivered in 0.2 s.

Ans: charge = 20 C, I = 100 A, P = 5 GW

3. A copper wire of $10^{-6}$ m$^2$ area of cross section, carries a current of 2 A. If the number of electrons per cubic meter is $8 \times 10^{28}$, calculate the current density and average drift velocity.

Ans: $J = 2 \times 10^6$ Am$^{-2}$; $v_d = 15.6 \times 10^{-5}$ m s$^{-1}$

4. The resistance of a nichrome wire at 0 $^\circ$C is 10 Ω. If its temperature coefficient of resistance is 0.004/°C, find its resistance at boiling point of water. Comment on the result.

Ans: $R_T = 14$ Ω.

As the temperature increases the resistance of the wire also increases.

5. The rod given in the figure is made up of two different materials.

Both have square cross sections of 3 mm side. The resistivity of the first material is $4 \times 10^{-3}$ Ω.m and it is 25 cm long while second material has resistivity of $5 \times 10^{-3}$ Ω.m and is of 70 cm long. What is the resistivity of rod between its ends?

Ans: 500 Ω

6. Three identical lamps each having a resistance R are connected to the battery of emf as shown in the figure.

Suddenly the switch S is closed. (a) Calculate the current in the circuit when S is open and closed (b) What happens to the intensities of the bulbs A, B and C. (c) Calculate the voltage across the three bulbs when S is open and closed (d) Calculate the power delivered to the circuit when S is opened and closed (e) Does the power delivered to the circuit decreases, increases or remain same?

Ans:

<table>
<thead>
<tr>
<th>Electrical quantities</th>
<th>Switch S is open</th>
<th>Switch S is closed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td>$\frac{\xi}{3R}$</td>
<td>$\frac{\xi}{2R}$</td>
</tr>
<tr>
<td>Voltage</td>
<td>$V_A = \frac{\xi}{3R}$, $V_B = \frac{\xi}{3R}$, $V_C = \frac{\xi}{3R}$</td>
<td>$V_A = \frac{\xi}{2R}$, $V_B = \frac{\xi}{2R}$, $V_C = 0$</td>
</tr>
<tr>
<td>Power</td>
<td>$P_A = \frac{\xi^2}{9R}$, $P_B = \frac{\xi^2}{9R}$, $P_C = \frac{\xi^2}{9R}$</td>
<td>$P_A = \frac{\xi^2}{4R}$, $P_B = \frac{\xi^2}{4R}$, $P_C = 0$</td>
</tr>
<tr>
<td>Intensity</td>
<td>All the bulbs glow with equal intensity</td>
<td>The intensities of the bulbs A and B equally increase. Bulb C will not glow since no current pass through it.</td>
</tr>
</tbody>
</table>

As the temperature increases the resistance of the wire also increases.
7. The current through an element is shown in the figure. Determine the total charge that pass through the element at a) \( t = 0 \) s, b) \( t = 2 \) s, c) \( t = 5 \) s

\[ I(A) \]

\[ 1 \quad 2 \quad 3 \quad 4 \quad 5 \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \]

Ans: At \( t = 0 \) s, \( dq = 0 \) C, At \( t = 2 \) s, \( dq = 10 \) C; At \( t = 5 \) s, \( dq = 0 \) C

8. An electronics hobbyist is building a radio which requires 150 \( \Omega \) in her circuit, but she has only 220 \( \Omega \), 79 \( \Omega \) and 92 \( \Omega \) resistors available. How can she connect the available resistors to get desired value of resistance?

Ans: Parallel combination of 220 \( \Omega \) and 79 \( \Omega \) in series with 92 \( \Omega \)

9. A cell supplies a current of 0.9 A through a 2 \( \Omega \) resistor and a current of 0.3 A through a 7 \( \Omega \) resistor. Calculate the internal resistance of the cell.

Ans: 0.5 \( \Omega \)

10. Calculate the currents in the following circuit.

Ans: \( I_1 = 0.070 \) A, \( I_2 = -0.010 \) A and \( I_3 = 0.080 \) A

11. A potentiometer wire has a length of 4 m and resistance of 20 \( \Omega \). It is connected in series with resistance of 2980 \( \Omega \) and a cell of emf 4 V. Calculate the potential along the wire.

Ans: Potential = \( 0.65 \times 10^{-2} \) V m

12. Determine the current flowing through the galvanometer (G) as shown in the figure.

Ans: \( I_g = \frac{1}{11} \) A

13. Two cells each of 5V are connected in series across a 8 \( \Omega \) resistor and three parallel resistors of 4 \( \Omega \), 6 \( \Omega \) and 12 \( \Omega \). Draw a circuit diagram for the above arrangement. Calculate i) the current drawn from the cell (ii) current through each resistor

Ans: The current at 4 \( \Omega \), \( I = \frac{2}{4} = 0.5A \),

the current at 6 \( \Omega \), \( I = \frac{2}{6} = 0.33A \),

the current at 12 \( \Omega \), \( I = \frac{2}{12} = 0.17A \)

14. Four light bulbs P, Q, R, S are connected in a circuit of unknown arrangement. When each bulb is removed one at a time and replaced, the following behavior is observed.
15. In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63 cm, what is the emf of the second cell?

Ans: emf of the second cell is 2.25 V

**BOOKS FOR REFERENCE:**

In this activity you will be able to
(a) measure the potential difference of cells
(b) measure the internal resistance of a given primary cell

STEPS:
• Open the browser and type “olabs.edu.in” in the address bar. Click physics tab and then click “Potentiometer-Internal Resistance of a Cell” in class 12 section. Go to “simulator” tab to do the experiment.
• Construct the electric circuit as per the connection diagram by clicking “show circuit diagram” tab. You can connect wires between electric component by dragging the mouse between the component.
• To check whether the connections are correct or not, drag the jockey and place it at the two end points of the wire. If the galvanometer shows opposite deflections, the connections are correct. (keep both keys on)

Find the balancing length. Calculate the internal resistance for the observed balancing lengths. Repeat the experiment for five times and take the average.

Note:
1. One time sign up is needed to do simulation. Then login using that username and password.
2. Read theory, procedure and animation to get the theory by clicking the corresponding tab.

URL:
http://amrita.olabs.edu.in/?sub=1&brch=6&sim=147&cnt=4

* Pictures are indicative only.
* If browser requires, allow Flash Player or Java Script to load the page.
UNIT 3
MAGNETISM AND MAGNETIC EFFECTS OF ELECTRIC CURRENT

“The magnetic force is animate, or imitates a soul; in many respects it surpasses the human soul while it is united to an organic body” – William Gilbert

In this unit, the student is exposed to

- Earth's magnetic field and magnetic elements
- Basic property of magnets
- Statement of Coulomb inverse square law of magnetism
- Magnetic dipole
- Magnetic induction at a point due to axial line and equatorial line
- Torque acting on a bar magnet in a uniform magnetic field
- Potential energy of a bar magnet placed in a uniform magnetic field
- Tangent law and tangent Galvanometer
- Magnetic properties – permeability, susceptibility etc
- Classification of magnetic materials – dia, para and ferro magnetic materials
- Concept of Hysteresis
- Magnetic effects of electric current – long straight conductor and circular coil
- Right hand thumb rule and Maxwell's right hand cork screw rule
- Biot-Savart's law – applications
- Current loop as a magnetic dipole
- Magnetic dipole moment of revolving electron
- Ampère's circuital law – applications
- Solenoid and toroid
- Lorentz force – charged particle moving in an electromagnetic field
- Cyclotron
- Force on a current carrying conductor in a magnetic field
- Force between two long parallel current carrying conductor
- Torque on a current loop in a magnetic field
- Moving coil Galvanometer
3.1 INTRODUCTION TO MAGNETISM

Magnetism is everywhere from tiny particles like electrons to the entire universe. Historically the word ‘magnetism’ was derived from iron ore magnetite (Fe₃O₄). In olden days, magnets were used as magnetic compass for navigation, magnetic therapy for treatment and also used in magic shows.

In modern days, most of the things we use in our daily life contain magnets (Figure 3.2). Motors, cycle dynamo, loudspeakers, magnetic tapes used in audio and video recording, mobile phones, head phones, CD, pen-drive, hard disc of laptop, refrigerator door, generator are a few examples.

Earlier, both electricity and magnetism were thought to be two independent branches in physics. In 1820, H.C. Oersted observed the deflection of magnetic compass needle kept near a current carrying wire. This unified the two different branches, electricity and magnetism as a single subject ‘electromagnetism’ in physics.

In this unit, basics of magnets and their properties are given. Later, how a current carrying conductor (here only steady current, not time-varying current is considered) behaves like a magnet is presented.

Magnetic levitation

Many birds and animals have magnetic sense in their eyes using Earth’s magnetic field for navigation.

Magnetic sensing in eyes - for Zebra finches bird, due to protein cryptochromes Cry4 present in retina, it uses Earth magnetic field for navigation.

Figure 3.2 Uses of magnets in modern world – (a) speakers (b) head phones (c) MRI scan (d) Hard disc of laptop
3.1.1 Earth’s magnetic field and magnetic elements

From the activities performed in lower classes, we have noticed that the needle in a magnetic compass or freely suspended magnet comes to rest in a position which is approximately along the geographical north-south direction of the Earth. William Gilbert in 1600 proposed that Earth itself behaves like a gigantic powerful bar magnet. But this theory is not successful because the temperature inside the Earth is very high and so it will not be possible for a magnet to retain its magnetism.

Gover suggested that the Earth’s magnetic field is due to hot rays coming out from the Sun. These rays will heat up the air near equatorial region. Once air becomes hotter, it rises above and will move towards northern and southern hemispheres and get electrified. This may be responsible to magnetize the ferromagnetic materials near the Earth’s surface. Till date, so many theories have been proposed. But none of the theory completely explains the cause for the Earth’s magnetism.

The north pole of magnetic compass needle is attracted towards the magnetic south pole of the Earth which is near the geographic north pole (Figure 3.3). Similarly, the south pole of magnetic compass needle is attracted towards the geographic north pole of the Earth which is near magnetic north pole. The branch of physics which deals with the Earth’s magnetic field is called Geomagnetism or Terrestrial magnetism.

There are three quantities required to specify the magnetic field of the Earth on its surface, which are often called as the elements of the Earth’s magnetic field. They are
(a) magnetic declination (D)
(b) magnetic dip or inclination (I)
(c) the horizontal component of the Earth’s magnetic field ($B_{H}$)

![Figure 3.3 Earth’s magnetic field](image)

![Figure 3.4 Declination angle](image)
Day and night occur because Earth spins about an axis called geographic axis. A vertical plane passing through the geographic axis is called geographic meridian and a great circle perpendicular to Earth’s geographic axis is called geographic equator.

The straight line which connects magnetic poles of Earth is known as magnetic axis. A vertical plane passing through magnetic axis is called magnetic meridian and a great circle perpendicular to Earth’s magnetic axis is called magnetic equator.

When a magnetic needle is freely suspended, the alignment of the magnet does not exactly lie along the geographic meridian as shown in Figure 3.4. The angle between magnetic meridian at a point and geographical meridian is called the declination or magnetic declination (D). At higher latitudes, the declination is greater whereas near the equator, the declination is smaller. In India, declination angle is very small and for Chennai, magnetic declination angle is -1° 8’ (which is negative (west)).

The angle subtended by the Earth’s total magnetic field $\vec{B}$ with the horizontal direction in the magnetic meridian is called dip or magnetic inclination (I) at that point (Figure 3.5). For Chennai, inclination angle is 14° 16’. The component of Earth’s magnetic field along the horizontal direction in the magnetic meridian is called horizontal component of Earth’s magnetic field, denoted by $B_H$.

Let $B_E$ be the net Earth’s magnetic field at a point P on the surface of the Earth. $B_E$ can be resolved into two perpendicular components.

<table>
<thead>
<tr>
<th>Horizontal component</th>
<th>$B_H = B_E \cos I$ (3.1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical component</td>
<td>$B_V = B_E \sin I$ (3.2)</td>
</tr>
</tbody>
</table>

Dividing equation (3.2) and (3.1), we get

$$\tan I = \frac{B_V}{B_H} \quad (3.3)$$

(i) At magnetic equator

The Earth’s magnetic field is parallel to the surface of the Earth (i.e., horizontal) which implies that the needle of magnetic compass rests horizontally at an angle of dip, $I = 0^\circ$ as shown in figure 3.6.

$$B_H = B_E$$

$B_V = 0$

This implies that the horizontal component is maximum at equator and vertical component is zero at equator.

(ii) At magnetic poles

The Earth’s magnetic field is perpendicular to the surface of the Earth (i.e., vertical) which implies that the needle
of magnetic compass rests vertically at an angle of dip, $I = 90^\circ$ as shown in Figure 3.7. Hence,

$B_H = 0$

$B_V = B_E$

This implies that the vertical component is maximum at poles and horizontal component is zero at poles.

**EXAMPLE 3.1**

The horizontal component and vertical components of Earth’s magnetic field at a place are 0.15 G and 0.26 G respectively. Calculate the angle of dip and resultant magnetic field.

**Solution:**

$B_H = 0.15 \text{ G}$ and $B_V = 0.26 \text{ G}$

$$\tan I = \frac{0.26}{0.15} \Rightarrow I = \tan^{-1}(1.732) = 60^\circ$$

The resultant magnetic field of the Earth is

$$B = \sqrt{B_H^2 + B_V^2} = 0.3 \text{ G}$$

### 3.1.2 Basic properties of magnets

Some basic terminologies and properties used in describing bar magnet.

(a) Magnetic dipole moment

Consider a bar magnet as shown in Figure 3.8. Let $q_m$ be the pole strength (it is also called as magnetic charge) of the magnetic pole and let $l$ be the distance between the geometrical center of bar magnet O and one end of the pole. The **magnetic dipole moment is defined as the product of its pole strength and magnetic length**. It is a vector quantity, denoted by $\vec{p}_m$.

$$\vec{p}_m = q_m \vec{d} \quad (3.4)$$

where $\vec{d}$ is the vector drawn from south pole to north pole and its magnitude $|\vec{d}| = 2l$. The magnitude of magnetic dipole moment is $p_m = 2q_m l$.
The SI unit of magnetic moment is A m². Note that the direction of magnetic moment is from South pole to North pole.

(b) Magnetic field

Magnetic field is the region or space around every magnet within which its influence can be felt by keeping another magnet in that region. The magnetic field $\vec{B}$ at a point is defined as a force experienced by the bar magnet of unit pole strength.

$$\vec{B} = \frac{1}{q_m} \vec{F}$$  \hspace{1cm} (3.5)

Its unit is N A⁻¹ m⁻¹.

Aurora Borealis and Aurora Australis

People living at high latitude regions (near Arctic or Antarctic) might experience dazzling coloured natural lights across the night sky. This ethereal display on the sky is known as aurora borealis (northern lights) or aurora australis (southern lights). These lights are often called as polar lights. The lights are seen above the magnetic poles of the northern and southern hemispheres. They are called as “Aurora borealis” in the north and “Aurora australis” in the south. This occurs as a result of interaction between the gaseous particles in the Earth’s atmosphere with highly charged particles released from the Sun’s atmosphere through solar wind. These particles emit light due to collision and variations in colour are due to the type of the gas particles that take part in the collisions. A pale yellowish – green colour is produced when the ionized oxygen takes part in the collision and a blue or purplish – red aurora is produced due to ionized nitrogen molecules.
(c) Types of magnets
Magnets are classified into natural magnets and artificial magnets. For example, iron, cobalt, nickel, etc. are natural magnets. Strengths of natural magnets are very weak and the shapes of the magnet are irregular. Artificial magnets are made by us in order to have desired shape and strength. If the magnet is in the form of rectangular shape or cylindrical shape, then it is known as bar magnet.

Properties of magnet
The following are the properties of bar magnet (Figure 3.9)
1. A freely suspended bar magnet will always point along the north-south direction.
2. A magnet attracts another magnet or magnetic substances towards itself. The attractive force is maximum near the end of the bar magnet. When a bar magnet is dipped into iron filling, they cling to the ends of the magnet.

Figure 3.9 Properties of bar magnet
3. When a magnet is broken into pieces, each piece behaves like a magnet with poles at its ends.

4. Two poles of a magnet have pole strength equal to one another.

5. The length of the bar magnet is called geometrical length and the length between two magnetic poles in a bar magnet is called magnetic length. Magnetic length is always slightly smaller than geometrical length. The ratio of magnetic length and geometrical length is \( \frac{5}{6} \).

\[
\frac{\text{Magnetic length}}{\text{Geometrical length}} = \frac{5}{6} = 0.833
\]

**EXAMPLE 3.2**

Let the magnetic moment of a bar magnet be \( \vec{p}_m \) whose magnetic length is \( d = 2l \) and pole strength is \( q_m \). Compute the magnetic moment of the bar magnet when it is cut into two pieces

(a) along its length

(b) perpendicular to its length.

**Solution**

(a) a bar magnet cut into two pieces along its length:

When the bar magnet is cut along the axis into two pieces, new magnetic pole strength is \( q'_m = \frac{q_m}{2} \) but magnetic length does not change. So, the magnetic moment is

\[
q'_m = \frac{q_m}{2}
\]

\[
p'_m = \frac{q_m 2l}{2} = \frac{1}{2}(q_m 2l) = \frac{1}{2}p_m
\]

In vector notation, \( \vec{p}'_m = \frac{1}{2}\vec{p}_m \)

(b) a bar magnet cut into two pieces perpendicular to the axis:

When the bar magnet is cut perpendicular to the axis into two pieces, magnetic pole strength will not change but magnetic length will be halved. So the magnetic moment is

\[
p'_m = q_m \times \frac{1}{2}(2l) = \frac{1}{2}(q_m 2l) = \frac{1}{2}p_m
\]

In vector notation \( \vec{p}'_m = \frac{1}{2}\vec{p}_m \)
EXAMPLE 3.3

Compute the magnetic length of a uniform bar magnet if the geometrical length of the magnet is 12 cm. Mark the positions of magnetic pole points.

**Solution**

Geometrical length of the bar magnet is 12 cm

\[
\text{Magnetic length} = \frac{5}{6} \times \text{(geometrical length)}
\]

\[
= \frac{5}{6} \times 12 = 10 \text{ cm}
\]

In this figure, the dot implies the pole points.

**Magnetic field lines**

1. Magnetic field lines are continuous closed curves. The direction of magnetic field lines is from North pole to South pole outside the magnet (Figure 3.10) and South pole to North pole inside the magnet.

2. The direction of magnetic field at any point on the curve is known by drawing tangent to the magnetic line of force at that point. In the Figure No. 3.10 (b), the tangent drawn at points P, Q and R gives the direction of magnetic field **B** at that point.

3. Magnetic field lines never intersect each other. Otherwise, the magnetic compass needle would point towards two directions, which is not possible.

4. The degree of closeness of the field lines determines the relative strength of the magnetic field. The magnetic field is strong where magnetic field lines crowd and weak where magnetic field lines thin out.

(d) **Magnetic flux**

*The number of magnetic field lines crossing per unit area is called magnetic flux \( \Phi_B \).* Mathematically, the magnetic flux through a surface of area \( A \) in a uniform magnetic field is defined as
Figure 3.10 Properties of magnetic field lines— unlike poles attracts each other shown in picture (a) and (b) like poles repel each other— shown in picture (c) and (d)

\[ \Phi_B = \vec{B} \cdot \vec{A} = BA \cos \theta = B_A A \]  

where \( \theta \) is the angle between \( \vec{B} \) and \( \vec{A} \) as shown in Figure 3.11.

**Figure 3.11** Magnetic flux

**Special cases**

(a) When \( \vec{B} \) is normal to the surface i.e., \( \theta = 0^\circ \), the magnetic flux is \( \Phi_B = BA \) (maximum).

(b) When \( \vec{B} \) is parallel to the surface i.e., \( \theta = 90^\circ \), the magnetic flux is \( \Phi_B = 0 \).

Suppose the magnetic field is not uniform over the surface, the equation (3.6) can be written as

\[ \Phi_B = \int \vec{B} \cdot d\vec{A} \]

Magnetic flux is a scalar quantity. The SI unit for magnetic flux is weber, which is denoted by symbol Wb. Dimensional formula for magnetic flux is \( [ML^2T^{-2}A^{-1}] \). The CGS unit of magnetic flux is Maxwell.

1 weber = \( 10^8 \) maxwell

The **magnetic flux density** can also be defined as the number of magnetic field lines crossing unit area kept normal to the direction of line of force. Its unit is Wb m\(^{-2}\) or tesla.
Non-uniform magnetic field
Magnetic field is said to be non-uniform if the magnitude or direction or both varies at all its points. Example: magnetic field of a bar magnet

Uniform magnetic field
Magnetic field is said to be uniform if it has same magnitude and direction at all the points in a given region. Example, locally Earth’s magnetic field is uniform.

The magnetic field of Earth has same value over the entire area of your school!

**Figure 3.12** Uniform magnetic field

**Figure 3.13** Non-uniform magnetic field – (a) direction is constant (b) direction is not a constant (c) both magnitude and direction are not constant (d) magnetic field of a bar magnet

**EXAMPLE 3.4**
Calculate the magnetic flux coming out from the surface containing magnetic dipole (say, a bar magnet) as shown in figure.
Unit 3 Magnetism and magnetic effects of electric current

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Solution
Magnetic dipole is kept, the total flux emanating from the closed surface S is zero. So,

$$\Phi_B = \oint B \cdot d\vec{A} = 0$$

Here the integral is taken over closed surface. Since no isolated magnetic pole (called magnetic monopole) exists, this integral is always zero,

$$\oint B \cdot d\vec{A} = 0$$

This is similar to Gauss's law in electrostatics. (Refer unit 1)

3.2 COULOMB’S INVERSE SQUARE LAW OF MAGNETISM

Consider two bar magnets A and B as shown in Figure 3.14.

When the north pole of magnet A and the north pole of magnet B or the south pole of magnet A and the south pole of magnet B are brought closer, they repel each other. On the other hand, when the north pole of magnet A and the south pole of magnet B or the south pole of magnet A and the north pole of magnet B are brought closer, their poles attract each other. This looks similar to Coulomb’s law for static charges studied in Unit I (opposite charges attract and like charges repel each other). So analogous to Coulomb’s law in electrostatics, (Refer unit 1) we can state Coulomb’s law for magnetism (Figure 3.15) as follows:

**Figure 3.14**: Magnetic poles behave like electric charges – like poles repel and unlike poles attract

**Figure 3.15** Coulomb’s law – force between two magnetic pole strength
The force of attraction or repulsion between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them.

Mathematically, we can write

\[ \vec{F} \propto \frac{q_m q_m}{r^2} \hat{r} \]

where \( q_m \) and \( q_m \) are pole strengths of two poles and \( r \) is the distance between two magnetic poles.

\[ \vec{F} = k \frac{q_m q_m}{r^2} \hat{r} \quad (3.7) \]

In magnitude, \( \vec{F} = k \frac{q_m q_m}{r^2} \quad (3.8) \)

where \( k \) is a proportionality constant whose value depends on the surrounding medium. In S.I. unit, the value of \( k \) for free space is \( k = \frac{\mu_0}{4\pi} \approx 10^{-7} \text{ H m}^{-1} \), where \( \mu_0 \) is the absolute permeability of free space (air or vacuum).

**EXAMPLE 3.5**

The repulsive force between two magnetic poles in air is \( 9 \times 10^{-3} \text{ N} \). If the two poles are equal in strength and are separated by a distance of 10 cm, calculate the pole strength of each pole.

**Solution:**

The force between two poles are given by

\[ \vec{F} = k \frac{q_m q_m}{r^2} \hat{r} \]

The magnitude of the force is

\[ F = k \frac{q_m q_m}{r^2} \]

Given: \( F = 9 \times 10^{-3} \text{ N}, r = 10 \text{ cm} = 10 \times 10^{-2} \text{ m} \)

Therefore,

\[ 9 \times 10^{-3} = 10^{-7} \times \frac{q_m^2}{\left(10 \times 10^{-2}\right)^2} \Rightarrow q_m = 30 \text{ NT}^{-1} \]

### 3.2.1 Magnetic field at a point along the axial line of the magnetic dipole (bar magnet)

Consider a bar magnet NS as shown in Figure 3.16. Let N be the North Pole and S be the south pole of the bar magnet, each of pole strength \( q_m \) and separated by a distance of \( 2l \). The magnetic field at a point C (lies along the axis of the magnet) at a distance from the geometrical center O of the bar magnet can be computed by keeping unit north pole \( (q_m = 1 \text{ A m}) \) at C. The force experienced by the unit north pole at C due to pole strength can be computed using Coulomb’s law of magnetism as follows:

The force of repulsion between north pole of the bar magnet and unit north pole at point C (in free space) is

\[ \vec{F}_N = \frac{\mu_0}{4\pi} \frac{q_m}{(r-l)^2} \hat{l} \quad (3.9) \]

where \( r - l \) is the distance between north pole of the bar magnet and unit north pole at C.

The force of attraction between South Pole of the bar magnet and unit North Pole at point C (in free space) is

\[ \vec{F}_S = -\frac{\mu_0}{4\pi} \frac{q_m}{(r+l)^2} \hat{l} \quad (3.10) \]

where \( r + l \) is the distance between south pole of the bar magnet and unit north pole at C.
Therefore, using equation (3.13) in equation (3.12), we get

$$B_{\text{axial}} = \frac{\mu}{4\pi} \left( \frac{2r p_m}{r^3} \right) \hat{i} = \frac{\mu}{4\pi} \frac{2}{r^3} p_m \hat{i} \quad (3.14)$$

where $\hat{p}_m = p_m \hat{i}$.

### 3.2.2. Magnetic field at a point along the equatorial line due to a magnetic dipole (bar magnet)

Consider a bar magnet NS as shown in Figure 3.17. Let N be the north pole and S be the south pole of the bar magnet, each with pole strength $q_m$ and separated by a distance of $2l$. The magnetic field at a point C (lies along the equatorial line) at a distance $r$ from the geometrical center O of the bar magnet can be computed by keeping unit north pole ($q_{nc} = 1$ A m) at C. The force experienced by the unit north pole at C due to pole strength N-S can be computed using Coulomb's law of magnetism as follows:
Unit 3
Magnetism and magnetic effects of electric current

\[ F = -\mu_0 \frac{q_m}{4\pi r^2} \cos \theta \mathbf{i} + \mu_0 \frac{q_m}{4\pi (r^2 + l^2)^{3/2}} \cos \theta \mathbf{j} \]

where, \( F_s = \frac{\mu_0 q_m}{4\pi r^2} \)

From equation (3.15) and equation (3.16), the net force at point C is \( \vec{F} = F_N + F_s \). This net force is equal to the magnetic field at the point C.

\[ \vec{B} = -(F_N + F_s) \cos \theta \mathbf{i} \]

Since, \( F_N = F_s \)

\[ \vec{B} = -\frac{2\mu_0 q_m}{4\pi r^2} \cos \theta \mathbf{i} = -\frac{2\mu_0 q_m}{4\pi (r^2 + l^2)^{3/2}} \cos \theta \mathbf{i} \]

In a right angle triangle NOC as shown in the Figure 3.17

\[ \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{l}{r'} = \frac{l}{(r^2 + l^2)^{1/2}} \]

Substituting equation (3.18) in equation (3.17) we get

\[ \vec{B} = -\frac{\mu_0 q_m \times (2l)}{4\pi (r^2 + l^2)^{3/2}} \mathbf{i} \]

Since, magnitude of magnetic dipole moment is \( |\vec{p_m}| = p_m = q_m \cdot 2l \) and substituting in equation (3.19), the magnetic field at a point C is

\[ \vec{B}_{\text{equatorial}} = -\frac{\mu_0 p_m}{4\pi (r^2 + l^2)^{3/2}} \mathbf{i} \]

If the distance between two poles in a bar magnet are small (looks like short magnet) when compared to the distance between geometrical center O of bar magnet and the location of point C i.e., \( r >> l \), then,

\[ \vec{F}_s = -F_s \cos \theta \mathbf{i} - F_s \sin \theta \mathbf{j} \]
Therefore, using equation (3.21) in equation (3.20), we get

\[
\vec{B}_{\text{equatorial}} = -\frac{\mu_0}{4\pi} \frac{P_m}{r^3} \hat{i}
\]

Since \( P_m \hat{i} = \vec{P}_m \). In general, the magnetic field at equatorial point is given by

\[
\vec{B}_{\text{equatorial}} = -\frac{\mu_0}{4\pi} \frac{P_m}{r^3} \hat{i}
\]

Note that magnitude of \( B_{\text{axial}} \) is twice that of magnitude of \( B_{\text{equatorial}} \) and the direction of \( B_{\text{axial}} \) and \( B_{\text{equatorial}} \) are opposite.

**EXAMPLE 3.6**

A short bar magnet has a magnetic moment of 0.5 J T\(^{-1}\). Calculate magnitude and direction of the magnetic field produced by the bar magnet which is kept at a distance of 0.1 m from the center of the bar magnet along (a) axial line of the bar magnet and (b) normal bisector of the bar magnet.

**Solution**

Given magnetic moment 0.5 J T\(^{-1}\) and distance \( r = 0.1 \) m

(a) When the point lies on the axial line of the bar magnet, the magnetic field for short magnet is given by

\[
\vec{B}_{\text{axial}} = \frac{\mu_0}{4\pi} \left( 2\frac{P_m}{r^3} \right) \hat{i}
\]

\[
\vec{B}_{\text{axial}} = 10^{-7} \times \left( \frac{2 \times 0.5}{(0.1)^3} \right) \hat{i} = 1 \times 10^{-4} T \hat{i}
\]

Hence, the magnitude of the magnetic field along axial is \( B_{\text{axial}} = 1 \times 10^{-4} \) T and direction is towards South to North.

(b) When the point lies on the normal bisector (equatorial) line of the bar magnet, the magnetic field for short magnet is given by

\[
\vec{B}_{\text{equatorial}} = -\frac{\mu_0}{4\pi} \frac{P_m}{r^3} \hat{i}
\]

Hence, the magnitude of the magnetic field along axial is \( B_{\text{equatorial}} = 0.5 \times 10^{-4} \) T and direction is towards North to South.

Note that magnitude of \( B_{\text{axial}} \) is twice that of magnitude of \( B_{\text{equatorial}} \) and the direction of \( B_{\text{axial}} \) and \( B_{\text{equatorial}} \) are opposite.

### 3.3 TORSION ACTING ON A BAR MAGNET IN UNIFORM MAGNETIC FIELD

Consider a magnet of length 2\( l \) of pole strength \( q_m \) kept in a uniform magnetic field \( \vec{B} \) as shown in Figure 3.19. Each pole experiences a force of magnitude \( q_m \vec{B} \) but acts in opposite direction. Therefore, the net force exerted on the magnet is zero, so that there is no translatory motion. These two forces constitute a couple (about midpoint of bar magnet) which will rotate and try to align in the direction of the magnetic field \( \vec{B} \).

The force experienced by north pole,

\[
\vec{F}_N = q_m \vec{B} \quad (3.23)
\]

The force experienced by south pole,

\[
\vec{F}_S = -q_m \vec{B} \quad (3.24)
\]

Adding equations (3.23) and (3.24), we get the net force acting on the dipole as

\[
\vec{F} = \vec{F}_N + \vec{F}_S = \vec{0}
\]
represents moment of inertia of the bar magnet, \( p_m \) is the magnetic moment and is the magnetic field.

**Solution**

The magnitude of deflecting torque (the torque which makes the object rotate) acting on the bar magnet which will tend to align the bar magnet parallel to the direction of the uniform magnetic field \( \vec{B} \) is

\[
|\vec{\tau}| = p_m B \sin \theta
\]

The magnitude of restoring torque acting on the bar magnet can be written as

\[
|\vec{\tau}| = l \frac{d^2 \theta}{dt^2}
\]

Under equilibrium conditions, both magnitude of deflecting torque and restoring torque will be equal but act in the opposite directions, which means

\[
l \frac{d^2 \theta}{dt^2} = -p_m B \sin \theta
\]

**EXAMPLE 3.7**

Show the time period of oscillation when a bar magnet is kept in a uniform magnetic field is

\[
T = 2\pi \sqrt{\frac{1}{l p_m B}}
\]

in second, where \( l \)

(a) Why a freely suspended bar magnet in your laboratory experiences only torque (rotational motion) but not any translatory motion even though Earth has non-uniform magnetic field?

It is because Earth’s magnetic field is locally (physics laboratory) uniform.

(b) Suppose we keep a freely suspended bar magnet in a non-uniform magnetic field. What will happen?

It will undergo translatory motion (net force) and rotational motion (torque).
The negative sign implies that both are in opposite directions. The above equation can be written as

\[
\frac{d^2 \theta}{dt^2} = -\frac{p_m B}{I} \sin \theta
\]

This is non-linear second order homogeneous differential equation. In order to make it linear, we use small angle approximation as we did in XI volume II (Unit 10 – oscillations, Refer section 10.4.4) i.e., \( \sin \theta \approx \theta \), we get

\[
\frac{d^2 \theta}{dt^2} = -\frac{p_m B}{I} \theta
\]

This linear second order homogeneous differential equation is a Simple Harmonic differential equation. Therefore,

\[\omega^2 = \frac{p_mB}{I} \Rightarrow \omega = \sqrt{\frac{p_mB}{I}}\]

\[T = 2\pi \sqrt{\frac{I}{p_mB}}\]

\[T = 2\pi \sqrt{\frac{I}{p_mB_{H}}} \text{ in second}\]

where, \( B_{H} \) is the horizontal component of Earth’s magnetic field.

### 3.3.1. Potential energy of a bar magnet in a uniform magnetic field

Figure 3.20: A bar magnet (magnetic dipole) in a uniform magnetic field

When a bar magnet (magnetic dipole) of dipole moment \( \vec{p}_m \) is held at an angle \( \theta \) with the direction of a uniform magnetic field \( \vec{B} \), as shown in Figure 3.20 the magnitude of the torque acting on the dipole is

\[|\tau_\theta| = |\vec{p}_m| |\vec{B}| \sin \theta\]

If the dipole is rotated through a very small angular displacement \( d\theta \) against the torque \( \tau_\theta \) at constant angular velocity, then the work done by external torque \( \vec{\tau}_{ext} \) for this small angular displacement is given by

\[dW = |\vec{\tau}_{ext}| d\theta\]

The bar magnet has to be moved at constant angular velocity, which implies that \( |\vec{\tau}_{\theta}| = |\vec{\tau}_{\text{ext}}| \)

\[dW = p_mB \sin \theta d\theta\]

Total work done in rotating the dipole from \( \theta' \) to \( \theta \) is

\[W = \int_{\theta'}^{\theta} \tau d\theta = \int_{\theta'}^{\theta} p_mB \sin \theta d\theta = p_mB \left[ -\cos \theta \right]_{\theta'}^{\theta} \]

\[W = -p_mB (\cos \theta - \cos \theta')\]
This work done is stored as potential energy in a bar magnet at an angle \( \theta \) when it is rotated from \( \theta' \) to \( \theta \) and it can be written as

\[
U = -p_m B (\cos \theta - \cos \theta')
\]  

(3.26)

In fact, the equation (3.26) gives the difference in potential energy between the angular positions \( \theta' \) and \( \theta \). We can choose the reference point \( \theta' = 90^\circ \), so that second term in the equation becomes zero and the equation (3.26) can be written as

\[
U = -p_m B (\cos \theta)
\]  

(3.27)

The potential energy stored in a bar magnet in a uniform magnetic field is given by

\[
U = -\vec{p}_m \cdot \vec{B}
\]  

(3.28)

**Case 1**

(i) If \( \theta = 0^\circ \), then

\[
U = -p_m B (\cos 0^\circ) = -p_m B
\]

(ii) If \( \theta = 180^\circ \), then

\[
U = -p_m B (\cos 180^\circ) = p_m B
\]

We can infer from the above two results, the potential energy of the bar magnet is minimum when it is aligned along the external magnetic field and maximum when the bar magnet is aligned anti-parallel to external magnetic field.

**EXAMPLE 3.8**

Consider a magnetic dipole which on switching ON external magnetic field orient only in two possible ways i.e., one along the direction of the magnetic field (parallel to the field) and another anti-parallel to magnetic field. Compute the energy for the possible orientation. Sketch the graph.

---

**Solution**

Let \( \vec{p}_m \) be the dipole and before switching ON the external magnetic field, there is no orientation. Therefore, the energy \( U = 0 \).

As soon as external magnetic field is switched ON, the magnetic dipole orient parallel \((\theta = 0^\circ)\) to the magnetic field with energy,

\[
U_{\text{parallel}} = U_{\text{maximum}} = -p_m B \cos 0
\]

\[
U_{\text{parallel}} = -p_m B
\]

Since \( \cos 0^\circ = 1 \)

Otherwise, the magnetic dipole orients anti-parallel \((\theta = 180^\circ)\) to the magnetic field with energy,

\[
U_{\text{anti-parallel}} = U_{\text{maximum}} = -p_m B \cos 180
\]

\[
U_{\text{anti-parallel}} = p_m B
\]

Since \( \cos 180^\circ = -1 \)

---

**3.3.2 Tangent law and Tangent Galvanometer**

Tangent Galvanometer (Figure 3.21) is a device used to measure very small currents. It is a moving magnet type galvanometer. Its working is based on tangent law.
of the equipment we use in laboratory consists of 2 turns, 5 turns and 50 turns which are of different thickness and are used for measuring currents of different strengths.

At the center of turn table, a small upright projection is seen on which compass box (also known as magnetometre box) is placed. Compass box consists of a small magnetic needle which is pivoted at the center, such that arrangement shows the center of both magnetic needle and circular coil exactly coincide. A thin aluminium pointer is attached to the magnetic needle normally and moves over circular scale. The circular scale is divided into four quadrants and graduated in degrees which are used to measure the deflection of aluminium pointer on a circular degree scale. In order to avoid parallax error in measurement, a mirror is placed below the aluminium pointer.

**Tangent law**

*When a magnetic needle or magnet is freely suspended in two mutually perpendicular uniform magnetic fields, it will come to rest in the direction of the resultant of the two fields.*

Let $B$ be the magnetic field produced by passing current through the coil of the tangent Galvanometer and $B_H$ be the horizontal component of earth's magnetic field. Under the action of two magnetic fields, the needle comes to rest making angle $\theta$ with $B_H$, such that

$$B = B_H \tan \theta \quad (3.29)$$

**Construction**

Tangent Galvanometer (TG) consists of copper coil wound on a non-magnetic circular frame. The frame is made up of brass or wood which is mounted vertically on a horizontal base table (turn table) with three levelling screws as shown in Figure 3.22. The TG is provided with two or more coils of different number of turns. Most

**Precautions**

1. All the nearby magnets and magnetic materials are kept away from the instrument.
2. Using spirit level, the levelling screws at the base are adjusted so that the small
magnetic needle is exactly horizontal and also coil (mounted on the frame) is exactly vertical.

3. The plane of the coil is kept parallel to the small magnetic needle by rotating the coil about its vertical axis. So, the coil remains in magnetic meridian.

4. The compass box (as shown in Figure 3.23) is rotated such that the pointer reads $0^\circ - 0^\circ$.

**Theory**

The circuit connection for Tangent Galvanometer (TG) experiment is shown in Figure 3.24. When no current is passed through the coil, the small magnetic needle lies along horizontal component of Earth’s magnetic field. When the circuit is switched ON, the electric current will pass through the circular coil and produce magnetic field. The magnetic field produced due to the circulatory electric current is discussed (in section 3.8.3). Now there are two fields which are acting mutually perpendicular to each other. They are:

1. the magnetic field ($B$) due to the electric current in the coil acting normal to the plane of the coil.
2. the horizontal component of Earth’s magnetic field ($B_H$)

Because of these crossed fields, the pivoted magnetic needle deflects through an angle $\theta$. From tangent law (equation 3.29),

$$B = B_H \tan \theta$$

When an electric current is passed through a circular coil of radius $R$ having $N$ turns, the magnitude of magnetic field at the center is

$$B = \frac{\mu N I}{2R}$$

From equation (3.29) and equation (3.30), we get

$$\mu \frac{N I}{2R} = B_H \tan \theta$$

The horizontal component of Earth’s magnetic field can be determined as

$$B_H = \mu \frac{N I}{2R \tan \theta} \text{ in tesla}$$
3.4 MAGNETIC PROPERTIES

All the materials we use are not magnetic materials. Further, all the magnetic materials will not behave identically. So, in order to differentiate one magnetic material from another, we need to know some basic parameters. They are:

(a) Magnetising field

The magnetic field which is used to magnetize a sample or specimen is called the magnetising field. Magnetising field is a vector quantity and it denoted by \( \vec{H} \) and its unit is A m\(^{-1}\).

(b) Magnetic permeability

The magnetic permeability can be defined as the measure of ability of the material to allow the passage of magnetic field lines through it or measure of the capacity of the substance to take magnetisation or the degree of penetration of magnetic field through the substance.

In free space, the permeability (or absolute permeability) is denoted by \( \mu_0 \) and for any medium it is denoted by \( \mu \). The relative permeability \( \mu_r \) is defined as the ratio between absolute permeability of the medium to the permeability of free space.

\[
\mu_r = \frac{\mu}{\mu_0} \quad (3.32)
\]

Relative permeability is a dimensionless number and has no units. For free space (air or vacuum), the relative permeability is unity i.e., \( \mu_r = 1 \). In isotropic medium, \( \mu \) is a scalar but for non-isotropic medium, \( \mu \) is a tensor.

**EXAMPLE 3.9**

A coil of a tangent galvanometer of diametre 0.24 m has 100 turns. If the horizontal component of Earth’s magnetic field is \( 25 \times 10^{-6} \) T then, calculate the current which gives a deflection of 60°.

**Solution**

The diameter of the coil is 0.24 m. Therefore, radius of the coil is 0.12 m.

Number of turns is 100 turns.

Earth’s magnetic field is \( 25 \times 10^{-6} \) T

Deflection is

\[
\theta = 60° \Rightarrow \tan 60° = \sqrt{3} = 1.732
\]

\[
I = \frac{2RB}{\mu N} \tan \theta
\]

\[
= \frac{2 \times 0.12 \times 25 \times 10^{-6}}{4 \times 10^{-7} \times 3.14 \times 100} \times 1.732 = 0.82 \times 10^{-1} \text{ A.}
\]

\[
I = 0.082 \text{ A}
\]
The unit of magnetisation is ampere metre$^{-1}$. For a bar magnet of pole strength $q_m$ length $2l$ and area of cross-section $A$, the magnetic moment of the bar magnet is $\vec{p}_m = q_m \vec{2l}$ and volume of the bar magnet is $V = A \frac{2l}{2} = 2lA$. The intensity of magnetisation for a bar magnet is

$$\vec{M} = \frac{\text{magnetic moment}}{\text{volume}} = \frac{q_m \vec{2l}}{2lA} \quad (3.34)$$

In magnitude, equation (3.34) is

$$|\vec{M}| = M = \frac{q_m \times 2l}{2l \times A} \Rightarrow M = \frac{q_m}{A}$$

This means, for a bar magnet the intensity of magnetisation can be defined as the pole strength per unit area (face area).

(d) Magnetic induction or total magnetic field

When a substance like soft iron bar is placed in an uniform magnetising field $\vec{H}$, it becomes a magnet, which means that the substance gets magnetised. The magnetic induction (total magnetic field) inside the specimen $\vec{B}$ is equal to the sum of the magnetic field $\vec{B}_m$ produced in vacuum due to the magnetising field and the magnetic field $\vec{B}_m$ due to the induced magnetisation of the substance.

$$\vec{B} = \vec{B}_o + \vec{B}_m = \mu_0 \vec{H} + \mu_1 \vec{I}$$

$$\Rightarrow \vec{B} = \vec{B}_o + \vec{B}_m = \mu_1 (\vec{H} + \vec{I}) \quad (3.35)$$
(e) Magnetic susceptibility

When a substance is kept in a magnetising field \( \vec{H} \), magnetic susceptibility gives information about how a material respond to the external (applied) magnetic field. In other words, the magnetic susceptibility measures, how easily and how strongly a material can be magnetised. It is defined as the ratio of the intensity of magnetisation \( (\vec{M}) \) induced in the material due to the magnetising field \( (\vec{H}) \)

\[
\chi_m = \frac{\vec{M}}{\vec{H}} \quad (3.36)
\]

It is a dimensionless quantity. For an isotropic medium, susceptibility is a scalar but for non-isotropic medium, susceptibility is a tensor. Magnetic susceptibility for some of the isotropic substances is given in Table 3.1.

<table>
<thead>
<tr>
<th>Material</th>
<th>Magnetic susceptibility ( (\chi_m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium</td>
<td>( 2.3 \times 10^{-5} )</td>
</tr>
<tr>
<td>Copper</td>
<td>( -0.98 \times 10^{-5} )</td>
</tr>
<tr>
<td>Diamond</td>
<td>( -2.2 \times 10^{-5} )</td>
</tr>
<tr>
<td>Gold</td>
<td>( -3.6 \times 10^{-5} )</td>
</tr>
<tr>
<td>Mercury</td>
<td>( -3.2 \times 10^{-5} )</td>
</tr>
<tr>
<td>Silver</td>
<td>( -2.6 \times 10^{-5} )</td>
</tr>
<tr>
<td>Titanium</td>
<td>( 7.06 \times 10^{-5} )</td>
</tr>
<tr>
<td>Tungsten</td>
<td>( 6.8 \times 10^{-5} )</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>( -2.3 \times 10^{-9} )</td>
</tr>
<tr>
<td>(1 atm)</td>
<td></td>
</tr>
<tr>
<td>Oxygen (1 atm)</td>
<td>( 2090 \times 10^{-9} )</td>
</tr>
</tbody>
</table>

**EXAMPLE 3.10**

Compute the intensity of magnetisation of the bar magnet whose mass, magnetic moment and density are 200 g, 2 A m\(^2\) and 8 g cm\(^{-3}\), respectively.

**Solution**

Density of the magnet is

\[
\text{Density} = \frac{\text{Mass}}{\text{Volume}} \Rightarrow \text{Volume} = \frac{\text{Mass}}{\text{Density}}
\]

\[
\text{Volume} = \frac{200 \times 10^{-3} \text{ kg}}{(8 \times 10^{-3} \text{ kg}) \times 10^6 \text{ m}^3} = 25 \times 10^{-6} \text{ m}^3
\]

Magnitude of magnetic moment \( p_m = 2 \text{ A m}^2 \)

Intensity of magnetization,

\[
I = \frac{\text{Magnetic moment}}{\text{Volume}} = \frac{2}{25 \times 10^{-6}}
\]

\[ M = 0.8 \times 10^5 \text{ A m}^{-1} \]

**EXAMPLE 3.11**

Using the relation \( \vec{B} = \mu_r (\vec{H} + \vec{M}) \), show that \( \chi_m = \mu_r - 1 \).

**Solution**

\[ \vec{B} = \mu_r (\vec{H} + \vec{M}) \]

But from equation (3.36), in vector form,

\[ \vec{M} = \chi_m \vec{H} \]

Hence,

\[ \vec{B} = \mu_r (\chi_m + 1)\vec{H} \Rightarrow \vec{B} = \mu \vec{H} \]

where,

\[ \mu = \mu_r (\chi_m + 1) \Rightarrow \chi_m + 1 = \frac{\mu}{\mu_r} = \mu_r \]

\[ \Rightarrow \chi_m = \mu_r - 1 \]
planes are oriented in random manner, the vector sum of magnetic moments is zero and there is no resultant magnetic moment for each atom.

In the presence of an external magnetic field, some electrons are speeded up and some are slowed down. The electrons whose moments were anti-parallel are speeded up according to Lenz’s law and this produces an induced magnetic moment in a direction opposite to the field. The induced moment disappears as soon as the external field is removed.

When placed in a non-uniform magnetic field, the interaction between induced magnetic moment and the external field creates a force which tends to move the material from stronger part to weaker part of the external field. It means that diamagnetic material is repelled by the field.

This action is called diamagnetic action and such materials are known as diamagnetic materials. Examples: Bismuth, Copper and Water etc.

The properties of diamagnetic materials are

i) Magnetic susceptibility is negative.
ii) Relative permeability is slightly less than unity.
iii) The magnetic field lines are repelled or expelled by diamagnetic materials when placed in a magnetic field.
iv) Susceptibility is nearly temperature independent.

3.5 CLASSIFICATION OF MAGNETIC MATERIALS

The magnetic materials are generally classified into three types based on the behaviour of materials in a magnetising field. They are diamagnetic, paramagnetic and ferromagnetic materials which are dealt with in this section.

(a) Diamagnetic materials

The orbital motion of electrons around the nucleus produces a magnetic field perpendicular to the plane of the orbit. Thus each electron orbit has finite orbital magnetic dipole moment. Since the orbital planes are oriented in random manner, the vector sum of magnetic moments is zero and there is no resultant magnetic moment for each atom.

In the presence of an external magnetic field, some electrons are speeded up and some are slowed down. The electrons whose moments were anti-parallel are speeded up according to Lenz’s law and this produces an induced magnetic moment in a direction opposite to the field. The induced moment disappears as soon as the external field is removed.

When placed in a non-uniform magnetic field, the interaction between induced magnetic moment and the external field creates a force which tends to move the material from stronger part to weaker part of the external field. It means that diamagnetic material is repelled by the field.

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The properties of diamagnetic materials are

i) Magnetic susceptibility is negative.
ii) Relative permeability is slightly less than unity.
iii) The magnetic field lines are repelled or expelled by diamagnetic materials when placed in a magnetic field.
iv) Susceptibility is nearly temperature independent.

EXAMPLE 3.12

Two materials X and Y are magnetised, whose intensity of magnetisation are 500 A m⁻¹ and 2000 A m⁻¹, respectively. If the magnetising field is 1000 A m⁻¹, then which one among these materials can be easily magnetized?

Solution

The susceptibility of material X is

\[ \chi_{m,X} = \frac{M}{H} = \frac{500}{1000} = 0.5 \]

The susceptibility of material Y is

\[ \chi_{m,Y} = \frac{M}{H} = \frac{2000}{1000} = 2 \]

Since, susceptibility of material Y is greater than that of material X, material Y can be easily magnetized than X.
Magnetic levitated train

Magnetic levitated train is also called as Maglev train. This train floats above few centimetre from the guideway because of electromagnet used. Maglev train does not need wheels and also achieve greater speed. The basic mechanism of working of Maglev train involves two sets of magnets. One set is used to repel which makes train to float above the track and another set is used to move the floating train ahead at very great speed. These trains are quieter, smoother and environmental friendly compared conventional trains and have potential for moving with much higher speeds with technology in future.

(b) Paramagnetic materials

In some magnetic materials, each atom or molecule has net magnetic dipole moment which is the vector sum of orbital and spin magnetic moments of electrons. Due to the random orientation of these magnetic moments, the net magnetic moment of the materials is zero.

In the presence of an external magnetic field, the torque acting on the atomic dipoles will align them in the field direction. As a result, there is net magnetic dipole moment induced in the direction of the applied field. The induced dipole moment is present as long as the external field exists.

When placed in a non-uniform magnetic field, the paramagnetic materials will have a tendency to move from weaker to stronger part of the field. Materials which exhibit weak magnetism in the direction of the applied field are known as paramagnetic materials. Examples: Aluminium, Platinum and chromium etc.

The properties of paramagnetic materials are:

i) Magnetic susceptibility is positive and small.

ii) Relative permeability is greater than unity.

iii) The magnetic field lines are attracted into the paramagnetic materials when placed in a magnetic field.

iv) Susceptibility is inversely proportional to temperature.

Curie’s law

When temperature is increased, thermal vibration will upset the alignment of magnetic dipole moments. Therefore, the magnetic susceptibility decreases with increase in temperature. In many cases, the susceptibility of the materials is
This relation is called Curie’s law. Here \( C \) is called Curie constant and temperature \( T \) is in kelvin. The graph drawn between magnetic susceptibility and temperature is shown in Figure 3.26, which is a rectangular hyperbola.

\[
\chi_m \propto \frac{1}{T} \quad \text{or} \quad \chi_m = \frac{C}{T}
\]

In the presence of external magnetic field, two processes take place

1. the domains having magnetic moments parallel to the field grow in size
2. the other domains (not parallel to field) are rotated so that they are aligned with the field.

As a result of these mechanisms, there is a strong net magnetisation of the material in the direction of the applied field (Figure 3.28).

(c) Ferromagnetic materials

An atom or a molecule in a ferromagnetic material possesses net magnetic dipole moment as in a paramagnetic material. A ferromagnetic material is made up of smaller regions, called ferromagnetic domain (Figure 3.27). Within each domain, the magnetic moments are spontaneously aligned in a direction. This alignment is caused by strong interaction arising from electron spin which depends on the inter-atomic distance. Each domain has net magnetisation in a direction. However the direction of magnetisation varies from domain to domain and thus net magnetisation of the specimen is zero.

When placed in a non-uniform magnetic field, the ferromagnetic materials will have a strong tendency to move from weaker to stronger part of the field. Materials which exhibit strong magnetism in the direction of applied field are called ferromagnetic materials. Examples: Iron, Nickel and Cobalt.
The properties of ferromagnetic materials are:

i) Magnetic susceptibility is positive and large.

ii) Relative permeability is large.

iii) The magnetic field lines are strongly attracted into the ferromagnetic materials when placed in a magnetic field.

iv) Susceptibility is inversely proportional to temperature.

Curie-Weiss law

As temperature increases, the ferromagnetism decreases due to the increased thermal agitation of the atomic dipoles. At a particular temperature, ferromagnetic material becomes paramagnetic. This temperature is known as Curie temperature $T_C$. The susceptibility of the material above the Curie temperature is given by

$$\chi_m = \frac{C}{T - T_C}$$

This relation is called Curie-Weiss law. The constant $C$ is called Curie constant and temperature $T$ is in kelvin. A plot of magnetic susceptibility with temperature is as shown in Figure 3.29.

**Figure 3.29** Curie-Weiss law – Susceptibility vs temperature

**Spin**

Like mass and charge for particles, spin is also another important attribute for an elementary particle. Spin is a quantum mechanical phenomenon (this is discussed in Volume 2) which is responsible for magnetic properties of the material. Spin in quantum mechanics is entirely different from spin we encounter in classical mechanics. Spin in quantum mechanics does not mean rotation; it is intrinsic angular momentum which does not have classical analogue. For historical reason, the name spin is retained. Spin of a particle takes only positive values but the orientation of the spin vector takes plus or minus values in an external magnetic field. For an example, electron has spin $s = \frac{1}{2}$. In the presence of magnetic field, the spin will orient either parallel or anti-parallel to the direction of magnetic field.

This implies that the magnetic spin $m_s$ takes two values for an electron, such as $m_s = \frac{1}{2}$ (spin up) and $m_s = -\frac{1}{2}$ (spin down). Spin for proton and neutron is $s = \frac{1}{2}$. For a photon is spin $s = 1$. 
### Magnetism and Magnetic Effects of Electric Current

<table>
<thead>
<tr>
<th>Type of Magnetism</th>
<th>Magnetising Field is Absent ((H = 0))</th>
<th>Magnetising Field is Present ((H \neq 0))</th>
<th>Magnetisation of the Material</th>
<th>Susceptibility</th>
<th>Relative Permeability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diamagnetism</td>
<td><img src="image" alt="Diamagnetism" /></td>
<td><img src="image" alt="Diamagnetism" /></td>
<td>(M)</td>
<td>Negative</td>
<td>Less than unity</td>
</tr>
<tr>
<td></td>
<td>(Zero magnetic moment)</td>
<td>(Aligned opposite to the field)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paramagnetism</td>
<td><img src="image" alt="Paramagnetism" /></td>
<td><img src="image" alt="Paramagnetism" /></td>
<td>(M)</td>
<td>Positive and small</td>
<td>Greater than unity</td>
</tr>
<tr>
<td></td>
<td>(Net magnetic moment but random alignment)</td>
<td>(Aligned with the field)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ferromagnetism</td>
<td><img src="image" alt="Ferromagnetism" /></td>
<td><img src="image" alt="Ferromagnetism" /></td>
<td>(M)</td>
<td>Positive and large</td>
<td>Very large</td>
</tr>
<tr>
<td></td>
<td>(Net magnetic moment in a domain but random alignment of domains)</td>
<td>(Aligned with the field)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

with magnetising field \(\vec{H}\) is not linear. It means that the ratio \(\frac{B}{H} = \mu\) is not a constant.

Let us study this behaviour in detail.

A ferromagnetic material (example, Iron) is magnetised slowly by a magnetising field \(\vec{H}\). The magnetic induction \(\vec{B}\) of the material increases from point A with the magnitude of the magnetising field and then attains a saturated level. This response of the material is depicted by the path AC as shown in Figure 3.30. Saturation magnetization is defined as the maximum point up to which the material can be magnetised by applying the magnetising field.

If the magnetising field is now reduced, the magnetic induction also decreases but does not retrace the original path CA. It takes different path CD. When the magnetising field is zero, the magnetic induction is not zero and it has positive value. This implies that some

**HYSTERESIS**

When a ferromagnetic material is kept in a magnetising field, the material gets magnetised by induction. An important characteristic of ferromagnetic material is that the variation of magnetic induction \(\vec{B}\)
of the reverse magnetising field for which the residual magnetism of the material vanishes is called its coercivity.

Further increase of $\vec{H}$ in the reverse direction, the magnetic induction increases along EF until it reaches saturation at F in the reverse direction. If magnetising field is decreased and then increased with direction reversed, the magnetic induction traces the path FGKC. This closed curve ACDEFGKC is called hysteresis loop and it represents a cycle of magnetisation.

In the entire cycle, the magnetic induction $B$ lags behind the magnetising field $H$. This phenomenon of lagging of magnetic induction behind the magnetising field is called hysteresis. Hysteresis means ‘lagging behind’.

Hysteresis loss

During the magnetisation of the specimen through a cycle, there is loss of energy in the form of heat. This loss is attributed to the rotation and orientation of molecular magnets in various directions. It is found that the energy lost (or dissipated) per unit volume of the material when it is carried through one cycle of magnetisation is equal to the area of the hysteresis loop. Thus, the loss of energy for a complete cycle is $\Delta E$,

$$\Delta E = \oint \vec{H} \cdot d\vec{B}$$

where $\vec{B}$ is in ampere – metre$^2$ and $\vec{H}$ is in ampere per meter. The loss in energy is measured in joules.

Hard and soft magnetic materials

Based on the shape and size of the hysteresis loop, ferromagnetic materials are classified as soft magnetic materials with smaller area and hard magnetic materials

**Figure 3.30** Hysteresis – plot for $B$ vs $H$
Applications of hysteresis loop

The significance of hysteresis loop is that it provides information such as retentivity, coercivity, permeability, susceptibility and energy loss during one cycle of magnetisation for each ferromagnetic material. Therefore, the study of hysteresis loop will help us in selecting proper and suitable material for a given purpose. Some examples:

i) Permanent magnets:

The materials with high retentivity, high coercivity and high permeability are suitable for making permanent magnets.
Examples: Steel and Alnico

ii) Electromagnets:

The materials with high initial permeability, low retentivity, low coercivity and thin hysteresis loop with smaller area are preferred to make electromagnets.

![Comparison of two ferromagnetic materials – hysteresis loop](image)

### Table 3.2 Difference between soft and hard ferromagnetic materials

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Properties</th>
<th>Soft ferromagnetic materials</th>
<th>Hard ferromagnetic materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>When external field is removed</td>
<td>Magnetisation disappears</td>
<td>Magnetisation persists</td>
</tr>
<tr>
<td>2</td>
<td>Area of the loop</td>
<td>Small</td>
<td>Large</td>
</tr>
<tr>
<td>3</td>
<td>Retentivity</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>4</td>
<td>Coercivity</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>5</td>
<td>Susceptibility and magnetic permeability</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>6</td>
<td>Hysteresis loss</td>
<td>Less</td>
<td>More</td>
</tr>
<tr>
<td>7</td>
<td>Uses</td>
<td>Solenoid core, transformer core and electromagnets</td>
<td>Permanent magnets</td>
</tr>
<tr>
<td>8</td>
<td>Examples</td>
<td>Soft iron, Mumetal, Stalloy etc.</td>
<td>Steel, Alnico, Lodestone etc.</td>
</tr>
</tbody>
</table>
Examples: Soft iron and Mumetal (Nickel Iron alloy).

iii) Core of the transformer:
The materials with high initial permeability, large magnetic induction and thin hysteresis loop with smaller area are needed to design transformer cores.

Examples: Soft iron

**EXAMPLE 3.13**

The following figure shows the variation of intensity of magnetisation with the applied magnetic field intensity for three magnetic materials X, Y and Z. Identify the materials X, Y and Z.

---

**Solution**
The slope of M-H graph measures the magnetic susceptibility, which is

\[ \chi_m = \frac{M}{H} \]

Material X: Slope is positive and larger value. So, it is a ferromagnetic material.

Material Y: Slope is positive and lesser value than X. So, it could be a paramagnetic material.

Material Z: Slope is negative and hence, it is a diamagnetic material.

---

**3.3 MAGNETIC EFFECTS OF CURRENT**

**3.7.1 Oersted experiment**

In 1820, Hans Christian Oersted while preparing for his lecture in physics noticed that electric current passing through a wire deflects the nearby magnetic compass. By proper investigation, he observed that the deflection of magnetic compass is due to the change in magnetic field produced around current carrying conductor (Figure 3.32). When the direction of current is reversed, the magnetic compass deflects in opposite direction. This lead to the development of the theory ‘electromagnetism’ which unifies the two branches in physics, namely electricity and magnetism.
3.7.2 Magnetic field around a straight current carrying conductor and circular loop

(a) Current carrying straight conductor:

Suppose we keep a magnetic compass near a current carrying straight conductor, then the needle of the magnetic compass experiences a torque and deflects to align in the direction of the magnetic field at that point. Tracing out the direction shown by magnetic compass, we can draw the magnetic field lines at a distance. For a straight current carrying conductor, the nature of magnetic field is like concentric circles having their center at the axis of the conductor as shown in Figure 3.33 (a).

The direction of circular magnetic field lines will be clockwise or anticlockwise depending on the direction of current in the conductor. If the strength (or magnitude) of the current is increased then the density of the magnetic field will also increase. The strength of the magnetic field \( B \) decreases as the distance \( r \) from the conductor increases are shown in Figure 3.33 (b).

(b) Circular coil carrying current

Suppose we keep a magnetic compass near a current carrying circular conductor, then the needle of the magnetic compass experiences a torque and deflects to align in the direction of the magnetic field at that point. We can notice that at the points A and B in the vicinity of the coil, the magnetic field lines are circular. The magnetic field lines are nearly parallel to each other near...
the center of the loop, indicating that the field present near the center of the coil is almost uniform (Figure 3.34).

The strength of the magnetic field is increased if either the current in the coil or the number of turns or both are increased. The polarity (north pole or south pole) depends on the direction of current in the loop.

### 3.7.3 Right hand thumb rule

The right hand rule is a mnemonic to find the direction of magnetic field when the direction of current in a conductor is known.

### 3.7.4 Maxwell’s right hand cork screw rule

This rule is used to determine the direction of the magnetic field. If we rotate a right-handed screw using a screw driver, then the direction of current is same as the direction in which screw advances and the direction of rotation of the screw gives the direction of the magnetic field. (Figure 3.36)
EXAMPLE 3.14

The magnetic field shown in the figure is due to the current carrying wire. In which direction does the current flow in the wire?

Solution

Using right hand rule, current flows upwards.

3.8

BIOT - SAVART LAW

Soon after the Oersted’s discovery, both Jean-Baptiste Biot and Felix Savart in 1819 did quantitative experiments on the force experienced by a magnet kept near current carrying wire and arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the magnetic field. This is true for any shape of the conductor.

3.8.1 Definition and explanation of Biot- Savart law

Biot and Savart experimentally observed that the magnitude of magnetic field \( dB \) at a point \( P \) (Figure 3.37) at a distance \( r \) from the small elemental length taken on a conductor carrying current varies

(i) directly as the strength of the current \( I \)

(ii) directly as the magnitude of the length element \( dl \)

(iii) directly as the sine of the angle (say,\( \theta \)) between \( dl \) and \( \hat{r} \).

(iv) inversely as the square of the distance between the point \( P \) and length element \( dl \).

This is expressed as

\[
\frac{dB}{dl} \propto \frac{I}{r^2} \sin \theta
\]

\[
dB = k \frac{I}{r^2} \sin \theta
\]

where \( k = \frac{\mu_0}{4\pi} \) in SI units and \( k = 1 \) in CGS units. In vector notation,

\[
d\vec{B} = \frac{\mu_0}{4\pi} \frac{I}{r^2} \hat{d} \times \hat{r}
\]

Here vector \( d\vec{B} \) is perpendicular to both \( I \hat{d} \) (pointing the direction of current flow)
and the unit vector \( \hat{r} \) directed from \( dl \) toward point P (Figure 3.38).

\[ \vec{B} = \int d\vec{B} = \frac{\mu I}{4\pi} \int \frac{dl \times \hat{r}}{r^2} \quad (3.38) \]

where the integral is taken over the entire current distribution.

**Cases**

1. If the point P lies on the conductor, then \( \theta = 0^\circ \). Therefore, \( d\vec{B} \) is zero.
2. If the point lies perpendicular to the conductor, then \( \theta = 90^\circ \). Therefore, \( d\vec{B} \)

**Note**

Electric current is not a vector quantity. It is a scalar quantity. But electric current in a conductor has direction of flow. Therefore, the electric current flowing in a small elemental conductor can be taken as vector quantity i.e. \( I dl \)

**Similarities between Coulomb’s law and Biot-Savort’s law**

Electric and magnetic fields
- obey inverse square law, so they are long range fields.
- obey the principle of superposition and are linear with respect to source. In magnitude,

\[ E \propto q \]
\[ B \propto I dl \]

**Difference between Coulomb’s law and Biot-Savort’s law**

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Electric field</th>
<th>Magnetic field</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Produced by a scalar source i.e., an electric charge ( q )</td>
<td>Produced by a vector source i.e., current element ( I dl )</td>
</tr>
<tr>
<td>2</td>
<td>It is directed along the position vector joining the source and the point at which the field is calculated</td>
<td>It is directed perpendicular to the position vector ( \hat{r} ) and the current element ( I dl )</td>
</tr>
<tr>
<td>3</td>
<td>Does not depend on angle</td>
<td>Depends on the angle between the position vector ( \hat{r} ) and the current element ( I dl )</td>
</tr>
</tbody>
</table>
Note that the exponent of charge \( q \) (source) and exponent of electric field \( E \) is unity. Similarly, the exponent of current element \( Id \) (source) and exponent of magnetic field \( B \) is unity. In other words, electric field \( \vec{E} \) is proportional only to charge (source) and not on higher powers of charge \( (q^2, q^3, \text{etc}) \). Similarly, magnetic field \( \vec{B} \) is proportional to current element \( Idl \) (source) and not on square or cube or higher powers of current element. The cause and effect have linear relationship.

### 3.8.2 Magnetic field due to long straight conductor carrying current

**Figure 3.39** Magnetic field due to a long straight current carrying conductor

Consider a long straight wire \( NM \) with current \( I \) flowing from \( N \) to \( M \) as shown in Figure 3.39. Let \( P \) be the point at a distance \( a \) from point \( O \). Consider an element of length \( dl \) of the wire at a distance \( l \) from point \( O \) and \( \vec{r} \) be the vector joining the element \( dl \) with the point \( P \). Let \( \theta \) be the angle between \( dl \) and \( \vec{r} \). Then, the magnetic field at \( P \) due to the element is

\[
d\vec{B} = \frac{\mu I}{4\pi} \frac{dl}{r^2} \sin \theta \hat{n}
\]

The direction of the field is perpendicular to the plane of the paper and going into it. This can be determined by taking the cross product between two vectors \( dl \) and \( \vec{r} \) (let it be \( \vec{n} \)). The net magnetic field can be determined by integrating equation (3.38) with proper limits.

From the Figure 3.39, in a right angle triangle \( PAO \),

\[
\tan(\pi - \theta) = \frac{a}{l}
\]

\[
l = -\frac{a}{\tan \theta} \quad \text{(since} \tan(\pi - \theta) = -\tan \theta \text{)}
\]

\[
l = -acot \theta \quad \text{and} \quad r = asec \theta
\]

Differentiating,

\[
dl = acosec^2 \theta d\theta
\]

\[
d\vec{B} = \frac{\mu I}{4\pi} \frac{(acosec^2 \theta d\theta)}{(acosec \theta)^2} \sin \theta d\theta \hat{n}
\]

\[
d\vec{B} = \frac{\mu I}{4\pi} \frac{(acosec^2 \theta d\theta)}{a^2 \cos ec^2 \theta} \sin \theta d\theta \hat{n}
\]

\[
d\vec{B} = \frac{\mu I}{4\pi a} \sin \theta d\theta \hat{n}
\]

This is the magnetic field at a point \( P \) due to the current in small elemental length. Note that we have expressed the magnetic field \( OP \) in terms of angular coordinate i.e. \( \theta \). Therefore, the net magnetic field at the point \( P \) which can be obtained by integrating \( d\vec{B} \) by varying the angle from \( \theta = \phi_1 \) to \( \theta = \phi_2 \) is

\[
\vec{B} = \frac{\mu I}{4\pi a} \int_{\phi_1}^{\phi_2} \sin \theta d\theta \hat{n} = \frac{\mu I}{4\pi a} (\cos \phi_1 - \cos \phi_2) \hat{n}
\]

For a an infinitely long straight wire, \( \phi_1 = 0 \) and \( \phi_2 = \pi \), the magnetic field is
\[ \vec{B} = \frac{\mu I}{2\pi a} \hat{n} \quad (3.39) \]

Note that here \( \hat{n} \) represents the unit vector from the point O to P.

**EXAMPLE 3.15**

Calculate the magnetic field at a point P which is perpendicular bisector to current carrying straight wire as shown in figure.

![Diagram](image)

**Solution**

Let the length MN = \( y \) and the point P is on its perpendicular bisector. Let O be the point on the conductor as shown in figure. Therefore, \( OM = ON = \frac{y}{2} \), then

\[ \cos \phi_1 = \frac{\frac{y}{2}}{\sqrt{\frac{y^2}{4} + a^2}} = \frac{\text{adjacent length}}{\text{hypotenuse length}} \]

\[ = \frac{ON}{PN} = -\frac{\frac{y}{2}}{\sqrt{\frac{y^2}{4} + a^2}} = -\frac{y}{\sqrt{y^2 + 4a^2}} \]

\[ \cos \phi_2 = \frac{\text{adjacent length}}{\text{hypotenuse length}} = \frac{OM}{PM} \]

\[ = -\frac{\frac{y}{2}}{\sqrt{\frac{y^2}{4} + a^2}} = -\frac{y}{\sqrt{y^2 + 4a^2}} \]

Hence,

\[ \vec{B} = \frac{\mu I}{4\pi a} \frac{2y}{\sqrt{y^2 + 4a^2}} \hat{n} \]

For long straight wire, \( y \to \infty \),

\[ \vec{B} = \frac{\mu I}{2\pi a} \hat{n} \]

The result obtained is same as we obtained in equation (3.39).

**EXAMPLE 3.16**

Show that for a straight conductor, the magnetic field

\[ \vec{B} = \frac{\mu I}{4\pi a} (\cos \phi_1 - \cos \phi_2) \hat{n} \]

\[ = \frac{\mu I}{4\pi a} (\sin \theta_1 + \sin \theta_2) \hat{n} \]

![Diagram](image)

**Solution:**

In a right angle triangle OPN, let the angle \( \angle OPN = \theta_1 \) which implies, \( \phi_1 = \frac{\pi}{2} - \theta_1 \)
According to Biot-Savart’s law, the magnetic field at P due to the current element \( I \, dl \) is

\[
d\vec{B} = \frac{\mu_0 \, I \, dl \times \hat{r}}{4\pi \, r^2}
\]

The magnitude of magnetic field due to current element \( I \, dl \) at C and D are equal because of equal distance from the coil. The magnetic field \( dB \) due to each current element \( I \, dl \) is resolved into two components; \( dB \sin \theta \) along y-direction and \( dB \cos \theta \) along z-direction. Horizontal components of each current element cancels out while the vertical components (\( dB \cos \theta \) \( \hat{k} \)) alone contribute to total magnetic field at the point P.

If we integrate \( dl \) around the loop, \( dB \) sweeps out a cone as shown in Figure 3.40, then the net magnetic field \( \vec{B} \) at point P is

\[
\vec{B} = \int d\vec{B} = \int dB \cos \theta \, \hat{k}
\]

But \( \cos \theta = \frac{R}{(R^2 + Z^2)^{\frac{1}{2}}} \), using Pythagorus theorem \( r^2 = R^2 + Z^2 \) and integrating line element from 0 to \( 2\pi R \), we get

\[ dB = dB \sin \theta \]

**3.8.3 Magnetic field produced along the axis of the current carrying circular coil**

Consider a current carrying circular loop of radius R and let I be the current flowing through the wire in the direction as shown in Figure 3.40. The magnetic field at a point P on the axis of the circular coil at a distance \( z \) from its center of the coil O. It is computed by taking two diametrically opposite line elements of the coil each of length \( dl \) at C and D. Let \( \hat{r} \) be the vector joining the current element (\( I \, dl \)) at C to the point P.

\[ PC = PD = r = \sqrt{R^2 + Z^2} \text{ and } \]

angle \( \angle CPO = \angle DPO = \theta \]

![Figure 3.40 Current carrying circular loop using Biot-Savart's law](image-url)
Let A be the area of the circular loop
\[ A = \pi R^2. \]
So rewriting the equation (3.41) in terms of area of the loop, we have
\[ \vec{B} = \frac{\mu I A}{2\pi (R^2 + Z^2)^{3/2}} \hat{k} \]  
(3.40)

Note that the magnetic field \( \vec{B} \) points along the direction from the point O to P. Suppose if the current flows in clockwise direction, then magnetic field points in the direction from the point P to O.

**EXAMPLE 3.17**

What is the magnetic field at the center of the loop shown in figure?

![Image of a loop with magnetic field](image)

**Solution**

The magnetic field due to current in the upper hemisphere and lower hemisphere of the circular coil are equal in magnitude but opposite in direction. Hence, the net magnetic field at the center of the loop (at point O) is zero \( \vec{B} = \vec{0} \).

### 3.8.4 Current loop as a magnetic dipole

The magnetic field from the centre of a circular loop of radius R along the axis is given by
\[ \vec{B} = \frac{\mu I}{2\pi} \frac{R^2}{(R^2 + Z^2)^{3/2}} \hat{k} \]

At larger distance \( Z \gg R \), therefore \( R^2 + Z^2 \approx Z^2 \), we have
\[ \vec{B} = \frac{\mu I R^2}{2Z^3} \hat{k} \]  
(3.41)

This implies that a current carrying circular loop behaves as a magnetic dipole of magnetic moment \( \vec{p}_m \). So, the magnetic dipole moment of any current loop is equal to the product of the current and area of the loop.

**Right hand thumb rule**

In order to determine the direction of magnetic moment, we use right hand thumb rule (mnemonic) which states that

*If we curl the fingers of right hand in the direction of current in the loop, then the stretched thumb gives the direction of the magnetic moment associated with the loop.*

**Table 3.3** End rule – polarity with direction of current in circular loop

<table>
<thead>
<tr>
<th>Current in circular loop</th>
<th>Polarity</th>
<th>Picture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anti-clockwise current</td>
<td>North Pole</td>
<td><img src="image" alt="Anti-clockwise current" /></td>
</tr>
<tr>
<td>Clockwise current</td>
<td>South Pole</td>
<td><img src="image" alt="Clockwise current" /></td>
</tr>
</tbody>
</table>

**Unit 3 Magnetism and magnetic effects of electric current**

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3.8.5 Magnetic dipole moment of revolving electron

Suppose an electron undergoes circular motion around the nucleus as shown in Figure 3.41. The circulating electron in a loop is like current in a circular loop (since flow of charge is current). The magnetic dipole moment due to current carrying circular loop is

\[ \vec{\mu}_L = I \vec{A} \]  \hspace{1cm} (3.44)

In magnitude,

\[ \mu_L = IA \]

**Figure 3.41** (a) Electron revolving in a circular orbit (b) Direction of magnetic dipole moment vector and orbital angular momentum vector are opposite

If \( T \) is the time period of an electron, the current due to circular motion of the electron is

\[ I = -\frac{e}{T} \]  \hspace{1cm} (3.45)

where \(-e\) is the charge of an electron. If \( R \) is the radius of the circular orbit and \( v \) is the velocity of the electron in the circular orbit, then

\[ T = \frac{2\pi R}{v} \]  \hspace{1cm} (3.46)

Using equation (3.45) and equation (3.46) in equation (3.44), we get

\[ \mu_L = -\frac{e}{2\pi R} \pi R^2 = -\frac{evR}{2} \] \hspace{1cm} (3.47)

where \( A = \pi R^2 \) is the area of the circular loop. By definition, angular momentum of the electron about \( O \) is

\[ \vec{L} = \vec{R} \times \vec{p} \]

In magnitude,

\[ L = Rp = mvR \] \hspace{1cm} (3.48)

Using equation (3.47) and equation (3.48), we get

\[ \frac{\mu_L}{L} = \frac{evR}{2mvR} = -\frac{e}{2m} \Rightarrow \vec{\mu}_L = -\frac{e}{2m} \vec{L} \] \hspace{1cm} (3.49)

The negative sign indicates that the magnetic moment and angular momentum are in opposite direction.

In magnitude,

\[ \frac{\mu_L}{L} = \frac{e}{2m} \cdot \frac{1.60 \times 10^{-19}}{2 \times 9.11 \times 10^{-31}} = 0.0878 \times 10^{12} \]

\[ \frac{\mu_L}{L} = 8.78 \times 10^{10} \text{ C kg}^{-1} = \text{constant} \]

The ratio \( \frac{\mu_L}{L} \) is a constant and also known as gyro-magnetic ratio \( \left( \frac{e}{2m} \right) \). It must be noted that the gyro-magnetic ratio is a constant of proportionality which connects angular momentum of the electron and the magnetic moment of the electron.

According to Neil’s Bohr quantization rule, the angular momentum of an electron moving in a stationary orbit is quantized, which means,

\[ L = nh = n \frac{\hbar}{2\pi} \]
where, \( h \) is the Planck’s constant (\( h = 6.63 \times 10^{-34} \text{ J s} \)) and number \( n \) takes natural numbers

(i.e., \( n = 1, 2, 3, \ldots \)). Hence,

\[
\mu_I = \frac{e}{2m} = n \frac{eh}{4\pi m} \text{ Am}^2
\]

\[
\mu_I = n \left( \frac{1.60 \times 10^{-19}}{4\pi} \right) h \text{ Am}^2
\]

\[
= n \left( \frac{1.60 \times 10^{-19}}{4\pi} \right) (6.63 \times 10^{-34})
\]

\[
= \frac{4 \times 3.14 \times (9.11 \times 10^{-31})}{4 \times 3.14 \times (9.11 \times 10^{-31})}
\]

\[
\mu_I = n \times 9.27 \times 10^{-24} \text{ Am}^2
\]

The minimum magnetic moment can be obtained by substituting \( n = 1 \),

\[
\mu_I = 9.27 \times 10^{-24} \text{ Am}^2 = 9.27 \times 10^{-24} \text{ J T}^{-1}
\]

\[
= (\mu_L)_{\text{min}} = \mu_B
\]

where, \( \mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24} \text{ Am}^2 \) is called Bohr magneton. This is a convenient unit with which one can measure atomic magnetic moments.

Note: Bohr quantization rule will be discussed in unit 8 of second volume

### 3.9.1 Definition and explanation of Ampère’s circuital law

**Ampère’s law:** The line integral of magnetic field over a closed loop is \( \mu_0 \) times net current enclosed by the loop.

\[
\oint_B \overrightarrow{dl} = \mu_0 I_{\text{enclosed}} \tag{3.50}
\]

where \( I_{\text{enclosed}} \) is the net current linked by the closed loop \( C \). Note that the line integral does not depend on the shape of the path or the position of the conductor with the magnetic field.

**Note:** Line integral means integral over a line or curve, symbol used is \( \oint \).

Closed line integral means integral over a closed curve (or line), symbol is \( \int_C \) or \( \int_C \).

### 3.9.2 Magnetic field due to the current carrying wire of infinite length using Ampère’s law

![Figure 3.42 Ampèrian loop for current carrying straight wire](image)

Consider a straight conductor of infinite length carrying current \( I \) and the direction of
magnetic field lines is shown in Figure 3.42. Since the wire is geometrically cylindrical in shape and symmetrical about its axis, we construct an Ampèrian loop in the form of a circular shape at a distance \( r \) from the centre of the conductor as shown in Figure 3.42. From the Ampère’s law, we get

\[
\oint_c \mathbf{B} \cdot d\mathbf{l} = \mu_0 I
\]

where \( d\mathbf{l} \) is the line element along the amperian loop (tangent to the circular loop). Hence, the angle between magnetic field vector and line element is zero. Therefore,

\[
\oint_c B \cdot dl = \mu_0 I
\]

where \( I \) is the current enclosed by the Ampèrian loop. Due to the symmetry, the magnitude of the magnetic field is uniform over the Ampèrian loop, we can take \( B \) out of the integration.

\[
B \oint_c dl = \mu_0 I
\]

For a circular loop, the circumference is \( 2\pi r \), which implies,

\[
B \int_0^{2\pi r} dl = \mu_0 I
\]

\[
\mathbf{B} \cdot \mathbf{n} \cdot 2\pi r = \mu_0 I
\]

\[
B = \frac{\mu_0 I}{2\pi r}
\]

In vector form, the magnetic field is

\[
\mathbf{B} = \frac{\mu_0 I}{2\pi r} \mathbf{n}
\]

where \( \mathbf{n} \) is the unit vector along the tangent to the Ampèrian loop as shown in the Figure 3.42. This perfectly agrees with the result obtained from Biot-Savart’s law as given in equation (3.39).

**Example 3.18**

Compute the magnitude of the magnetic field of a long, straight wire carrying a current of 1 A at distance of 1 m from it. Compare it with Earth’s magnetic field.

**Solution**

Given that \( I = 1 \) A and radius \( r = 1 \) m

\[
B_{\text{straightwire}} = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 1} = 2 \times 10^{-7} \text{T}
\]

But the Earth’s magnetic field is \( B_{\text{Earth}} = 10^{-5} \text{T} \)

So, \( B_{\text{straightwire}} \) is one hundred times smaller than \( B_{\text{Earth}} \).

**Solenoid**

A solenoid is a long coil of wire closely wound in the form of helix as shown in Figure 3.43. When electric current is passed through the solenoid, the magnetic field is produced. The magnetic field of the solenoid is due to the superposition of magnetic fields of each turn of the solenoid. The direction of magnetic field due to solenoid is given by right hand palm-rule (mnemonic).

Inside the solenoid, the magnetic field is nearly uniform and parallel to its axis whereas, outside the solenoid the field is negligibly small. Based on the direction of the current, one end of the solenoid behaves like North Pole and the other end behaves like South Pole.

The current carrying solenoid is held in right hand. If the fingers curl in the direction of current, then extended thumb gives the direction of magnetic field of current carrying solenoid. It is shown in Figure 3.44. Hence, the magnetic field of a
solenoid looks like the magnetic field of a bar magnet.

The solenoid is assumed to be long which means that the length of the solenoid is large when compared to its diameter. The winding need not to be always circular, it can also be in other shapes. We consider here only circularly wound solenoid as shown in Figure 3.45.

3.9.3 Magnetic field due to a long current carrying solenoid

Consider a solenoid of length L having N turns. The diameter of the solenoid is assumed to be much smaller when compared to its length and the coil is wound very closely.

In order to calculate the magnetic field at any point inside the solenoid, we use Ampere's circuital law. Consider a rectangular loop abcd as shown in Figure 3.46. Then from Ampère's circuital law,
\[ \int_C B \cdot d\vec{l} = \mu_0 I_{\text{enclosed}} = \mu_0 \times (\text{total current enclosed by Amperian loop}) \]

The left hand side of the equation is

\[ \int_C B \cdot d\vec{l} = \int_{a}^{b} \int_{b}^{c} B \cdot d\vec{l} + \int_{c}^{d} B \cdot d\vec{l} = \int_{d}^{a} B \cdot d\vec{l} \]

Since the elemental lengths along bc and da are perpendicular to the magnetic field which is along the axis of the solenoid, the integrals

\[ \int_{a}^{b} B \cdot d\vec{l} = \int_{b}^{c} B \cdot d\vec{l} \cos 90^\circ = 0 \]
\[ \int_{d}^{a} B \cdot d\vec{l} = 0 \]

Since the magnetic field outside the solenoid is zero, the integral \( \int_{d}^{a} B \cdot d\vec{l} = 0 \)

For the path along ab, the integral is

\[ \int_{a}^{b} B \cdot d\vec{l} = B \int_{a}^{b} dl \cos 0^\circ = B \int_{a}^{b} dl \]

where the length of the loop ab as shown in the Figure 3.46 is h. But the choice of length of the loop ab is arbitrary. We can take very large loop such that it is equal to the length of the solenoid L. Therefore the integral is

\[ \int_{a}^{b} B \cdot d\vec{l} = BL \]

Let \( N \) I be the current passing through the solenoid of \( N \) turns, then

\[ \int_{a}^{b} B \cdot d\vec{l} = BL = \mu_0 NI \Rightarrow B = \mu_0 \frac{NI}{L} \]

The number of turns per unit length is given by \( \frac{N}{L} = n \), Then

\[ B = \mu_0 \frac{nLI}{L} = \mu_0 nI \quad (3.51) \]

Since \( n \) is a constant for a given solenoid and \( \mu_0 \) is also constant. For a fixed current I, the magnetic field inside the solenoid is also a constant.

**Note**

Solenoid can be used as electromagnets. It produces strong magnetic field that can be turned ON or OFF. This is not possible in case of permanent magnet. Further the strength of the magnetic field can be increased by keeping an iron bar inside the solenoid. This is because the magnetic field of the solenoid magnetizes the iron bar and hence the net magnetic field is the sum of magnetic field of the solenoid and magnetic field of magnetized iron. Because of these properties, solenoids are useful in designing variety of electrical appliances.

**EXAMPLE 3.19**

Calculate the magnetic field inside a solenoid, when

(a) the length of the solenoid becomes twice and fixed number of turns

(b) both the length of the solenoid and number of turns are double

(c) the number of turns becomes twice for the fixed length of the solenoid

Compare the results.

**Solution**

The magnetic field of a solenoid (inside) is

\[ B_{L,N} = \mu_0 \frac{NI}{L} \]
MRI is Magnetic Resonance Imaging which helps the physicians to diagnose or monitor treatment for a variety of abnormal conditions happening within the head, chest, abdomen and pelvis. It is a non invasive medical test. The patient is placed in a circular opening (actually interior of a solenoid which is made up of superconducting wire) and large current is sent through the superconducting wire to produce a strong magnetic field. So, it uses more powerful magnet, radio frequency pulses and a computer to produce pictures of organs which helps the physicians to examine various parts of the body.

(a) length of the solenoid becomes twice and fixed number of turns
   \( L \rightarrow 2L \) (length becomes twice)
   \( N \rightarrow N \) (number of turns are fixed)
   The magnetic field is
   \[
   B_{2L,N} = \mu_0 \frac{NI}{2L} = \frac{1}{2} B_{L,N}
   \]

(b) both the length of the solenoid and number of turns are double
   \( L \rightarrow 2L \) (length becomes twice)
   \( N \rightarrow 2N \) (number of turns becomes twice)
   The magnetic field is
   \[
   B_{2L,2N} = \mu_0 \frac{2NI}{2L} = B_{L,N}
   \]

(c) the number of turns becomes twice but for the fixed length of the solenoid
   \( L \rightarrow L \) (length is fixed)
   \( N \rightarrow 2N \) (number of turns becomes twice)
   The magnetic field is
   \[
   B_{L,2N} = \mu_0 \frac{2NI}{L} = 2B_{L,N}
   \]

From the above results,

\[
B_{L,2N} > B_{2L,2N} > B_{2L,N}
\]

Thus, strength of the magnetic field is increased when we pack more loops into the same length for a given current.

### 3.9.5 Toroid

A solenoid is bent in such a way its ends are joined together to form a closed ring shape, is called a toroid which is shown in Figure 3.47. The magnetic field has constant magnitude inside the toroid whereas in the interior region (say, at point P) and exterior region (say, at point Q), the magnetic field is zero.
of radius $r_3$ around the point $Q$ as shown in Figure 3.48. The length of the loop is

$$L_3 = 2\pi r_3$$

Ampère’s circuit law for the loop 3 is

$$\oint_{loop3} \vec{B}_Q \cdot d\vec{l} = \mu_0 I_{enclosed}$$

Since, in each turn of the toroid loop, current coming out of the plane of paper is cancelled by the current going into the plane of paper. Thus, $I_{enclosed} = 0$

$$\oint_{loop3} \vec{B}_Q \cdot d\vec{l} = 0$$

This is possible only if the magnetic field at point $Q$ vanishes i.e.

$$\vec{B}_Q = 0$$

(c) Inside the toroid

Let us calculate the magnetic field $B_S$ at point $S$ by constructing an Amperian loop 2 of radius $r_2$ around the point $S$ as shown in Figure 3.48. The length of the loop is

$$L_2 = 2\pi r_2$$

Ampere’s circuital law for the loop 2 is

$$\oint_{loop2} \vec{B}_S \cdot d\vec{l} = \mu_0 I_{enclosed}$$

Let $I$ be the current passing through the toroid and $N$ be the number of turns of the toroid, then

$$I_{enclosed} = NI$$

and

$$\oint_{loop2} \vec{B}_S \cdot d\vec{l} = \oint_{loop2} Bdl\cos\theta = B2\pi r_2$$

$$\oint_{loop2} \vec{B}_S \cdot d\vec{l} = \mu_0 NI$$
The equations (3.54) and (3.55) imply
1. $\vec{F}_m$ is directly proportional to the magnetic field $\vec{B}$
2. $\vec{F}_m$ is directly proportional to the velocity $\vec{v}$
3. $\vec{F}_m$ is directly proportional to sine of the angle between the velocity and magnetic field
4. $\vec{F}_m$ is directly proportional to the magnitude of the charge $q$
5. The direction of $\vec{F}_m$ is always perpendicular to $\vec{v}$ and $\vec{B}$ as $\vec{F}_m$ is the cross product of $\vec{v}$ and $\vec{B}$

7. The direction of $\vec{F}_m$ on negative charge is opposite to the direction of $\vec{F}_m$ on positive charge provided other factors are identical as shown Figure 3.49
8. If velocity $\vec{v}$ of the charge $q$ is along magnetic field $\vec{B}$ then, $\vec{F}_m$ is zero

**Definition of tesla**

The strength of the magnetic field is one tesla if unit charge moving in it with unit velocity experiences unit force.

$$1 \ T = \frac{1 N \ s}{C \ m} = \frac{1 N}{A \ m} = 1 N \ A^{-1} \ m^{-1}$$
EXAMPLE 3.20

A particle of charge q moves with velocity $\vec{v}$ along positive y-direction in a magnetic field $\vec{B}$. Compute the Lorentz force experienced by the particle (a) when magnetic field is along positive y-direction (b) when magnetic field points in positive z-direction (c) when magnetic field is in zy-plane and making an angle $\theta$ with velocity of the particle. Mark the direction of magnetic force in each case.

Solution

Velocity of the particle is $\vec{v} = v\hat{j}$

(a) Magnetic field is along positive y-direction, this implies, $\vec{B} = v\hat{j}$

From Lorentz force, $\vec{F}_m = q(\vec{v} \times \vec{B}) = 0$

So, no force acts on the particle when it moves along the direction of magnetic field.

(b) Magnetic field points in positive z-direction, this implies, $\vec{B} = B\hat{k}$

From Lorentz force, $\vec{F}_m = q(\vec{v} \times \vec{B}) = qvB\hat{i}$

Therefore, the magnitude of the Lorentz force is $qvB$ and direction is along positive x-direction.

(c) Magnetic field is in zy-plane and making an angle $\theta$ with the velocity of the particle, which implies $\vec{B} = B\cos\theta\hat{j} + B\sin\theta\hat{k}$

From Lorentz force,

$$\vec{F}_m = q(\vec{v} \times (B\cos\theta\hat{j} + B\sin\theta\hat{k}))$$

$$= qvB\sin\theta\hat{i}$$

EXAMPLE 3.21

Compute the work done and power delivered by the Lorentz force on the particle of charge q moving with velocity $\vec{v}$. Calculate the angle between Lorentz force and velocity of the charged particle and also interpret the result.

Solution

For a charged particle moving on a magnetic field, $\vec{F} = q(\vec{v} \times \vec{B})$

The work done by the magnetic field is

$$W = \int \vec{F} \cdot d\vec{r} = \int \vec{F} \cdot \vec{v} dt$$

$$W = q\int (\vec{v} \times \vec{B}) \cdot \vec{v} dt = 0$$

Since $\vec{v} \times \vec{B}$ is perpendicular to $\vec{v}$ and hence $(\vec{v} \times \vec{B}) \cdot \vec{v} = 0$ This means that Lorentz force do no work on the particle. From work-kinetic energy theorem, (Refer section 4.2.6, XI th standard Volume I)

$$\frac{dW}{dt} = P = 0$$
The Lorentz force on the charged particle is given by
\[ \vec{F} = q(\vec{v} \times \vec{B}) \]

Since Lorentz force alone acts on the particle, the magnitude of the net force on the particle is
\[ \sum F_i = F_n = qvB \]

This Lorentz force acts as centripetal force for the particle to execute circular motion. Therefore,
\[ qvB = m \frac{v^2}{r} \]

The radius of the circular path is
\[ r = \frac{mv}{qB} = \frac{p}{qB} \quad (3.56) \]

where \( p = mv \) is the magnitude of the linear momentum of the particle. Let \( T \) be the time taken by the particle to finish one complete circular motion, then
\[ T = \frac{2\pi r}{v} \quad (3.57) \]

Hence substituting (3.56) in (3.57), we get
\[ T = \frac{2\pi m}{qB} \quad (3.58) \]

Equation (3.58) is called the cyclotron period. The reciprocal of time period is the frequency \( f \), which is
\[ f = \frac{1}{T} \]

\[ f = \frac{qB}{2\pi m} \quad (3.59) \]

In terms of angular frequency \( \omega \),
\[ \omega = 2\pi f = \frac{qB}{m} \quad (3.60) \]
EXAMPLE 3.22

An electron moving perpendicular to a uniform magnetic field 0.500 T undergoes circular motion of radius 2.80 mm. What is the speed of electron?

Solution

Charge of an electron \( q = -1.60 \times 10^{-19} \text{ C} \)

\[ q = -1.60 \times 10^{-19} \text{ C} \]

Magnitude of magnetic field \( B = 0.500 \text{ T} \)

Mass of the electron, \( m = 9.11 \times 10^{-31} \text{ kg} \)

Radius of the orbit, \( r = 2.50 \text{ mm} = 2.50 \times 10^{-3} \text{ m} \)

Velocity of the electron,

\[ v = \frac{qB}{m} \]

\[ v = 1.60 \times 10^{-19} \times \frac{2.50 \times 10^{-3} \times 0.500}{9.11 \times 10^{-31}} \]

\[ v = 2.195 \times 10^{8} \text{ m/s} \]

EXAMPLE 3.23

A proton moves in a uniform magnetic field of strength 0.500 T magnetic field is directed along the x-axis. At initial time, \( t = 0 \text{ s} \), the proton has velocity \( \vec{v} = (1.95 \times 10^{5} \hat{i} + 2.00 \times 10^{5} \hat{k}) \text{ m/s} \). Find

(a) At initial time, what is the acceleration of the proton.

(b) Is the path circular or helical? If helical, calculate the radius of helical trajectory and also calculate the pitch of the helix (Note: Pitch of the helix is the distance travelled along the helix axis per revolution).

Solution

Magnetic field \( \vec{B} = 0.500\hat{i} \text{ T} \)
Velocity of the particle
\[ \vec{v} = (1.95 \times 10^5 \hat{i} + 2.00 \times 10^5 \hat{k}) \text{ m s}^{-1} \]

Charge of the proton \( q = 1.60 \times 10^{-19} \text{ C} \)

Mass of the proton \( m = 1.67 \times 10^{-27} \text{ kg} \)

(a) The force experienced by the proton is
\[ \vec{F} = q(\vec{v} \times \vec{B}) = 1.60 \times 10^{-19} \times (1.95 \times 10^5 \hat{i} + 2.00 \times 10^5 \hat{k} \times 0.500 \hat{j}) \]
\[ \vec{F} = 1.60 \times 10^{-14} \text{ N} \hat{j} \]

Therefore, from Newton’s second law,
\[ \vec{a} = \frac{1}{m} \vec{F} = \frac{1}{1.67 \times 10^{-27}} (1.60 \times 10^{-14}) \]
\[ = 9.58 \times 10^{12} \text{ m s}^{-2} \]

(b) Trajectory is helical

Radius of helical path is
\[ R = \frac{qv}{|q|B} = \frac{1.67 \times 10^{-27} \times 2.00 \times 10^5}{1.60 \times 10^{-19} \times 0.500} \]
\[ = 4.175 \times 10^{-3} \text{ m} = 4.18 \text{ mm} \]

Pitch of the helix is the distance travelled along \( x \)-axis in a time \( T \), which is \( P = v_x T \)

But time,
\[ T = \frac{2\pi}{\omega} = \frac{2\pi m}{|q|B} = \frac{2 \times 3.14 \times 1.67 \times 10^{-27}}{1.60 \times 10^{-19} \times 0.500} \]
\[ = 13.1 \times 10^{-8} \text{ s} \]

Hence, pitch of the helix is
\[ P = v_x T = (1.95 \times 10^5)(13.1 \times 10^{-8}) \]
\[ = 25.5 \times 10^{-3} \text{ m} = 25.5 \text{ mm} \]

The proton experiences appreciable acceleration in the magnetic field, hence the pitch of the helix is almost six times greater than the radius of the helix.

**EXAMPLE 3.24**

Two singly ionized isotopes of uranium \( ^{235}\text{U} \) and \( ^{238}\text{U} \) (isotopes have same atomic number but different mass number) are sent with velocity \( 1.00 \times 10^5 \text{ m s}^{-1} \) into a magnetic field of strength \( 0.500 \text{ T} \) normally. Compute the distance between the two isotopes after they complete a semi-circle. Also compute the time taken by each isotope to complete one semi-circular path. (Given: masses of the isotopes: \( m_{235} = 3.90 \times 10^{-25} \text{ kg} \) and \( m_{238} = 3.95 \times 10^{-25} \text{ kg} \))

**Solution**

Since isotopes are singly ionized, they have equal charge which is equal to the charge of an electron, \( q = -1.6 \times 10^{-19} \text{ C} \). Mass of uranium \( ^{235}\text{U} \) and \( ^{238}\text{U} \) are \( 3.90 \times 10^{-25} \text{ kg} \) and \( 3.95 \times 10^{-25} \text{ kg} \) respectively. Magnetic field applied, \( B = 0.500 \text{ T} \). Velocity of the electron is \( 1.00 \times 10^5 \text{ m s}^{-1} \), then

(a) The radius of the path of \( ^{235}\text{U} \) is
\[ r_{235} = \frac{mv}{|q|B} = \frac{3.90 \times 10^{-25} \times 1.00 \times 10^5}{1.6 \times 10^{-19} \times 0.500} \]
\[ = 48.8 \times 10^{-3} \text{ m} \]
\[ r_{235} = 48.8 \text{ cm} \]

The diameter of the semi-circle due to \( ^{235}\text{U} \) is \( d_{235} = 2r_{235} = 97.6 \text{ cm} \)

The radius of the path of \( ^{238}\text{U} \) is \( r_{238} \) then
\[ r_{238} = \frac{mv}{|q|B} = \frac{3.90 \times 10^{-25} \times 1.00 \times 10^5}{1.6 \times 10^{-19} \times 0.500} \]
\[ = 49.4 \times 10^{-3} \text{ m} \]
\[ r_{238} = 49.4 \text{ cm} \]

The diameter of the semi-circle due to \( ^{238}\text{U} \) is \( d_{238} = 2r_{238} = 98.8 \text{ cm} \)
Therefore the separation distance between the isotopes is \( \Delta d = d_{238} - d_{235} = 1.2 \text{ cm} \)

(b) The time taken by each isotope to complete one semi-circular path are

\[
t_{235} = \frac{\text{magnitude of the displacement}}{\text{velocity}} = \frac{9.8 \times 10^{-2}}{1.00 \times 10^5} = 9.76 \times 10^{-6} \text{ s} = 9.76 \mu \text{s}
\]

\[
t_{238} = \frac{\text{magnitude of the displacement}}{\text{velocity}} = \frac{9.88 \times 10^{-2}}{1.00 \times 10^5} = 9.88 \times 10^{-6} \text{ s} = 9.88 \mu \text{s}
\]

Note that even though the difference between mass of two isotopes are very small, this arrangement helps us to convert this small difference into an easily measurable distance of separation. This arrangement is known as mass spectrometer. A mass spectrometer is used in many areas in sciences, especially in medicine, in space science, in geology etc. For example, in medicine, anaesthesiologists use it to measure the respiratory gases and biologist use it to determine the reaction mechanisms in photosynthesis.

### 3.10.3 Motion of a charged particle under crossed electric and magnetic field (velocity selector)

Consider an electric charge \( q \) of mass \( m \) which enters into a region of uniform magnetic field \( B \) with velocity \( \vec{v} \) such that velocity is not perpendicular to the magnetic field. Then the path of the particle is a helix. The Lorentz force on the charged particle moving in a uniform magnetic field can be balanced by Coulomb force by proper arrangement of electric and magnetic fields.

The Coulomb force acts along the direction of electric field (for a positive charge \( q \)) whereas the Lorentz force is perpendicular to the direction of magnetic field. Therefore in order to balance these forces, both electric and magnetic fields must be perpendicular to each other. Such an arrangement of perpendicular electric and magnetic fields are known as cross fields.

For illustration, consider an experimental arrangement as shown in Figure 3.53. In the region of space between parallel plates of a capacitor (which produces uniform electric field), uniform magnetic field is maintained perpendicular to the direction of magnetic field. Therefore in order to balance these forces, both electric and magnetic fields must be perpendicular to each other. Such an arrangement of perpendicular electric and magnetic fields are known as cross fields.

For illustration, consider an experimental arrangement as shown in Figure 3.53. In the region of space between parallel plates of a capacitor (which produces uniform electric field), uniform magnetic field is maintained perpendicular to the direction of magnetic field. Suppose a charged particle enters this space from the left side as shown, the net force on the particle is

\[
\vec{F} = q \left( \vec{E} + \vec{v} \times \vec{B} \right)
\]

For a positive charge, the electric force on the charge acts in downward direction whereas the Lorentz force acts upwards. When these two forces balance one another, then

\[
qE = qvB \implies v = \frac{E}{B} \quad \text{(3.61)}
\]
This means, for a given magnitude of $\vec{E}$-field and $\vec{B}$-field, the forces act only for the particle moving with particular speed $v = \frac{\vec{E}}{\vec{B}}$. This speed is independent of mass and charge.

If the charge enters into the crossed fields with velocity $v$, other than $v_o$, it results in any of the following possibilities (Table 3.4).

**Table 3.4 Deflection based on the velocity – velocity selector**

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Velocity</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$v &gt; v_o$</td>
<td>Charged particle deflects in the direction of Lorentz force</td>
</tr>
<tr>
<td>2</td>
<td>$v &lt; v_o$</td>
<td>Charged particle deflects in the direction of Coulomb force</td>
</tr>
<tr>
<td>3</td>
<td>$v = v_o$</td>
<td>No deflection and particle moves straight</td>
</tr>
</tbody>
</table>

So by proper choice of electric and magnetic fields, the particle with particular speed can be selected. Such an arrangement of fields is called a **velocity selector**.

**Note**: This principle is used in Bainbridge mass spectrograph to separate the isotopes.

**EXAMPLE 3.25**

Let $E$ be the electric field of magnitude $6.0 \times 10^6 \text{ N C}^{-1}$ and $B$ be the magnetic field magnitude $0.83 \text{ T}$. Suppose an electron is accelerated with a potential of $200 \text{ V}$, will it show zero deflection?. If not, at what potential will it show zero deflection.

**Solution:**

Electric field, $E = 6.0 \times 10^6 \text{ N C}^{-1}$ and magnetic field, $B = 0.83 \text{ T}$.

Then

$$v = \frac{E}{B} = \frac{6.0\times10^6}{0.83} = 7.23\times10^6 \text{ m s}^{-1}$$

When an electron goes with this velocity, it shows null deflection. Since the accelerating potential is $200 \text{ V}$, the electron acquires kinetic energy because of this accelerating potential. Hence,

$$\frac{1}{2}mv^2 = eV \Rightarrow v = \sqrt{\frac{eV}{2m}}$$

Since the mass of the electron, $m = 9.1\times10^{-31} \text{ kg}$ and charge of an electron, $|q| = e = 1.6\times10^{-19} \text{ C}$. The velocity due to accelerating potential $200 \text{ V}$

$$v_{200} = \sqrt{\frac{2(1.6\times10^{-19})(200)}{(9.1\times10^{-31})}} = 8.39\times10^6 \text{ m s}^{-1}$$

Since the speed $v_{200} > v$, the electron is deflected towards direction of Lorentz force. So, in order to have null deflection, the potential, we have to supply is

$$V = \frac{1}{2} \frac{mv^2}{e} = \frac{\left(9.1\times10^{-31}\right) \times \left(7.23\times10^6\right)^2}{2 \times (1.6\times10^{-19})}$$

$$V = 148.65 \text{ V}$$

**3.10.4 Cyclotron**

Cyclotron (Figure 3.54) is a device used to accelerate the charged particles to gain large kinetic energy. It is also called as high energy accelerator. It was invented by Lawrence and Livingston in 1934.

**Principle**

When a charged particle moves normal to the magnetic field, it experiences magnetic Lorentz force.
Construction

The schematic diagram of a cyclotron is shown in Figure 3.55. The particles are allowed to move in between two semi-circular metal containers called Dees (hollow D - shaped objects). Dees are enclosed in an evacuated chamber and it is kept in a region with uniform magnetic field controlled by an electromagnet. The direction of magnetic field is normal to the plane of the Dees. The two Dees are kept separated with a gap and the source S (which ejects the particle to be accelerated) is placed at the center in the gap between the Dees. Dees are connected to high frequency alternating potential difference.

Working

Let us assume that the ion ejected from source S is positively charged. As soon as ion is ejected, it is accelerated towards a Dee (say, Dee – 1) which has negative potential at that time. Since the magnetic field is normal to the plane of the Dees, the ion undergoes circular path. After one semi-circular path in Dee-1, the ion reaches the gap between Dees. At this time, the polarities of the Dees are reversed so that the ion is now accelerated towards Dee-2 with a greater velocity. For this circular motion, the centripetal force of the charged particle q is provided by Lorentz force.

\[
\frac{m v^2}{r} = qvB \\
\Rightarrow r = \frac{m}{qB} v \\
\Rightarrow r \propto v
\]

(3.62)

From the equation (3.62), the increase in velocity increases the radius of circular path. This process continues and hence the particle undergoes spiral path of increasing
radius. Once it reaches near the edge, it is taken out with the help of deflector plate and allowed to hit the target T.

Very important condition in cyclotron operation is the resonance condition. It happens when the frequency \( f \) at which the positive ion circulates in the magnetic field must be equal to the constant frequency of the electrical oscillator \( f_{osc} \)

From equation (3.59), we have

\[
f_{osc} = \frac{qB}{2\pi m}
\]

The time period of oscillation is

\[
T = \frac{2\pi m}{qB}
\]

The kinetic energy of the charged particle is

\[
KE = \frac{1}{2} mv^2 = \frac{q^2 B^2 r^2}{2m}
\] (3.63)

**Limitations of cyclotron**

(a) the speed of the ion is limited
(b) electron cannot be accelerated
(c) uncharged particles cannot be accelerated

**Note**

Deutrons (bundles of one proton and one neutron) can be accelerated because it has same charge as that of proton. But neutron (electrically neutral particle) cannot be accelerated by the cyclotron. When a deuteron is bombarded with a beryllium target, a beam of high energy neutrons are produced. These high-energy neutrons are sent into the patient’s cancerous region to break the bonds in the DNA of the cancer cells (killing the cells). This is used in treatment of fast-neutron cancer therapy.

**EXAMPLE 3.26**

Suppose a cyclotron is operated to accelerate protons with a magnetic field of strength 1 T. Calculate the frequency in which the electric field between two Dees could be reversed.

**Solution**

Magnetic field \( B = 1 \text{ T} \)

Mass of the proton, \( m_p = 1.67 \times 10^{-27} \text{ kg} \)

Charge of the proton, \( q = 1.60 \times 10^{-19} \text{ C} \)

\[
f = \frac{qB}{2\pi m_p} = \frac{(1.60 \times 10^{-19})(1)}{2(3.14)(1.67 \times 10^{-27})} = 15.3 \times 10^6 \text{ Hz} = 15.3 \text{ MHz}
\]

### 3.10.5 Force on a current carrying conductor placed in a magnetic field

![Figure 3.56 Current carrying conductor in a magnetic field](image)

When a current carrying conductor is placed in a magnetic field, the force experienced by the wire is equal to the sum of Lorentz forces on the individual charge carriers in the wire. Consider a small segment of wire of length \( dl \), with cross-sectional area \( A \) and current \( I \) as shown in Figure 3.56. The free electrons drift opposite to the direction of current. So the relation between current \( I \) and magnitude of drift velocity \( v_d \) (Refer Unit 2) is

\[
I = neAv_d
\] (3.64)
If the wire is kept in a magnetic field \( \vec{B} \), then average force experienced by the charge (here, electron) in the wire is

\[
\vec{F} = -e(\vec{v}_d \times \vec{B})
\]

Let \( n \) be the number of free electrons per unit volume, therefore

\[
n = \frac{N}{V}
\]

where \( N \) is the number of free electrons in the small element of volume \( V = A \, dl \).

Hence Lorentz force on the wire of length \( dl \) is the product of the number of the electrons

\[
(N = nA \, dl)
\]

and the force acting on an electron.

\[
d\vec{F} = -enAdl(\vec{v}_d \times \vec{B})
\]

The length \( dl \) is along the length of the wire and hence the current element in the wire is

\[
I \, dl = -enA \, v \, dl
\]

Therefore the force on the wire is

\[
d\vec{F} = (I \, dl \times \vec{B})
\]  
(3.65)

The force in a straight current carrying conducting wire of length \( l \) placed in a uniform magnetic field is

\[
\vec{F} = (Il \times \vec{B})
\]  
(3.66)

In magnitude,

\[
F = Bll\sin\theta
\]

(a) If the conductor is placed along the direction of the magnetic field, the angle between them is \( \theta = 0^\circ \). Hence, the force experienced by the conductor is zero.

(b) If the conductor is placed perpendicular to the magnetic field, the angle between them is \( \theta = 90^\circ \). Hence, the force experienced by the conductor is maximum, which is \( F = Bll \).

**Fleming’s left hand rule (mnemonic)**

When a current carrying conductor is placed in a magnetic field, the direction of the force experienced by it is given by Fleming’s Left Hand Rule (FLHR) as shown in Figure 3.57.

Stretch forefinger, the middle finger and the thumb of the left hand such that they are in mutually perpendicular directions. If forefinger points the direction of magnetic field, the middle finger points the direction of the electric current, then thumb will point the direction of the force experienced by the conductor.

**EXAMPLE 3.27**

A metallic rod of linear density is 0.25 kg m\(^{-1}\) is lying horizontally on a smooth inclined plane which makes an angle of 45\(^\circ\) with the horizontal. The rod is not allowed to slide down by flowing a current through it when a magnetic field of strength 0.25 T is acting on it in the vertical direction. Calculate the electric current flowing in the rod to keep it stationary.
Solution

The linear density of the rod i.e., mass per unit length of the rod is 0.25 kg m⁻¹

\[ \Rightarrow \frac{m}{l} = 0.25 \text{ kg m}^{-1} \]

Let \( I \) be the current flowing in the metallic rod. The direction of electric current is into the paper. The direction of magnetic force \( IBl \) is given by Fleming’s left hand rule.

![Diagram of magnetic field and forces]

For equilibrium,

\[ mg \sin 45° = IBl \cos 45° \]

\[ \Rightarrow I = \frac{m}{Bl} g \tan 45° \]

\[ = \frac{0.25 \text{ kg m}^{-1}}{0.25 \text{ T}} \times 1 \times 9.8 \text{ m s}^{-2} \]

\[ \Rightarrow I = 9.8 \text{ A} \]

So, we need to supply current of 9.8 A to keep the metallic rod stationary.

3.10.6 Force between two long parallel current carrying conductors

Two long straight parallel current carrying conductors separated by a distance \( r \) are kept in air as shown in Figure 3.58. Let \( I_1 \) and \( I_2 \) be the electric currents passing through the conductors A and B in same direction (i.e. along \( z \) - direction) respectively. The net magnetic field at a distance \( r \) due to current \( I_1 \) in conductor A is

\[ \vec{B}_1 = \frac{\mu_0 I_1}{2\pi r} (-\hat{i}) = -\frac{\mu_0 I_1}{2\pi r} \hat{i} \]

From thumb rule, the direction of magnetic field is perpendicular to the plane of the paper and inwards (arrow into the page \( \otimes \)) i.e. along negative \( \hat{i} \) direction.

Let us consider a small elemental length \( dl \) in conductor B at which the magnetic field \( \vec{B}_1 \) is present. From equation 3.65, Lorentz force on the element \( dl \) of conductor B is

\[ d\vec{F} = (I_1 dl \times \vec{B}_1) = -I_2 dl \frac{\mu_0 I_1}{2\pi r} (\hat{k} \times \hat{i}) \]

\[ = -\frac{\mu_0 I_1 I_2 dl}{2\pi r} \hat{j} \]

Therefore the force on \( dl \) of the wire B is directed towards the wire \( W_1 \). So the length \( dl \) is attracted towards the conductor A. The force per unit length of the conductor B due to the wire conductor A is

\[ \vec{F} = -\frac{\mu_0 I_1 I_2}{2\pi r} \hat{j} \]

In the same manner, we compute the magnitude of net magnetic induction due to current \( I_2 \) in conductor A at a distance \( r \) in the elemental length \( dl \) of conductor A is

\[ \vec{B}_2 = \frac{\mu_0 I_2}{2\pi r} \hat{i} \]

From the thumb rule, direction of magnetic field is perpendicular to the plane of the paper and outwards (arrow out of the page \( \odot \)) i.e., along positive \( \hat{i} \) direction.
Hence, the magnetic force at element $dl$ of the wire is $W_1$ is

$$
\vec{F} = (I_1 dl \times \vec{B}_1) = I_1 dl \frac{\mu}{2\pi r} (\hat{\kappa} \times \hat{i})
$$

$$
= \frac{\mu I_1 I_2 dl}{2\pi r} \hat{j}
$$

(3.67)

Therefore the force on $dl$ of conductor $A$ is directed towards the conductor $B$. So the length $dl$ is attracted towards the conductor $B$ as shown in Figure (3.59).

Thus the force experienced by two parallel current carrying conductors is attractive if the direction of electric current passing through them is same as shown in Figure 3.60.

Thus the force experienced by two parallel current carrying conductors is repulsive if they carry current in the opposite directions as shown in Figure 3.61.

**Definition of ampère**

One ampère is defined as that current when it is passed through each of the two infinitely long parallel straight conductors kept at a distance of one meter apart in vacuum causes each conductor to experience a force of $2 \times 10^{-7}$ newton per meter length of conductor.

**3.11 TORQUE ON A CURRENT LOOP**

The force on a current carrying wire in a magnetic field is responsible for the motor operation.
3.11.1 Expression for torque on a current loop placed in a magnetic field

Consider a single rectangular loop PQRS kept in a uniform magnetic field $B$. Let $a$ and $b$ be the length and breadth of the rectangular loop respectively. Let $\hat{n}$ be the unit vector normal to the plane of the current loop. This unit vector $\hat{n}$ completely describes the orientation of the loop. Let $B$ be directed from north pole to south pole of the magnet as shown in Figure 3.62.

When an electric current is sent through the loop, the net force acting is zero but there will be net torque acting on it. For the sake of understanding, we shall consider two configurations of the loop; (i) unit vector $\hat{n}$ pointing perpendicular to the field (ii) unit vector pointing at an angle $\theta$ with the field.

(i) when unit vector $\hat{n}$ is perpendicular to the field

In the simple configuration, the unit vector $\hat{n}$ is perpendicular to the field and plane of the loop is lying on xy plane as shown in Figure 3.62. Let the loop be divided into four sections PQ, QR, RS and SP. The Lorentz force on each loop can be calculated as follows:

(a) Force on section PQ

For section PQ, $\vec{I} = -a \hat{j}$ and $\vec{B} = B \hat{i}$

$$\vec{F}_{PQ} = \vec{I} \times \vec{B} = -IaB(\hat{j} \times \hat{i}) = IaB \hat{k}$$

Since the unit vector normal to the plane $\hat{n}$ is along the direction of $\hat{k}$.

(b) The force on section QR

$\vec{I} = b \hat{i}$ and $\vec{B} = B \hat{i}$

$$\vec{F}_{QR} = \vec{I} \times \vec{B} = IbB(\hat{i} \times \hat{i}) = 0$$

(c) The force on section RS

$\vec{I} = a \hat{j}$ and $\vec{B} = B \hat{i}$

$$\vec{F}_{RS} = \vec{I} \times \vec{B} = IaB(\hat{j} \times \hat{i}) = -IaB \hat{k}$$

Since, the unit vector normal to the plane is along the direction of $-\hat{k}$.

(d) The force on section SP

$\vec{I} = -b \hat{i}$ and $\vec{B} = B \hat{i}$

$$\vec{F}_{SP} = \vec{I} \times \vec{B} = -IbB(\hat{i} \times \hat{i}) = 0$$

The net force on the rectangular loop is

$$\vec{F}_{net} = \vec{F}_{PQ} + \vec{F}_{QR} + \vec{F}_{RS} + \vec{F}_{SP}$$

$$\vec{F}_{net} = IaB \hat{k} + 0 - IaB \hat{k} + 0 \Rightarrow \vec{F}_{net} = 0$$

Hence, the net force on the rectangular loop in this configuration is zero. Now let us calculate the net torque due to these forces about an axis passing through the center

$$\vec{\tau}_{net} = \sum_{i=1}^{4} \vec{\tau}_{i} = \sum_{i=1}^{4} \vec{r}_{i} \times \vec{F}_{i}$$

$$= \left( \frac{b}{2} IaB + 0 + \frac{b}{2} IaB + 0 \right) \hat{j}$$

$$\vec{\tau}_{net} = abIB \hat{j}$$

Since, $A = ab$ is the area of the rectangular loop PQRS, the net torque for this configuration is

$$\vec{\tau}_{net} = ABI \hat{j}$$
When the loop starts rotating due to this torque, the magnetic field \( \vec{B} \) is no longer in the plane of the loop. So the above equation is the special case.

When the loop starts rotating about z axis due to this torque, the magnetic field \( \vec{B} \) is no longer in the plane of the loop. So the above equation is the special case.

(ii) when unit vector \( \hat{n} \) is at an angle \( \theta \) with the field

In the general case, the unit normal vector \( \hat{n} \) and magnetic field \( \vec{B} \) is with an angle \( \theta \) as shown in Figure 3.63.

(a) The force on section PQ

\[
\vec{I} = -a \hat{j} \quad \text{and} \quad \vec{B} = B \hat{i}
\]

\[
\vec{F}_{PQ} = \vec{I} \times \vec{B} = -Ia(\hat{j} \times \hat{i}) = IaB \hat{k}
\]

Since, the unit vector normal to the plane \( \hat{n} \) is along the direction of \( \hat{k} \).

(b) The force on section QR

\[
\vec{I} = b \cos \left( \frac{\pi}{2} - \theta \right) \hat{i} - \sin \left( \frac{\pi}{2} - \theta \right) \hat{k} \quad \text{and} \quad \vec{B} = B \hat{i}
\]

\[
\vec{F}_{QR} = \vec{I} \times \vec{B} = -IbB \sin \left( \frac{\pi}{2} - \theta \right) \hat{j}
\]

\[
\vec{F}_{QR} = -IbB \cos \theta \hat{j}
\]

Figure 3.63 Unit vector makes an angle \( \theta \) with the field

(c) The force on section RS

\[
\vec{I} = a \hat{j} \quad \text{and} \quad \vec{B} = B \hat{i}
\]

\[
\vec{F}_{RS} = \vec{I} \times \vec{B} = IaB(\hat{j} \times \hat{i}) = -IaB \hat{k}
\]

Since, the unit vector normal to the plane is along the direction of \( -\hat{k} \).

(d) The force on section SP

\[
\vec{I} = b \cos \left( \frac{\pi}{2} + \theta \right) \hat{i} + \sin \left( \frac{\pi}{2} + \theta \right) \hat{k} \quad \text{and} \quad \vec{B} = B \hat{i}
\]

\[
\vec{F}_{SP} = \vec{I} \times \vec{B} = IbB \sin \left( \frac{\pi}{2} + \theta \right) \hat{j}
\]

\[
\vec{F}_{SP} = IbB \cos \theta \hat{j}
\]

The net force on the rectangular loop is

\[
\vec{F}_{net} = \vec{F}_{PQ} + \vec{F}_{QR} + \vec{F}_{RS} + \vec{F}_{SP}
\]

\[
\vec{F}_{net} = IaB \hat{k} - IbB \cos \theta \hat{j} - IaB \hat{k} + IbB \cos \theta \hat{j}
\]

\[
\Rightarrow \vec{F}_{net} = 0
\]

Hence, the net force on the rectangular loop in this configuration is also zero. Notice that the force on section QR and SP are not zero.

**Figure 3.64** Horizontal and vertical component of the sections - (a) QR (b) SP
produced by these two forces about the axis of the rectangular loop is given by

$$\tau_{net} = babI\sin\theta \hat{k} = ABI\sin\theta \hat{k}$$

From the Figure 3.65 (c),

$$\overrightarrow{OA} = \frac{b}{2}\cos\left(\frac{\pi}{2} - \theta\right)(-\hat{i}) + \frac{b}{2}\sin\left(\frac{\pi}{2} - \theta\right)(-\hat{k})$$

$$= \frac{b}{2}(-\sin\theta \hat{i} + \cos\theta \hat{k})$$

$$\overrightarrow{OB} = \frac{b}{2}\cos\left(\frac{\pi}{2} - \theta\right)(\hat{i}) + \frac{b}{2}\sin\left(\frac{\pi}{2} - \theta\right)(-\hat{k})$$

$$= \frac{b}{2}(-\sin\theta \hat{i} + \cos\theta \hat{k})$$

$$\overrightarrow{OA} \times \overrightarrow{F_{PQ}} = \left\{\frac{b}{2}(-\sin\theta \hat{i} + \cos\theta \hat{k})\right\} \times \{Iab \hat{k}\}$$

$$= \frac{1}{2}IabB\sin\theta \hat{j}$$

$$\overrightarrow{OB} \times \overrightarrow{F_{RS}} = \left\{\frac{b}{2}(\sin\theta \hat{i} + \cos\theta \hat{k})\right\} \times \{-Iab \hat{k}\}$$

$$= \frac{1}{2}IabB\sin\theta \hat{j}$$

The net torque \(\tau_{net} = IabB\sin\theta \hat{j}\)

Note that the net torque is in the positive y direction which tends to rotate the loop in clockwise direction about the y axis. If the current is passed in the other way (P→S→R→Q→P), then total torque will point in the negative y direction which tends to rotate the loop in anticlockwise direction about y axis.

Another important point is to note that the torque is less in this case compared to earlier case (where the \(\hat{n}\) is perpendicular to the magnetic field \(\vec{B}\)). It is because the perpendicular distance is reduced between the forces \(\vec{F}_{PQ}\) and \(\vec{F}_{RS}\) in this case.

The equation (3.68) can also be rewritten in terms of magnetic dipole moment \(\vec{p}_{m} = I\vec{A} = Iab\hat{n}\)

$$\tau_{net} = \vec{p} \times \vec{B}$$

Figure 3.65 Force on the rectangular loop – (a) top view and (b) side view (c) net torque on the loop

Even though the forces PQ and RS also are also equal and opposite, they are not collinear. So these two forces constitute a couple as shown in Figure 3.65 (a). Hence the net torque produced by these two sections is zero here. But, they have equal and opposite effects, but we assume the loop to be rigid, so no deformation. Hence, no torque produced by these two sections.
When the current is passed through the loop, the torque is produced. If the torque acting on the loop is increased then the loop will start to rotate. The loop will start to lift if and only if the magnitude of magnetic torque due to current applied equals to the gravitational torque as shown in Figure

\[ \tau_{\text{magnetic}} = \tau_{\text{gravitational}} \]

\[ IAB = mgR \]

But \( p_m = IA = I\left(\pi R^2\right) \)

\[ \pi IR^2B = mgR \]

\[ \Rightarrow I = \frac{mg}{\pi RB} \]

The current estimated using this equation should be applied so that one edge of loop is lifted off the surface.

### 3.11.2 Moving coil galvanometer

Moving coil galvanometer is a device which is used to indicate the flow of current in an electrical circuit.

**Principle** When a current carrying loop is placed in a uniform magnetic field it experiences a torque.

**Construction**

A moving coil galvanometer consists of a rectangular coil PQRS of insulated thin copper wire. The coil contains a large number of turns wound over a light metallic frame. A cylindrical soft-iron core is placed symmetrically inside the coil as shown in Figure 3.66. The rectangular coil is suspended freely between two pole pieces of a horse-shoe magnet.

The upper end of the rectangular coil is attached to one end of fine strip of
radial magnetic field. Due to this radial field, the sides QR and SP are always parallel to the B-field (magnetic field) and experience no force. The sides PQ and RS are always parallel to the B-field and experience force and due to this, torque is produced.

For single turn, the deflection couple as shown in Figure 3.68 is
\[ \tau = bF = bBI \]
since, area of the coil \( A = lb \)
For coil with N turns, we get
\[ \tau = NABI \] (3.69)

Due to this deflecting torque, the coil gets twisted and restoring torque (also known as restoring couple) is developed. Hence the magnitude of restoring couple is proportional to the amount of twist \( \theta \) (Refer Unit 10 of Std. XI Physics). Thus
\[ \tau = K\theta \] (3.70)

**Working**

Consider a single turn of the rectangular coil PQRS whose length be \( l \) and breadth \( b \). PQ = RS = \( l \) and QR = SP = \( b \). Let I be the electric current flowing through the rectangular coil PQRS as shown in Figure 3.67. The horse-shoe magnet has hemispherical magnetic poles which produces a

**Figure 3.66** Moving coil galvanometer – its parts

phosphor bronze and the lower end of the coil is connected to a hair spring which is also made up of phosphor bronze. In a fine suspension strip W, a small plane mirror is attached in order to measure the deflection of the coil with the help of lamp and scale arrangement. The other end of the mirror is connected to a torsion head T. In order to pass electric current through the galvanometer, the suspension strip W and the spring S are connected to terminals.

**Figure 3.67** Force acting on current carrying coil

**Figure 3.68** Deflection couple
where \( K \) is the restoring couple per unit twist or torsional constant of the spring.

At equilibrium, the deflection couple is equal to the restoring couple. Therefore by comparing equation (3.69) and (3.70), we get

\[
NABI = K \theta \\
\Rightarrow I = \frac{K}{NAB} \theta \quad (3.71)
\]

(or) \( I = G \theta \)

where, \( G = \frac{K}{NAB} \) is called galvanometer constant or current reduction factor of the galvanometer.

Since, suspended moving coil galvanometer is very sensitive, we have to handle with high care while doing experiments. Most of the galvanometer we use are pointer type moving coil galvanometer.

**Figure of merit of a galvanometer**

*It is defined as the current which produces a deflection of one scale division in the galvanometer.*

**Sensitivity of a galvanometer**

The galvanometer is said to be sensitive if it shows large scale deflection even though a small current is passed through it or a small voltage is applied across it.

**Current sensitivity:** *It is defined as the deflection produced per unit current flowing through it.*

\[
I_s = \frac{\theta}{I} = \frac{NAB}{K} \Rightarrow I_s = \frac{1}{G} \quad (3.72)
\]

The current sensitivity of a galvanometer can be increased

(a) by increasing

(1) the number of turns \( N \)

(2) the magnetic induction \( B \)

(3) the area of the coil \( A \)

(b) by decreasing

the couple per unit twist of the suspension wire \( k \). Phosphor - bronze wire is used as the suspension wire because the couple per unit twist is very small.

**Voltage sensitivity:** *It is defined as the deflection produced per unit voltage applied across it.*

\[
V_s = \frac{\theta}{V} \\
V_s = \frac{\theta}{IR_s} = \frac{NAB}{KR_s} \Rightarrow V_s = \frac{1}{GR_s} = \frac{I_s}{R_s} \quad (3.73)
\]

where \( R_s \) is the resistance of galvanometer.

**Example 3.29**

The coil of a moving coil galvanometer has 5 turns and each turn has an effective area of \( 2 \times 10^{-2} \) m\(^2\). It is suspended in a magnetic field whose strength is \( 4 \times 10^{-2} \) Wb m\(^{-2}\). If the torsional constant \( K \) of the suspension fibre is \( 4 \times 10^{-9} \) N m deg\(^{-1}\).

(a) Find its current sensitivity in degree per micro - ampere.

(b) Calculate the voltage sensitivity of the galvanometer for it to have full scale deflection of 50 divisions for 25 mV.

(c) Compute the resistance of the galvanometer.

**Solution**

The number of turns of the coil is 5 turns
The area of each coil is \( 2 \times 10^{-2} \) m\(^2\)
Strength of the magnetic field is \( 4 \times 10^{-2} \) Wb m\(^{-2}\)
Torsional constant is \( 4 \times 10^{-9} \) N m deg\(^{-1}\)

(a) Current sensitivity

\[
I_s = \frac{NAB}{K} = \frac{5 \times 2 \times 10^{-2} \times 4 \times 10^{-2}}{4 \times 10^{-9}}
\]

\[= 10^6 \text{ divisions per ampere} \]
1μA = 1microampere = 10⁻⁶ ampere

Therefore,
\[ I_s = 10^6 \text{div} \frac{\text{div}}{A} = \frac{1 \text{div}}{10^6 \text{A}} = 1 \text{div} \frac{\text{div}}{\mu A} \]

\[ I_s = 1 \text{div}(\mu A)^{-1} \]

(b) Voltage sensitivity
\[ V_s = \frac{\theta}{V} = \frac{50 \text{div}}{25 \text{mV}} = 2 \times 10^3 \text{div} V^{-1} \]

(c) The resistance of the galvanometer is
\[ R_g = \frac{I_s}{V_s} = \frac{10^6 \text{div}}{2 \times 10^7 \text{div} \frac{\text{div}}{V}} = 0.5 \times 10^1 \text{V} = 0.5 \text{kΩ} \]

**EXAMPLE 3.30**

The resistance of a moving coil galvanometer is made twice its original value in order to increase current sensitivity by 50%. Will the voltage sensitivity change? If so, by how much?

**Solution**

Yes, voltage sensitivity will change.

Voltage sensitivity is \( V_s = \frac{I_s}{R} \)

When the resistance is doubled, then new resistance is \( R' = 2R \)

Increase in current sensitivity is
\[ I'_s = \left(1 + \frac{50}{100}\right) I_s = \frac{3}{2} I_s \]

The new voltage sensitivity is
\[ V'_s = \frac{2}{2R} \frac{3}{V} = \frac{3}{4} V_s \]

Hence the voltage sensitivity decreases. The percentage decrease in voltage sensitivity is
\[ \frac{V_s - V'_s}{V_s} \times 100\% = 25\% \]

**Conversion of galvanometer into ammeter and voltmeter**

A galvanometer is very sensitive instrument to detect the current. It can be easily converted into ammeter and voltmeter.

**Galvanometer to an Ammeter**

Ammeter is an instrument used to measure current flowing in the electrical circuit. The ammeter must offer low resistance such that it will not change the current passing through it. So ammeter is connected in series to measure the circuit current.

A galvanometer is converted into an ammeter by connecting a low resistance in parallel with the galvanometer. This low resistance is called shunt resistance \( S \). The scale is now calibrated in amperes and the range of ammeter depends on the values of the shunt resistance.

Let \( I \) be the current passing through the circuit as shown in Figure 3.69. When current \( I \) reaches the junction \( A \), it divides into two components. Let \( I_e \) be the current passing through the galvanometer of resistance \( R_g \) through a path \( AGE \) and the remaining current \( (I - I_g) \) passes along the path \( ACDE \) through shunt resistance \( S \). The
value of shunt resistance is so adjusted that current \( I_g \) produces full scale deflection in
the galvanometer. The potential difference across galvanometer is same as the potential
difference across shunt resistance.

\[
V_{galvanometer} = V_{shunt}
\]

\[
\Rightarrow I_g R_g = (I - I_g) S
\]

\[
S = \frac{I_g}{I_g - I} R_g \quad \text{Or}
\]

\[
I_g = \frac{S}{S + R_g} I \Rightarrow I_g \propto I
\]

Since, the deflection in the galvanometer is proportional to the current passing through it.

\[
\theta = \frac{1}{G} I_g \Rightarrow \theta \propto I_g \Rightarrow \theta \propto I
\]

So, the deflection in the galvanometer measures the current \( I \) passing through the
circuit (ammeter).

Shunt resistance is connected in parallel to galvanometer. Therefore, resistance of
ammeter can be determined by computing the effective resistance, which is

\[
\frac{1}{R_{eff}} = \frac{1}{R_g} + \frac{1}{S} \Rightarrow R_{eff} = \frac{R_g S}{R_g + S} = R_g
\]

Since, the shunt resistance is a very low
resistance and the ratio \( \frac{S}{R_g} \) is also small. This
means, \( R_g \) is also small, i.e., the resistance offered by the ammeter is small. So, when
we connect ammeter in series, the ammeter will not change the resistance appreciably
and also the current in the circuit. For an ideal ammeter, the resistance must be equal
to zero. Hence, the reading in ammeter is always lesser than the actual current in the
circuit. Let \( I_{\text{ideal}} \) be current measured from
ideal ammeter and \( I_{\text{actual}} \) be the actual current measured in the circuit by the ammeter.
Then, the percentage error in measuring a
current through an ammeter is

\[
\frac{\Delta I}{I} \times 100\% = \frac{I_{\text{ideal}} - I_{\text{actual}}}{I_{\text{actual}}} \times 100\%
\]

Key points

1. An ammeter is a low resistance instrument
and it is always connected in series to the
circuit
2. An ideal ammeter has zero resistance
3. In order to increase the range of an
ammeter \( n \) times, the value of shunt
resistance to be connected in parallel is

\[
S = \frac{G}{n - 1}
\]

Galvanometer to a voltmeter

A voltmeter is an instrument used to
measure potential difference across any two
points in the electrical circuits. It should not
draw any current from the circuit otherwise
the value of potential difference to be
measured will change.

Voltmeter must have high resistance
and when it is connected in parallel, it will
not draw appreciable current so that it will
indicate the true potential difference.

![Figure 3.70 Shunt resistance connected in series](image-url)
A galvanometer is converted into a voltmeter by connecting high resistance \( R_h \) in series with galvanometer as shown in Figure 3.74. The scale is now calibrated in volt and the range of voltmeter depends on the values of the resistance connected in series i.e. the value of resistance is so adjusted that only current \( I_g \) produces full scale deflection in the galvanometer.

Let \( R_g \) be the resistance of galvanometer and \( I_g \) be the current with which the galvanometer produces full scale deflection. Since the galvanometer is connected in series with high resistance, the current in the electrical circuit is same as the current passing through the galvanometer.

\[
I = I_g
\]

\[
I = I_g \Rightarrow I_g = \frac{\text{potential difference}}{\text{total resistance}}
\]

Since the galvanometer and high resistance are connected in series, the total resistance or effective resistance gives the resistance of voltmeter. The voltmeter resistance is

\[
R_v = R_g + R_h
\]

Therefore,

\[
I_g = \frac{V}{R_g + R_h}
\]

\[\Rightarrow R_h = \frac{V}{I_g} - R_g\]

Note that \( I_g \propto V \)

The deflection in the galvanometer is proportional to current \( I_g \). But current \( I_g \) is proportional to the potential difference. Hence the deflection in the galvanometer is proportional to potential difference. Since the resistance of voltmeter is very large, a voltmeter connected in an electrical circuit will draw least current in the circuit. An ideal voltmeter is one which has infinite resistance.

**Key points**

1. Voltmeter is a high resistance instrument and it is always connected in parallel with the circuit element across which the potential difference is to be calculated.
2. An ideal voltmeter has infinite resistance.
3. In order to increase the range of voltmeter \( n \) times the value of resistance to be connected in series with galvanometer is \( R = (n-1)G \).
A vertical plane passing through geographic axis is called geographic meridian.
A vertical plane passing through magnetic axis is called magnetic meridian.
The angle between magnetic meridian at a point with the geographical meridian is called the declination or magnetic declination.
The angle subtended by the Earth’s total magnetic field \( \vec{B} \) with the horizontal direction in the magnetic meridian is called dip or magnetic inclination at that point.
The magnetic moment is defined as the product of its pole strength and magnetic length. It is a vector quantity, denoted by \( \vec{P}_m \).
The region surrounding magnet where magnetic pole of strength unity experiences a force is known as magnetic field. It is a vector quantity and denoted by \( \vec{B} \) Its unit is \( \text{N} \text{A}^{-1} \text{m}^{-1} \).
The number of magnetic field lines crossing per unit area is called magnetic flux \( \Phi_B \). It is a scalar quantity. In SI unit, magnetic flux \( \Phi_B \) is Weber, symbol Wb.
Statement of Coulomb’s law in magnetism “The force of attraction or repulsion between two magnetic poles is proportional to the product of their pole strengths and inversely proportional to the square of distance between them”.
Magnetic dipole kept in a uniform magnetic field experiences torque.
Tangent galvanometer is a device used to measure very small currents. It is a moving magnet type galvanometer. Its working is based on tangent law.
Tangent law is \( \vec{B} = \vec{H} \tan \theta \).
The magnetic field which is used to magnetize a sample or specimen is called the magnetising field. It is a vector quantity and denoted by \( \vec{H} \) and its unit is \( \text{A}^{-1} \text{m} \).
The measure of ability of the material to allow the passage of magnetic lines of force through it is known as magnetic permeability.
The net magnetic moment per unit volume of material is known as intensity of magnetisation or magnetisation vector or magnetisation.
Magnetic susceptibility is defined as the ratio of the intensity of magnetisation \( (\vec{I}) \) induced in the material due to the magnetising field \( (\vec{H}) \).
Magnetic materials are classified into three categories: diamagnetic, paramagnetic and ferromagnetic materials.
The lagging of magnetic induction \( \vec{B} \) behind the cyclic variation in magnetising field \( \vec{H} \) is defined as “Hysteresis”, which means “lagging behind”.
The right hand thumb rule “If we hold the current carrying conductor in our right hand such that the thumb points in the direction of current flow, then the rest of the fingers encircling the wire points in the direction of the magnetic field lines produced”.
Maxwell right hand cork screw rule “If we rotate a screw by a screw driver, then the direction of current is same as the direction in which screw advances, and the direction of rotation of the screw will determine the direction of the magnetic field”.

**SUMMARY:**
- A vertical plane passing through geographic axis is called geographic meridian.
- A vertical plane passing through magnetic axis is called magnetic meridian.
- The angle between magnetic meridian at a point with the geographical meridian is called the declination or magnetic declination.
- The angle subtended by the Earth’s total magnetic field \( \vec{B} \) with the horizontal direction in the magnetic meridian is called dip or magnetic inclination at that point.
- The magnetic moment is defined as the product of its pole strength and magnetic length. It is a vector quantity, denoted by \( \vec{P}_m \).
- The region surrounding magnet where magnetic pole of strength unity experiences a force is known as magnetic field. It is a vector quantity and denoted by \( \vec{B} \) Its unit is \( \text{N} \text{A}^{-1} \text{m}^{-1} \).
- The number of magnetic field lines crossing per unit area is called magnetic flux \( \Phi_B \). It is a scalar quantity. In SI unit, magnetic flux \( \Phi_B \) is Weber, symbol Wb.
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- Magnetic dipole kept in a uniform magnetic field experiences torque.
- Tangent galvanometer is a device used to measure very small currents. It is a moving magnet type galvanometer. Its working is based on tangent law.
- Tangent law is \( \vec{B} = \vec{H} \tan \theta \).
- The magnetic field which is used to magnetize a sample or specimen is called the magnetising field. It is a vector quantity and denoted by \( \vec{H} \) and its unit is \( \text{A}^{-1} \text{m} \).
- The measure of ability of the material to allow the passage of magnetic lines of force through it is known as magnetic permeability.
- The net magnetic moment per unit volume of material is known as intensity of magnetisation or magnetisation vector or magnetisation.
- Magnetic susceptibility is defined as the ratio of the intensity of magnetisation \( (\vec{I}) \) induced in the material due to the magnetising field \( (\vec{H}) \).
- Magnetic materials are classified into three categories: diamagnetic, paramagnetic and ferromagnetic materials.
- The lagging of magnetic induction \( \vec{B} \) behind the cyclic variation in magnetising field \( \vec{H} \) is defined as “Hysteresis”, which means “lagging behind”.
- The right hand thumb rule “If we hold the current carrying conductor in our right hand such that the thumb points in the direction of current flow, then the rest of the fingers encircling the wire points in the direction of the magnetic field lines produced”.
- Maxwell right hand cork screw rule “If we rotate a screw by a screw driver, then the direction of current is same as the direction in which screw advances, and the direction of rotation of the screw will determine the direction of the magnetic field”.

**UNIT 3  MAGNETISM AND MAGNETIC EFFECTS OF ELECTRIC CURRENT**
Ampère’s circuital law is \( \oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enclosed}} \).

Magnetic field inside the solenoid is \( B = \mu_0 nI \), where \( n \) is the number of turns per unit length.

Magnetic field interior to the toroid is \( B = \mu_0 nI \), where \( n \) is the number of turns per unit length.

Lorentz force is \( \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \).

Charged particle moving in a uniform magnetic field will undergo circular motion.

Fleming’s Left Hand Rule: Stretch forefinger, the middle finger and the thumb of the left hand such that they are in mutually perpendicular directions. If we keep the forefinger in the direction of magnetic field, the middle finger in the direction of the electric current, then the thumb points in the direction the force experienced by the conductors.

One ampere is defined as that current when it is passed through each of the two infinitely long parallel straight conductors kept at a distance of one meter apart in vacuum causes each conductor to experience a force of \( 2 \times 10^{-7} \) newton per meter length of the conductor.

When a current carrying coil is placed in a uniform magnetic field, the net force on it is always zero but net torque is not zero. The magnitude of net torque is \( \tau = NABI \sin \theta \).

Moving coil galvanometer is an instrument used for the detection and measurement of small currents.

In moving coil galvanometer, current passing through the galvanometer is directly proportional to the deflection. Mathematically, \( I = G\theta \), where \( G = \frac{K}{NAB} \) is called galvanometer constant or current reduction factor of the galvanometer.

Current sensitivity is defined as the deflection produced per unit current flowing through it, \( I_s = \frac{\theta}{I} = \frac{NAB}{K} \Rightarrow I_s = \frac{1}{G} \).

Voltage sensitivity is defined as the deflection produced per unit voltage which is applied across it, \( V_s = \frac{\theta}{V} = \frac{1}{GR_g} = \frac{I_s}{R_g} \), where, \( R_g \) is the resistance of galvanometer.

Ammeter is an instrument used to measure current in an electrical circuit.

A galvanometer can be converted into an ammeter of given range by connecting a suitable low resistance \( S \) called shunt in parallel to the given galvanometer.

An ideal ammeter has zero resistance.

Voltmeter is an instrument used to measure potential difference across any element in an electrical circuit.

A galvanometer can be converted into suitable voltmeter of given range by connecting a suitable resistance \( R \) in series with the given galvanometer.

An ideal voltmeter has infinite resistance.
Magnetism and Magnetic effects of electric current

- Magnetism
- Earth's Magnetism
- Magnetic materials
- Magnetic elements
- Properties
- Magnetic field axial line
- Magnetic field equatorial line
- Tangent Galvanometer
- Potential energy
- Torque

- Bar Magnetics
- Coulomb's law

- Magnetic effects of electric current
  - Biot Savart's law
  - Ampere law
  - magnetic field of current carrying conductor
  - Toroid
  - Solenoid
  - Torque
  - Lorentz force
  - Cyclotron
  - Moving coil Galvanometer
I Multiple choice questions

1. The magnetic field at the center O of the following current loop is

   ![Diagram of a current loop](image)

   (a) $\frac{\mu I}{4r}$
   (b) $\frac{\mu I}{4r}$
   (c) $\frac{\mu I}{2r}$
   (d) $\frac{\mu I}{2r}$

2. An electron moves straight inside a charged parallel plate capacitor of uniform charge density $\sigma$. The time taken by the electron to cross the parallel plate capacitor when the plates of the capacitor are kept under constant magnetic field of induction $B$ is

   ![Diagram of a charged parallel plate capacitor](image)

   (a) $\frac{e \sigma B}{\sigma}$
   (b) $\frac{e \sigma B}{\sigma l}$
   (c) $\frac{e \sigma B}{l \sigma}$
   (d) $\frac{e \sigma B}{\sigma}$

3. The force experienced by a particle having mass $m$ and charge $q$ accelerated through a potential difference $V$ when it is kept under perpendicular magnetic field $B$ is

   ![Diagram of a particle in a magnetic field](image)

   (a) $\frac{2q^2BV}{m}$
   (b) $\frac{q^2B^2V}{2m}$
   (c) $\frac{2q^2V}{m}$
   (d) $\frac{2q^2BV}{m^3}$

4. A circular coil of radius 5 cm and 50 turns carries a current of 3 ampere. The magnetic dipole moment of the coil is

   (a) 1.0 amp – m$^2$
   (b) 1.2 amp – m$^2$
   (c) 0.5 amp – m$^2$
   (d) 0.8 amp – m$^2$

5. A thin insulated wire forms a plane spiral of $N = 100$ tight turns carrying a current $I = 8$ m A (milli ampere). The radii of inside and outside turns are $a = 50$ mm and $b = 100$ mm respectively. The magnetic induction at the center of the spiral is

   (a) 5 $\mu$T
   (b) 7 $\mu$T
   (c) 8 $\mu$T
   (d) 10 $\mu$T

6. Three wires of equal lengths are bent in the form of loops. One of the loops is circle, another is a semi-circle and the third one is a square. They are placed in a uniform magnetic field and same electric current is passed through them. Which of the following loop configuration will experience greater torque?

   (a) circle
   (b) semi-circle
   (c) square
   (d) all of them

7. Two identical coils, each with $N$ turns and radius $R$ are placed coaxially at a distance $R$ as shown in the figure. If $I$ is the current passing through the loops in the same direction, then the
magnetic field at a point P which is at exactly at \( \frac{R}{2} \) distance between two coils is

\[
\mathbf{B} = \frac{\beta}{\sqrt{3}} (\hat{i} + \hat{j} + \hat{k}) \text{T.}
\]

The magnitude of Lorentz force acting on the wire is

(a) \( \frac{2}{\sqrt{3}} \beta Il \)  
(b) \( \frac{1}{\sqrt{3}} \beta Il \)  
(c) \( \sqrt{2} \beta Il \)  
(d) \( \frac{1}{2} \beta Il \)

8. A wire of length \( l \) carries a current \( I \) along the Y direction and magnetic field is given by \( \mathbf{B} = \frac{\beta}{\sqrt{3}} (\hat{i} + \hat{j} + \hat{k}) \text{T.} \)

The magnitude of Lorentz force acting on the wire is

(a) \( \frac{8N\mu I}{\sqrt{5R}} \)  
(b) \( \frac{8N\mu I}{5^{3/2} R} \)  
(c) \( \frac{8N\mu I}{5R} \)  
(d) \( \frac{4N\mu I}{\sqrt{5R}} \)

9. A bar magnet of length \( l \) and magnetic moment \( M \) is bent in the form of an arc as shown in figure. The new magnetic dipole moment will be

(a) \( M \)  
(b) \( \frac{3}{\pi} M \)  
(c) \( \frac{2}{\pi} M \)  
(d) \( \frac{1}{2} M \)

10. A non-conducting charged ring of charge \( q \), mass \( m \) and radius \( r \) is rotated with constant angular speed \( \omega \). Find the ratio of its magnetic moment with angular momentum is

(a) \( \frac{q}{m} \)  
(b) \( \frac{2q}{m} \)  
(c) \( \frac{q}{2m} \)  
(d) \( \frac{q}{4m} \)

11. The BH curve for a ferromagnetic material is shown in the figure. The material is placed inside a long solenoid which contains 1000 turns/cm. The current that should be passed in the solenoid to demagnetize the ferromagnet completely is

(a) 1.00 mA (milli ampere)  
(b) 1.25 mA  
(c) 1.50 mA  
(d) 1.75 mA

12. Two short bar magnets have magnetic moments 1.20 Am\(^2\) and 1.00 Am\(^2\)
respectively. They are kept on a horizontal table parallel to each other with their north poles pointing towards the south. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centers is (Horizontal components of Earth’s magnetic induction is $3.6 \times 10^{-5}$ Wb m$^{-2}$)

(NSEP 2000-2001)

(a) $3.60 \times 10^{-5}$ Wb m$^{-2}$
(b) $3.5 \times 10^{-5}$ Wb m$^{-2}$
(c) $2.56 \times 10^{-4}$ Wb m$^{-2}$
(d) $2.2 \times 10^{-4}$ Wb m$^{-2}$

13. The vertical component of Earth’s magnetic field at a place is equal to the horizontal component. What is the value of angle of dip at this place?
(a) $30^o$
(b) $45^o$
(c) $60^o$
(d) $90^o$

14. A flat dielectric disc of radius R carries an excess charge on its surface. The surface charge density is $\sigma$. The disc rotates about an axis perpendicular to its plane passing through the center with angular velocity $\omega$. Find the magnitude of the torque on the disc if it is placed in a uniform magnetic field whose strength is $B$ which is directed perpendicular to the axis of rotation

(a) $\frac{1}{4} \sigma \omega \pi BR$
(b) $\frac{1}{4} \sigma \omega \pi BR^2$
(c) $\frac{1}{4} \sigma \omega \pi BR^3$
(d) $\frac{1}{4} \sigma \omega \pi BR^4$

15. A simple pendulum with charged bob is oscillating with time period $T$ and let $\theta$ be the angular displacement. If the uniform magnetic field is switched ON in a direction perpendicular to the plane of oscillation then
(a) time period will decrease but $\theta$ will remain constant
(b) time period remain constant but $\theta$ will decrease
(c) both $T$ and $\theta$ will remain the same
(d) both $T$ and $\theta$ will decrease

Answers

1) a  2) d  3) c  4) b  5) b  6) a  7) b  8) a  9) b  10) c  11) b  12) c  13) b  14) d  15) c

II Short answer questions:

1. What is meant by magnetic induction?
2. Define magnetic flux.
3. Define magnetic dipole moment.
4. State Coulomb’s inverse law.
5. What is magnetic susceptibility?
7. What is magnetic permeability?
8. State Ampere’s circuital law.

9. Compare dia, para and ferromagnetism.

10. What is meant by hysteresis?
III Long answer questions

1. Discuss Earth's magnetic field in detail.
2. Deduce the relation for the magnetic induction at a point due to an infinitely long straight conductor carrying current.
3. Obtain a relation for the magnetic induction at a point along the axis of a circular coil carrying current.
4. Compute the torque experienced by a magnetic needle in a uniform magnetic field.
5. Calculate the magnetic induction at a point on the axial line of a bar magnet.
6. Obtain the magnetic induction at a point on the equatorial line of a bar magnet.
7. Find the magnetic induction due to a long straight conductor using Ampere's circuital law.
8. Discuss the working of cyclotron in detail.
9. What is tangent law? Discuss in detail.
10. Explain the principle and working of a moving coil galvanometer.
11. Discuss the conversion of galvanometer into an ammeter and also a voltmeter.
12. Calculate the magnetic field inside and outside of the long solenoid using Ampere's circuital law.

IV. Numerical problems

1. A bar magnet having a magnetic moment \( \vec{M} \) is cut into four pieces i.e., first cut in two pieces along the axis of the magnet and each piece is further cut into two pieces. Compute the magnetic moment of each piece.

Answer \( \vec{M}_{new} = \frac{1}{4} \vec{M} \)

2. A conductor of linear mass density 0.2 g m\(^{-1}\) suspended by two flexible wire as shown in figure. Suppose the tension in the supporting wires is zero when it is kept inside the magnetic field of 1 T whose direction is into the page. Compute the current inside the conductor and also the direction of the current. Assume g = 10 m s\(^{-2}\).

Answer 2 mA

3. A circular coil with cross-sectional area 0.1 cm\(^2\) is kept in a uniform magnetic field of strength 0.2 T. If the current passing in the coil is 3 A and plane of the loop is perpendicular to the direction of magnetic field. Calculate (a) total torque on the coil (b) total force on the coil (c) average force on each electron in the coil due to the magnetic field of the free electron density for the material of the wire is \(10^{28}\) m\(^{-3}\).
4. A bar magnet is placed in a uniform magnetic field whose strength is 0.8 T. Suppose the bar magnet orient at an angle 30° with the external field experiences a torque of 0.2 N m. Calculate:

(i) the magnetic moment of the magnet 
(ii) the work done by an applied force in moving it from most stable configuration to the most unstable configuration and also compute the work done by the applied magnetic field in this case.

Answer (i) 0.5 A m² (ii) W = 0.8 J and W_{mag} = -0.8 J

5. A non-conducting sphere has a mass of 100 g and radius 20 cm. A flat compact coil of wire with turns 5 is wrapped tightly around it with each turns concentric with the sphere. This sphere is placed on an inclined plane such that plane of coil is parallel to the inclined plane. A uniform magnetic field of 0.5 T exists in the region in vertically upward direction. Compute the current I required to rest the sphere in equilibrium.

Answer \( \frac{2}{\pi} A \)

6. Calculate the magnetic field at the center of a square loop which carries a current of 1.5 A, length of each loop is 50 cm. Answer 3.4 x 10^-6 T

7. Show that the magnetic field at any point on the axis of the solenoid having n turns per unit length is \( B = \frac{1}{2} \mu_0 n I (\cos \theta_1 - \cos \theta_2) \).

8. Let I₁ and I₂ be the steady currents passing through a long horizontal wire XY and PQ respectively. The wire PQ is fixed in horizontal plane and the wire XY be is allowed to move freely in a vertical plane. Let the wire XY is in equilibrium at a height d over the parallel wire PQ as shown in figure.

Show that if the wire XY is slightly displaced and released, it executes Simple Harmonic Motion (SHM). Also, compute the time period of oscillations.

Answer \( a_y = -\omega^2 y \) (SHM) and time period

\[ T = 2\pi \sqrt{\frac{d}{g}} \text{ sec} \]
BOOKS FOR REFERENCE

1. H. C. Verma, *Concepts of Physics – Volume 2*, Bharati Bhawan Publisher
Magnetism

In this activity you will be able to visualize and understand the working of cyclotron.

**STEPS:**
- Open the browser and type 'physics.bu.edu/~duffy/HTML5/cyclotron.html' in the address bar.
- Click ‘play’ to release the positively charged particle between the D-shaped sections.
- Observe trajectory of positively charged particle under the magnetic field between D-shaped sections.
- Note the kinetic energy of the particle after some time (say \( t = 20 \) s)

**URL:**

* Pictures are indicative only.
* If browser requires, allow Flash Player or Java Script to load the page.
In this unit, the student is exposed to

- the phenomenon of electromagnetic induction
- the application of Lenz’s law to find the direction of induced emf
- the concept of Eddy current and its uses
- the phenomenon of self-induction and mutual-induction
- the various methods of producing induced emfs
- the construction and working of AC generators
- the principle of transformers and its role in long distance power communication
- the notion of root mean square value of alternating current
- the idea of phasors and phase relationships in different AC circuits
- the insight about power in an AC circuit and wattless current
- the understanding of energy conservation during LC oscillations

Physicists then began to think of the converse effect. Is it possible to produce an electric current with the help of a magnetic field? A series of experiments were conducted to establish the converse effect. These experiments were done by Michael Faraday of UK and Joseph Henry of USA, almost simultaneously and independently. These attempts became successful and led to the discovery of the phenomenon, called Electromagnetic Induction. Michael Faraday is credited with the discovery of electromagnetic induction in 1831.
In this chapter, let us see a few experiments of Faraday, the results and the phenomenon of Electromagnetic Induction. Before that, we will recollect the concept of magnetic flux linked with a surface area.

### An anecdote!

Michael Faraday was enormously popular for his lectures as well. In one of his lectures, he demonstrated his experiments which led to the discovery of electromagnetic induction.

At the end of the lecture, one member of the audience approached Faraday and said, “Mr. Faraday, the behaviour of the magnet and the coil of wire was interesting, but what is the use of it?” Faraday answered politely, “Sir, what is the use of a newborn baby?”

**Note**: We will soon see the greatness of ‘that little child’ who has now grown as an adult to cater to the energy needs.

### 4.1.2 Magnetic Flux ($\Phi_B$)

The magnetic flux through an area $A$ in a magnetic field is defined as the number of magnetic field lines passing through that area normally and is given by the equation (Figure 4.1(a)).

$$\Phi_B = \int_A B \cdot dA = BA \cos \theta$$  
(4.1)

where the integral is taken over the area $A$ and $\theta$ is the angle between the direction of the magnetic field and the outward normal to the area.

### Figure 4.1 Magnetic flux

If the magnetic field $\vec{B}$ is uniform over the area $A$ and is perpendicular to the area as shown in Figure 4.1(b), then the above equation becomes

$$\Phi_B = BA$$  
(4.2)

since $\theta = 0^\circ$, $\cos 0^\circ = 1$

The SI unit of magnetic flux is $T \, m^2$. It is also measured in weber or $Wb$.

$$1 \, Wb = 1 \, T \, m^2$$

### Example 4.1

A circular antenna of area 3 m$^2$ is installed at a place in Madurai. The plane of the area of antenna is inclined at 47° with the direction of Earth’s magnetic field. If the magnitude of Earth’s field at that place is 40773.9 nT find the magnetic flux linked with the antenna.

**Solution**

$B = 40773.9$ nT; $\theta = 90^\circ - 47^\circ = 43^\circ$; $A = 3m^2$
We know that \( \Phi_B = BA \cos \theta \)

\[
\Phi_B = 40.773.9 \times 10^{-9} \times 3 \times \cos 43°
\]
\[
= 40.7739 \times 10^{-6} \times 3 \times 0.7314
\]
\[
= 89.47 \times 10^{-6} \text{ Wb}
\]
\[\Phi_B = 89.47 \mu \text{Wb}\]

(ii) \( \theta = 90° - 60° = 30°; \)

\[
\Phi_B = BA \cos \theta = 0.2 \times 5 \times 10^{-2} \times \cos 30°
\]
\[
\Phi_B = 1 \times 10^{-2} \times \frac{\sqrt{3}}{2} = 8.66 \times 10^{-3} \text{ Wb}
\]

(iii) \( \theta = 90°; \)

\[\Phi_B = BA \cos 90° = 0\]

4.1.3 Faraday’s Experiments on Electromagnetic Induction

First Experiment

Consider a closed circuit consisting of a coil \( C \) of insulated wire and a galvanometer \( G \) as shown in Figure 4.2(a). The galvanometer does not indicate deflection as there is no electric current in the circuit.

When a bar magnet is inserted into the stationary coil, with its north pole facing the coil, there is a momentary deflection in the galvanometer. This indicates that an electric current is set up in the coil (Figure 4.2(b)). If the magnet is kept stationary inside the coil, the galvanometer does not indicate deflection (Figure 4.2(c)).

The bar magnet is now withdrawn from the coil, the galvanometer again gives a momentary deflection but in the opposite direction. So the electric current flows in the opposite direction (Figure 4.2(d)). Now if the magnet is moved faster, it gives a larger deflection due to a greater current in the circuit (Figure 4.2(e)).

The bar magnet is reversed i.e., the south pole now faces the coil. When the above experiment is repeated, the deflections are opposite to that obtained in the case of north pole (Figure 4.2(f)).
If the primary circuit is closed, electric current starts flowing in the primary circuit. At that time, the galvanometer gives a momentary deflection (Figure 4.3(a)). After that, when the electric current reaches a certain steady value, no deflection is observed in the galvanometer.

Likewise if the primary circuit is broken, the electric current starts decreasing and there is again a sudden deflection but in the opposite direction (Figure 4.3(b)). When the electric current becomes zero, the galvanometer shows no deflection.

From the above observations, it is concluded that whenever the electric current in the primary circuit changes, the galvanometer shows a deflection.

**Second Experiment**

Consider two closed circuits as shown in Figure 4.3(a). The circuit consisting of a coil $P$, a battery $B$ and a key $K$ is called as primary circuit while the circuit with a coil $S$ and a galvanometer $G$ is known as secondary circuit. The coils $P$ and $S$ are kept at rest in close proximity with respect to one another.

If the magnet is kept stationary and the coil is moved towards or away from the coil, similar results are obtained. It is concluded that whenever there is a relative motion between the coil and the magnet, there is deflection in the galvanometer, indicating the electric current setup in the coil.
Faraday’s Law of Electromagnetic Induction

From the results of his experiments, Faraday realized that

*Whenever the magnetic flux linked with a closed coil changes, an emf (electromotive force) is induced and hence an electric current flows in the circuit. This current is called an induced current and the emf giving rise to such current is called an induced emf. This phenomenon is known as electromagnetic induction.*

Based on this idea, Faraday’s experiments are understood in the following way. In the first experiment, when a bar magnet is placed close to a coil, some of the magnetic field lines of the bar magnet pass through the coil i.e., the magnetic flux is linked with the coil. When the bar magnet and the coil approach each other, the magnetic flux linked with the coil increases. So this increase in magnetic flux induces an emf and hence a transient current.

**Activity**

*Exploring Electromagnetic Induction*

Make a circuit containing a coil of insulated wire wound around soft hollow core and a galvanometer as shown in Figure. It is better to use a thin wire for the coil so that we can wind many turns in the available space. Perform the steps described in first experiment of Faradad with the help of a strong bar magnet. Students will get hands-on experience about electromagnetic induction.
When the primary circuit is open, no electric current flows in it and hence the magnetic flux linked with the secondary coil is zero (Figure 4.5(a)). However, when the primary circuit is closed, the increasing current builds up a magnetic field around the primary coil. Therefore, the magnetic flux linked with the secondary coil increases. This increasing flux linked induces a transient electric current in the secondary coil (Figure 4.5(b)). When the electric current in the primary coil reaches a steady value, the magnetic flux linked with the secondary coil does not change and the electric current in the secondary coil will disappear.

When the primary circuit is open, no electric current flows in it and hence the magnetic flux linked with the secondary coil is zero (Figure 4.5(a)). However, when the primary circuit is closed, the increasing current builds up a magnetic field around the primary coil. Therefore, the magnetic flux linked with the secondary coil increases. This increasing flux linked induces a transient electric current in the secondary coil (Figure 4.5(b)). When the electric current in the primary coil reaches a steady value, the magnetic flux linked with the secondary coil does not change and the electric current in the secondary coil will disappear.

**Figure 4.4** Explanation of Faraday’s first experiment

At the same time, when they recede away from one another, the magnetic flux linked with the coil decreases. The decrease in magnetic flux again induces an emf in opposite direction and hence an electric current flows in opposite direction (Figure 4.4(b)). So there is deflection in the galvanometer when there is a relative motion between the coil and the magnet.

In the second experiment, when the primary coil $P$ carries an electric current, a magnetic field is established around it. The magnetic lines of this field pass through itself and the neighbouring secondary coil $S$.
Importance of Electromagnetic Induction!

The application of the phenomenon of Electromagnetic Induction is almost everywhere in the present day life. Right from home appliances to huge factory machineries, from cellphone to computers and internet, from electric guitar to satellite communication, all need electricity for their operation. There is an ever growing demand for electric power.

All these are met with the help of electric generators and transformers which function on electromagnetic induction. The modern, sophisticated human life would not be possible without the discovery of electromagnetic induction.

The conclusions of Faraday’s experiments are stated as two laws.

First law
Whenever magnetic flux linked with a closed circuit changes, an emf is induced in the circuit.

Second law
The magnitude of induced emf in a closed circuit is equal to the time rate of change of magnetic flux linked with the circuit.

If the magnetic flux linked with the coil changes by $d\Phi_B$ in a time $dt$, then the induced emf is given by

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

The negative sign in the above equation gives the direction of the induced current which will be dealt with in the next section.

If a coil consisting of $N$ turns is tightly wound such that each turn covers the same
area, then the flux through each turn will be the same. Then total emf induced in the coil is given by

\[
\varepsilon = -N \frac{d\Phi_B}{dt} = - \frac{d(N\Phi_B)}{dt}
\]

(4.3)

Here, \(N\Phi_B\) is called flux linkage, defined as the product of number of turns \(N\) of the coil and the magnetic flux linking each turn of the coil \(\Phi_B\).

**Example 4.3**

A cylindrical bar magnet is kept along the axis of a circular solenoid. If the magnet is rotated about its axis, find out whether an electric current is induced in the coil.

**Solution**

The magnetic field of a cylindrical magnet is symmetrical about its axis. As the magnet is rotated along the axis of the solenoid, there is no induced current in the solenoid because the flux linked with the solenoid does not change due to the rotation of the magnet.

**Example 4.4**

A closed coil of 40 turns and of area 200 \(\text{cm}^2\), is rotated in a magnetic field of flux density 2 \(\text{Wb m}^{-2}\). It rotates from a position where its plane makes an angle of 30° with the field to a position perpendicular to the field in time 0.2 sec. Find the magnitude of the emf induced in the coil due to its rotation.

**Solution**

\(N = 40\) turns; \(B = 2\ \text{Wb m}^{-2}\)
\(A = 200\ \text{cm}^2 = 200 \times 10^{-4} \text{m}^2\);

Initial flux, \(\Phi_i = BA\cos\theta\)
\[= 2 \times 200 \times 10^{-4} \times \cos 60°\]
\[= 2 \times 200 \times 10^{-4} \times 0.5\] since \(\theta = 90° - 30° = 60°\)
\[\Phi_i = 2 \times 10^{-2} \text{Wb}\]

Final flux, \(\Phi_f = BA\cos\theta\)
\[= 2 \times 200 \times 10^{-4} \times \cos 0°\] since \(\theta = 0°\)
\[\Phi_f = 4 \times 10^{-2} \text{Wb}\]

Magnitude of the induced emf is
\[
\varepsilon = N \frac{d\Phi_B}{dt} = \frac{40 \times (4 \times 10^{-2} - 2 \times 10^{-2})}{0.2} = 4V
\]

**Example 4.5**

A straight conducting wire is dropped horizontally from a certain height with its length along east – west direction. Will an emf be induced in it? Justify your answer.

**Solution**

Yes! An emf will be induced in the wire because it moves perpendicular to the horizontal component of Earth’s magnetic field.

**4.1.4** Lenz’s Law

A German physicist Heinrich Lenz performed his own experiments on electromagnetic induction and deduced a
law to determine the direction of the induced current. This law is known as Lenz’s law.

**Lenz’s law states that the direction of the induced current is such that it always opposes the cause responsible for its production.**

Faraday discovered that when magnetic flux linked with a coil changes, an electric current is induced in the circuit. Here the flux change is the cause while the induction of current is the effect. Lenz’s law says that the effect always opposes the cause. Therefore, the induced current would flow in a direction so that it could oppose the flux change.

To understand Lenz’s law, let us consider two illustrations in which we find the direction of the induced current in a circuit.

**Illustration 1**

Consider a uniform magnetic field, with its field lines perpendicular to the plane of the paper and pointing inwards. These field lines are represented by crosses (x) as shown in Figure 4.6(a). A rectangular metallic frame $ABCD$ is placed in this magnetic field, with its plane perpendicular to the field. The arm $AB$ is movable so that it can slide towards right or left.

If the arm $AB$ slides to our right side, the number of field lines (magnetic flux) passing through the frame $ABCD$ increases and a current is induced. As suggested by Lenz’s law, the induced current opposes this flux increase and it tries to reduce it by producing another magnetic field pointing outwards i.e., opposite to the existing magnetic field.

The magnetic lines of this induced field are represented by red-colored circles in the Figure 4.6(b). From the direction of the magnetic field thus produced, the direction of the induced current is found to be anti-clockwise by using right-hand thumb rule.
The leftward motion of arm $AB$ decreases magnetic flux. The induced current, this time, produces a magnetic field in the inward direction (red-colored crosses) i.e., in the direction of the existing magnetic field (Figure 4.6(c)). Therefore, the flux decrease is opposed by the flow of induced current. From this, it is found that induced current flows in clockwise direction.

**Illustration 2**

Let us move a bar magnet towards the solenoid, with its north pole pointing the solenoid as shown in Figure 4.7(b). This motion increases the magnetic flux of the coil which in turn, induces an electric current. Due to the flow of induced current, the coil becomes a magnetic dipole whose two magnetic poles are on either end of the coil.

In this case, the cause producing the induced current is the movement of the magnet. According to Lenz’s law, the induced current should flow in such a way that it opposes the movement of the north pole towards coil. It is possible if the end nearer to the magnet becomes north pole (Figure 4.7(b)). Then it repels the north pole of the bar magnet and opposes the movement of the magnet. Once pole ends are known, the direction of the induced current could be found by using right hand thumb rule.

When the bar magnet is withdrawn, the nearer end becomes south pole which attracts north pole of the bar magnet, opposing the receding motion of the magnet (Figure 4.7(c)).

Thus the direction of the induced current can be found from Lenz’s law.

**Conservation of energy**

The truth of Lenz’s law can be established on the basis of the law of conservation of energy. The explanation is as follows: According to Lenz’s law, when a magnet is moved either towards or away from a coil, the induced current produced opposes its motion. As a result, there will always be a...
Demonstration of Lenz’s law

Take a narrow copper pipe and a strongly magnetized button magnet as shown in figure. Keep the copper pipe vertical and drop the magnet into the pipe. Watch the motion of the magnet and note that magnet has become slower than its free fall. The reason is that an electric current generated by a moving magnet will always oppose the original motion of the magnet that produced the current.

resisting force on the moving magnet. Work has to be done by some external agency to move the magnet against this resisting force. Here the mechanical energy of the moving magnet is converted into the electrical energy which in turn, gets converted into Joule heat in the coil i.e., energy is converted from one form to another.

On the contrary to Lenz’s law, let us assume that the induced current helps the cause responsible for its production. Now when we push the magnet little bit towards the coil, the induced current helps the movement of the magnet towards the coil. Then the magnet starts moving towards the coil without any expense of energy. This, then, becomes a perpetual motion machine. In practice, no such machine is possible. Therefore, the assumption that the induced current helps the cause is wrong.

4.1.5 Fleming’s right hand rule

When a conductor moves in a magnetic field, the direction of motion of the conductor, the field and the induced current are given by Fleming’s right hand rule and is as follows:

The thumb, index finger and middle finger of right hand are stretched out in mutually perpendicular directions (as shown in Figure 4.8). If the index finger points the direction of the magnetic field and the thumb indicates the direction of motion of the conductor, then the middle finger will indicate the direction of the induced current.

Fleming’s right hand rule is also known as generator rule.
**EXAMPLE 4.6**

If the current $i$ flowing in the straight conducting wire as shown in the figure decreases, find out the direction of induced current in the metallic square loop placed near it.

![Image of a wire and loop](image)

**Solution**

From right hand rule, the magnetic field by the straight wire is directed into the plane of the square loop perpendicularly and its magnetic flux is decreasing. The decrease in flux is opposed by the current induced in the loop by producing a magnetic field in the same direction as the magnetic field of the wire. Again from right hand rule, for this inward magnetic field, the direction of the induced current in the loop is clockwise.

**EXAMPLE 4.7**

The magnetic flux passes perpendicular to the plane of the circuit and is directed into the paper. If the magnetic flux varies with respect to time as per the following relation: $\Phi_B = (2t^3 + 3t^2 + 8t + 5)\text{mWb}$, what is the magnitude of the induced emf in the loop when $t = 3s$? Find out the direction of current through the circuit.

![Image of a magnetic field and current](image)

**Solution**

\[ \Phi_B = (2t^3 + 3t^2 + 8t + 5)\text{mWb}; N = 1; t = 3\text{s} \]

(i) \[ \varepsilon = \frac{d}{dt}(N\Phi_B) = \frac{d}{dt}(2t^3 + 3t^2 + 8t + 5) \times 10^{-3} \]
\[ = (6t^2 + 6t + 8) \times 10^{-3} \]

At $t = 3\text{s}$,
\[ \varepsilon = [(6 \times 9) + (6 \times 3) + 8] \times 10^{-3} \]
\[ = 80 \times 10^{-3} \text{V} = 80\text{mV} \]

(ii) As time passes, the magnetic flux linked with the loop increases. According to Lenz’s law, the direction of the induced current should be in a way so as to oppose the flux increase. So, the induced current flows in such a way to produce a magnetic field opposite to the given field. This magnetic field is perpendicularly outwards. Therefore, the induced current flows in anti-clockwise direction.

**4.1.6 Motional emf from Lorentz force**

Consider a straight conducting rod $AB$ of length $l$ in a uniform magnetic field $\vec{B}$ which is directed perpendicularly into the plane of the paper as shown in Figure 4.9(a). The length of the rod is normal to the magnetic field. Let the rod move with a constant velocity $\vec{v}$ towards right side.

When the rod moves, the free electrons present in it also move with same
velocity $\vec{v}$ in $\vec{B}$. As a result, the Lorentz force acts on free electrons in the direction from B to A and is given by the relation

$$\vec{F}_B = -e(\vec{v} \times \vec{B})$$

(4.4)

The action of this Lorentz force is to accumulate the free electrons at the end A. This accumulation of free electrons produces a potential difference across the rod which in turn establishes an electric field $\vec{E}$ directed along BA (Figure 4.9(b)). Due to the electric field $\vec{E}$, the coulomb force starts acting on the free electrons along $AB$ and is given by

$$\vec{F}_E = -e\vec{E}$$

(4.5)

The magnitude of the electric field $\vec{E}$ keeps on increasing as long as accumulation of electrons at the end A continues. The force $\vec{F}_E$ also increases until equilibrium is reached. At equilibrium, the magnetic Lorentz force $\vec{F}_B$ and the coulomb force $\vec{F}_E$ balance each other and no further accumulation of free electrons at the end A takes place.

i.e.,

$$|\vec{F}_B| = |\vec{F}_E|$$

$$e(\vec{v} \times \vec{B}) = -e\vec{E}$$

$$vB \sin 90^\circ = E$$

$$vB = E$$

(4.6)

The potential difference between two ends of the rod is

$$V = El$$

$$V = vBl$$

Figure 4.9 Motional emf from Lorentz force

Thus the Lorentz force on the free electrons is responsible to maintain this potential difference and hence produces an emf

$$\varepsilon = Blv$$

(4.7)

As this emf is produced due to the movement of the rod, it is often called as motional emf. If the ends A and B are connected by an external circuit of total resistance $R$, then current $i = \frac{\varepsilon}{R} = \frac{Blv}{R}$ flows in it. The direction of the current is found from right-hand thumb rule.

4.1.7 Motional emf from Faraday’s law and Energy conservation

Let us consider a rectangular conducting loop of width $l$ in a uniform magnetic field $\vec{B}$ which is perpendicular to the plane of the loop and is directed inwards. A part of the loop is in the magnetic field while the remaining part is outside the field as shown in Figure 4.10.
where $v = \frac{dx}{dt}$ is the velocity of the loop. This emf is known as motional emf since it is produced due to the movement of the loop in the magnetic field.

From Lenz’s law, it is found that the induced current flows in clockwise direction. If $R$ is the resistance of the loop, then the induced current is given by

$$i = \frac{\varepsilon}{R} = \frac{Blv}{R} \quad (4.10)$$

**Energy conservation**

In order to move the loop with a constant velocity $\vec{v}$, a constant force that is equal and opposite to the magnetic force, must be applied. Therefore, mechanical work is done to move the loop. Then the rate of doing work or power is

$$P = \vec{F} \cdot \vec{v} = Fv\cos \theta$$

Here $\theta = 0^\circ$ (4.11)

Now, let us find the magnetic force acting on the loop due to its movement in the magnetic field. Let three deflecting forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ be acting on the three segments of the loop as shown in Figure 4.10. The general equation of such a deflecting force is given by

$$\vec{F}_d = i \vec{l} \times \vec{B}$$

Forces $\vec{F}_2$ and $\vec{F}_3$ are equal in magnitude and opposite in direction and cancel each other. Therefore, the force $\vec{F}_1$ alone acts on the left segment of the loop in a direction shown in Figure 4.10 and is given by

$$\vec{F}_1 = i \vec{l} \times \vec{B} \quad (4.12)$$

As this magnetic flux decreases due to the movement of the loop, the magnitude of the induced emf is given by

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{d}{dt}(Blx)$$

Here, both $B$ and $l$ are constants. Therefore,

$$\varepsilon = Blv \quad (4.9)$$

**Figure 4.10** Motional emf from Faraday’s law

When the loop is pulled with a constant velocity $\vec{v}$ to the right, the area of the portion of the loop within the magnetic field will decrease. Thus, the flux linked with the loop will also decrease. According to Faraday’s law, an electric current is induced in the loop which flows in a direction so as to oppose the pull of the loop.

Let $x$ be the length of the loop which is still within the magnetic field, then its area is $lx$. The magnetic flux linked with the loop is

$$\Phi_B = \int_B \cdot d\vec{A} = BA \cos \theta$$

Here $\theta = 0^\circ$ and $\cos 0^\circ = 1$

$$\Phi_B = Blx \quad (4.8)$$

As this magnetic flux decreases due to the movement of the loop, the magnitude of the induced emf is given by

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{d}{dt}(Blx)$$

Here, both $B$ and $l$ are constants. Therefore,

$$\varepsilon = Bl \frac{dx}{dt}$$

$$\varepsilon = Blv \quad (4.9)$$
EXAMPLE 4.8

A conducting rod of length 0.5 m falls freely from the top of a building of height 7.2 m at a place in Chennai where the horizontal component of Earth’s magnetic field is 40378.7 nT. If the length of the rod is perpendicular to Earth’s horizontal magnetic field, find the emf induced across the conductor when the rod is about to touch the ground. [Take \( g = 10 \text{ m s}^{-2} \)]

Solution

\( l = 0.5 \text{ m}; h = 7.2 \text{ m}; u = 0 \text{ m s}^{-1}; g = 10 \text{ m s}^{-2}; B_H = 40378.7 \text{ nT} \)

The final velocity of the rod is

\[ v^2 = u^2 + 2gh \]
\[ = 0 + (2 \times 10 \times 7.2) \]
\[ v^2 = 144 \]
\[ v = 12 \text{ m s}^{-1} \]

Induced emf when the rod is about to touch the ground, \( \varepsilon = B_H lv \)
\[ = 40.378.7 \times 10^{-9} \times 0.5 \times 12 \]
\[ = 242.27 \times 10^{-6} \text{ V} \]
\[ = 242.27 \mu \text{V} \]

EXAMPLE 4.9

A copper rod of length \( l \) rotates about one of its ends with an angular velocity \( \omega \) in a magnetic field \( B \) as shown in the figure. The plane of rotation is perpendicular to the field. Find the emf induced between the two ends of the rod.

Here \( \theta \) is the angle between \( \vec{B} \) and the length vector \( \vec{l} \) for the left segment and is \( 90^\circ \)

\[ \therefore F_i = ilB \sin 90^\circ = ilB \quad \text{since} \quad \sin 90^\circ = 1 \]

The applied force \( \vec{F} \) must be equal to \( F_1 \) in order to just move the loop with a constant velocity \( \vec{v} \)

\[ \therefore \vec{F} = -F_1 \]

(since \( \vec{F} \) and \( F_1 \) are in opposite direction)

Considering only the magnitudes,

\[ F = F_1 = ilB \]

Substituting for \( i \) from equation (4.10)

\[
F = \left( \frac{Blv}{R} \right) lB
\]

\[ F = \frac{B^2l^2v}{R} \]

From equation (4.11), the rate at which the mechanical work is done to pull the loop from the magnetic field or power is given by

\[ P = Fv = \left( \frac{B^2l^2v}{R} \right) v = \frac{B^2l^2v^2}{R} \]  

(4.13)

When the induced current flows in the loop, Joule heating takes place. The rate at which thermal energy is dissipated in the loop or power dissipated is

\[ P = i^2R = \left( \frac{Blv}{R} \right)^2 R = \frac{B^2l^2v^2}{R} \]

(4.14)

This equation is exactly same as the equation (4.13). Thus the mechanical work done in moving the loop appears as thermal energy in the loop.
Even for a conductor in the form of a sheet or plate, an emf is induced when magnetic flux linked with it changes. But the difference is that there is no definite loop or path for induced current to flow away. As a result, the induced currents flow in concentric circular paths (Figure 4.11). As these electric currents resemble eddies of water, these are known as Eddy currents. They are also called Foucault currents.

**Demonstration**

Here is a simple demonstration for the production of eddy currents. Consider a pendulum that can be made to oscillate between the poles of a powerful electromagnet as shown in Figure 4.12(a). First the electromagnet is switched off, the pendulum is slightly displaced and released. It begins to oscillate and it executes a large number of oscillations before stopping. The air friction is the only damping force.

When the electromagnet is switched on and the disc of the pendulum is made to oscillate, eddy currents are produced in it which will oppose the oscillation. A heavy damping force of eddy currents will bring the pendulum to rest within a few oscillations (Figure 4.12(b)).

**Solution**

Consider a small element of length $dx$ at a distance $x$ from the centre of the circle described by the rod. As this element moves perpendicular to the field with a linear velocity $v = x \omega$, the emf developed in the element $dx$ is

$$de = Bvdx = B(x \omega)dx$$

This rod is made up of many such elements, moving perpendicular to the field. The emf developed across two ends is

$$\varepsilon = \int_0^l B\omega x dx = B\omega \frac{x^2}{2} \bigg|_0^l$$

$$\varepsilon = \frac{1}{2} B\omega l^2$$

**4.2 EDDY CURRENTS**

According to Faraday’s law of electromagnetic induction, an emf is induced in a conductor when the magnetic flux passing through it changes. However, the conductor need not be in the form of a wire or coil.
Figure 4.12 Demonstration of eddy currents

However if some slots are cut in the disc as shown in Figure 4.12(c), the eddy currents are reduced. The pendulum now will execute several oscillations before coming to rest. This clearly demonstrates the production of eddy current in the disc of the pendulum.

Drawbacks of Eddy currents

When eddy currents flow in the conductor, a large amount of energy is dissipated in the form of heat. The energy loss due to the flow of eddy current is inevitable but it can be reduced to a greater extent with suitable measures.

The design of transformer core and electric motor armature is crucial in order to minimise the eddy current loss. To reduce these losses, the core of the transformer is made up of thin laminas insulated from one another (Figure 4.13 (a)) while for electric motor the winding is made up of a group of wires insulated from one another (Figure 4.13 (b)). The insulation used does not allow huge eddy currents to flow and hence losses are minimized.

Figure 4.13 (a) Insulated laminas of the core of a transformer

Figure 4.13 (b) Insulated winding of an electric motor
Application of eddy currents

Though the production of eddy current is undesirable in some cases, it is useful in some other cases. A few of them are

i. Induction stove
ii. Eddy current brake
iii. Eddy current testing
iv. Electromagnetic damping

i. Induction stove

Induction stove is used to cook the food quickly and safely with less energy consumption. Below the cooking zone, there is a tightly wound coil of insulated wire. The cooking pan made of suitable material, is placed over the cooking zone. When the stove is switched on, an alternating current flowing in the coil produces high frequency alternating magnetic field which induces very strong eddy currents in the pan. The eddy currents in the pan produce so much of heat due to Joule heating which is used to cook the food.

Note: The frequency of the domestic AC supply is increased from 50–60 Hz to around 20–40 KHz before giving it to the coil in order to produce high frequency alternating magnetic field.

Example

A spherical stone and a spherical metallic ball of same size and mass are dropped from the same height. Which one, a stone or a metal ball, will reach the earth’s surface first? Justify your answer. Assume that there is no air friction.

Answer

The stone will reach the earth’s surface earlier than the metal ball. The reason is that when the metal ball falls through the magnetic field of earth, the eddy currents are produced in it which opposes its motion. But in the case of stone, no eddy currents are produced and it falls freely.
ii. Eddy current brake

This eddy current braking system is generally used in high speed trains and roller coasters. Strong electromagnets are fixed just above the rails. To stop the train, electromagnets are switched on. The magnetic field of these magnets induces eddy currents in the rails which oppose or resist the movement of the train. This is Eddy current linear brake (Figure 4.15(a)).

In some cases, the circular disc, connected to the wheel of the train through a common shaft, is made to rotate in between the poles of an electromagnet. When there is a relative motion between the disc and the magnet, eddy currents are induced in the disc which stop the train. This is Eddy current circular brake (Figure 4.15(b)).

iii. Eddy current testing

It is one of the simple non-destructive testing methods to find defects like surface cracks, air bubbles present in a specimen. A coil of insulated wire is given an alternating electric current so that it produces an alternating magnetic field. When this coil is brought near the test surface, eddy current is induced in the test surface. The presence of defects causes the change in phase and amplitude of the eddy current that can be detected by some other means. In this way, the defects present in the specimen are identified (Figure 4.16).

iv. Electromagnetic damping

The armature of the galvanometer coil is wound on a soft iron cylinder. Once the
will study about self-induction and mutual induction in the next sections.

**Figure 4.18** Examples for inductor

Self-induction

An electric current flowing through a coil will set up a magnetic field around it. Therefore, the magnetic flux of the magnetic field is linked with that coil itself. If this flux is changed by changing the current, an emf is induced in that same coil (Figure 4.19). This phenomenon is known as self-induction. The emf induced is called self-induced emf.

**Figure 4.19** Self-Induction

**4.3 SELF–INDUCTION**

**4.3.1 Introduction**

Inductor is a device used to store energy in a magnetic field when an electric current flows through it. The typical examples are coils, solenoids and toroids shown in Figure 4.18.

Inductance is the property of inductors to generate emf due to the change in current flowing through that circuit (self-induction) or a change in current through a neighbouring circuit with which it is magnetically linked (mutual induction). We

Figure 4.17 Electromagnetic damping

armature is deflected, the relative motion between the soft iron cylinder and the radial magnetic field induces eddy current in the cylinder (Figure 4.17). The damping force due to the flow of eddy current brings the armature to rest immediately and then galvanometer shows a steady deflection. This is called electromagnetic damping.

**Figure 4.17** Electromagnetic damping

**Figure 4.18** Examples for inductor

**Figure 4.19** Self-Induction

UNIT 4 ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

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Let $\Phi_\beta$ be the magnetic flux linked with each turn of the coil of $N$ turns, then the total flux linked with the coil $N\Phi_\beta$ (flux linkage) is proportional to the current $i$ in the coil.

\[
N\Phi_\beta \propto i
\]

\[
N\Phi_\beta = Li
\]  \hspace{1cm} (4.15)

(or)

\[
L = \frac{N\Phi_\beta}{i}
\]

The constant of proportionality $L$ is called self-inductance of the coil. It is also referred to as the coefficient of self-induction. If $i = 1 A$, then $L = N\Phi_\beta$. **Self-inductance or simply inductance of a coil is defined as the flux linkage of the coil when 1A current flows through it.**

When the current $i$ changes with time, an emf is induced in it. From Faraday’s law of electromagnetic induction, this self-induced emf is given by

\[
\varepsilon = -\frac{d(N\Phi_\beta)}{dt} = -\frac{d(Li)}{dt}
\]

(4.16)

The negative sign in the above equation means that the self-induced emf always opposes the change in current with respect to time. If $\frac{di}{dt} = 1 A s^{-1}$, then $L = -\varepsilon$. **Inductance of a coil is also defined as the opposing emf induced in the coil when the rate of change of current through the coil is 1 $A s^{-1}$.**

**Unit of inductance**

Inductance is a scalar and its unit is $Wb A^{-1}$ or $Vs A^{-1}$. It is also measured in henry (H).

\[
1 H = 1 Wb A^{-1} = 1 Vs A^{-1}
\]

The dimensional formula of inductance is $MLTA^{-2}$. If $i = 1 A$ and $N\Phi_\beta = 1 Wb$ *turns*, then $L = 1H$.

Therefore, the **inductance of the coil is said to be one henry if a current of 1 A produces unit flux linkage in the coil.**

If $\frac{di}{dt} = 1 A s^{-1}$ and $\varepsilon = -1 V$, then $L = 1H$.

Therefore, the **inductance of the coil is one henry if a current changing at the rate of 1 $A s^{-1}$ induces an opposing emf of 1 $V$ in it.**

**Physical significance of inductance**

We have learnt about inertia in XI standard. In translational motion, mass is a measure of inertia; in the same way, for rotational motion, moment of inertia is a measure of rotational inertia (Refer sections 3.2.1 and 5.4 of XI physics text book). Generally, inertia means opposition to change its state.

The inductance plays the same role in a circuit as mass and moment of inertia play in mechanical motion. When a circuit is switched on, the increasing current induces an emf which opposes the growth of current in a circuit (Figure 4.20(a)). Likewise, when circuit is broken, the decreasing current induces an emf in the reverse direction. This emf now opposes the decay of current (Figure 4.20(b)).

Thus, inductance of the coil opposes any change in current and tries to maintain the original state.
**4.3.2 Self-inductance of a long solenoid**

Consider a long solenoid of length \( l \) and cross-sectional area \( A \). Let \( n \) be the number of turns per unit length (or turn density) of the solenoid. When an electric current \( i \) is passed through the solenoid, a magnetic field is produced by it which is almost uniform and is directed along the axis of the solenoid as shown in Figure 4.21. The magnetic field at any point inside the solenoid is given by (Refer section 3.9.3)

\[ B = \mu_n i \]

As this magnetic field passes through the solenoid, the windings of the solenoid are linked by the field lines. The magnetic flux passing through each turn is

\[ \Phi_B = \int_A B \cdot dA = B A \cos \theta = B A \text{ since } \theta = 0^\circ = (\mu_n i)A \]

The total magnetic flux linked or flux linkage of the solenoid with \( N \) turns (the total number of turns \( N \) is given by \( N = n l \)) is

\[ N \Phi_B = (nl)(\mu_n i)A \]

\[ N \Phi_B = (\mu \mu_n^2 Al) i \quad \text{(4.17)} \]

The equation (4.15) is

\[ N \Phi_B = Li \]

Comparing equations (4.15) and (4.17), we have

\[ L = \mu_n \mu^2 Al \]

From the above equation, it is clear that inductance depends on the geometry of the solenoid (turn density \( n \), cross-sectional area \( A \), length \( l \)) and the medium present inside the solenoid. If the solenoid is filled with a dielectric medium of relative permeability \( \mu_r \), then

\[ L = \mu n^2 Al \quad \text{or} \quad L = \mu \mu_r n^2 Al \]
Energy stored in an inductor

Whenever a current is established in the circuit, the inductance opposes the growth of the current. In order to establish a current in the circuit, work is done against this opposition by some external agency. This work done is stored as magnetic potential energy.

Let us assume that electrical resistance of the inductor is negligible and inductor effect alone is considered. The induced emf $\varepsilon$ at any instant $t$ is

$$\varepsilon = -L \frac{di}{dt}$$

Let $dW$ be work done in moving a charge $dq$ in a time $dt$ against the opposition, then

$$dW = -\varepsilon dq$$

$$= -\varepsilon idt$$

$\therefore dq = idt$

Substituting for $\varepsilon$ from equation (4.16),

$$= -\left( -L \frac{di}{dt} \right) idt$$

$$dW = L idi$$

Total work done in establishing the current $i$ is

$$W = \int dW = \int L idi = L \int \left[ \frac{i^2}{2} \right]_0^i$$

$$W = \frac{1}{2} Li^2$$

This work done is stored as magnetic potential energy.

$$\therefore U_B = \frac{1}{2} Li^2$$  \hspace{1cm} (4.18)

The energy density is the energy stored per unit volume of the space and is given by

$$u_B = \frac{U_B}{Al}$$ \hspace{1cm} Volume of the solenoid = $Al$

**Example 4.10**

A solenoid of 500 turns is wound on an iron core of relative permeability 800. The length and radius of the solenoid are 40 cm and 3 cm respectively. Calculate the average emf induced in the solenoid if the current in it changes from 0 to 3 A in 0.4 second.

**Solution**

$N = 500$ turns; \hspace{0.5cm} $\mu_r = 800$;

$l = 40$ cm = 0.4 m; \hspace{0.5cm} $r = 3$ cm = 0.03 m;

$di = 3 - 0 = 3$ A; \hspace{0.5cm} $dt = 0.4$ s

Self inductance,

$$L = \mu \mu_0 n^2 Al$$

$$= \frac{\mu \mu_0 N^2 \pi r^2}{l}$$

$$= \frac{4 \times 3.14 \times 10^{-7} \times 800 \times 500^2 \times 3.14 \times (3 \times 10^{-2})^2}{0.4}$$

$L = 1.77$ H

Magnitude of induced emf, $\varepsilon = L \frac{di}{dt}$

$$= \frac{1.77 \times 3}{0.4}$$

$\varepsilon = 13.275$ V

**Example 4.11**

The self-inductance of an air-core solenoid is 4.8 mH. If its core is replaced by iron core, then its self-inductance becomes 1.8 H. Find out the relative permeability of iron.
Consider two coils which are placed close to each other. If an electric current $i_1$ is sent through coil 1, the magnetic field produced by it is also linked with coil 2 as shown in Figure 4.22(a).

Let $\Phi_{21}$ be the magnetic flux linked with each turn of the coil 2 of $N_2$ turns due to coil 1, then the total flux linked with coil 2

$$(N_2 \Phi_{21})$$

is proportional to the current $i_1$ in the coil 1.

$$
N_2 \Phi_{21} \propto i_1
$$

(or)

$$
M_{21} = \frac{N_2 \Phi_{21}}{i_1}
$$

The constant of proportionality $M_{21}$ is the mutual inductance of the coil 2 with respect to coil 1. It is also called as coefficient of mutual induction. If $i_1 = 1A$, then $M_{21} = N_2 \Phi_{21}$.

Therefore, the mutual inductance $M_{21}$ is defined as the flux linkage of the coil 2 when 1A current flows through coil 1.

When the current $i_1$ changes with time, an emf $\varepsilon_2$ is induced in coil 2. From Faraday’s law of electromagnetic induction, this mutually induced emf $\varepsilon_2$ is given by

$$
\varepsilon_2 = -\frac{d(N_2 \Phi_{21})}{dt} = -\frac{d(M_{21} i_1)}{dt}
$$

(or)

$$
M_{21} = -\frac{\varepsilon_2}{di_1/dt}
$$

The negative sign in the above equation shows that the mutually induced emf always opposes the change in current $i_1$ with respect to time. If $di_1/dt = 1A$ s$^{-1}$, then $M_{21} = -\varepsilon_2$. 

**Solution**

$$L_{\text{air}} = 4.8 \times 10^{-3} H$$

$$L_{\text{iron}} = 1.8 H$$

$$L_{\text{air}} = \mu_n n^2 Al = 4.8 \times 10^{-3} H$$

$$L_{\text{iron}} = \mu n^2 Al = \mu \mu_n n^2 Al = 1.8 H$$

$$\therefore \mu_r = \frac{L_{\text{iron}}}{L_{\text{air}}} = \frac{1.8}{4.8 \times 10^{-3}} = 375$$

**4.3.3 Mutual induction**

When an electric current passing through a coil changes with time, an emf is induced in the neighbouring coil. This phenomenon is known as mutual induction and the emf is called mutually induced emf.

---

**Figure 4.22** Mutual induction
Mutual inductance $M_{21}$ is also defined as the opposing emf induced in the coil 2 when the rate of change of current through the coil 1 is $1A/s$.

Similarly, if an electric current $i_1$ through coil 2 changes with time, then emf $\varepsilon_1$ is induced in coil 1. Therefore,

$$M_{12} = \frac{N_1 \Phi_{12}}{i_2} \quad \text{and} \quad M_{12} = \frac{-\varepsilon_1}{di_2/dt}$$

where $M_{12}$ is the mutual inductance of the coil 1 with respect to coil 2. It can be shown that for a given pair of coils, the mutual inductance is same.

i.e., \[ M_{21} = M_{12} = M \]

In general, the mutual induction between two coils depends on size, shape, the number of turns of the coils, their relative orientation and permeability of the medium.

**Unit of mutual-inductance**

The unit of mutual inductance is also henry (H).

If $i_1 = 1A$ and $N_2 \Phi_{21} = 1 Wb$ turns, then $M_{21} = 1H$.

Therefore, the mutual inductance between two coils is said to be one henry if a current of $1A$ in coil 1 produces unit flux linkage in coil 2.

If \( di_1/dt = 1A/s \) and \( \varepsilon_2 = -1 V \), then $M_{21} = 1H$.

Therefore, the mutual inductance between two coils is one henry if a current changing at the rate of $1A/s$ in coil 1 induces an opposing emf of $1V$ in coil 2.
The flux linkage of solenoid 2 with total turns $N_2$ is

$$N_2 \Phi_{21} = (n_1 l) (\mu N_2 i_2) A_2 \quad \text{since} \quad N_2 = n_2 l$$

$$N_2 \Phi_{21} = (\mu N_1 n_2 A_2) i_1$$  \hspace{1cm} (4.20)

From equation (4.19),

$$N_2 \Phi_{21} = M_{21} i_1$$  \hspace{1cm} (4.21)

Comparing the equations (4.20) and (4.21),

$$M_{21} = \mu N_1 n_2 A_2$$  \hspace{1cm} (4.22)

This gives the expression for mutual inductance $M_{21}$ of the solenoid 2 with respect to solenoid 1. Similarly, we can find mutual inductance $M_{12}$ of solenoid 1 with respect to solenoid 2 as given below.

The magnetic field produced by the solenoid 2 when carrying a current $i_2$ is

$$B_2 = \mu N_2 i_2$$

This magnetic field $B_2$ is uniform inside the solenoid 2 but outside the solenoid 2, it is almost zero. Therefore for solenoid 1, the area $A_2$ is the effective area over which the magnetic field $B_2$ is present; not area $A_1$. Then the magnetic flux $\Phi_{12}$ linked with each turn of solenoid 1 due to solenoid 2 is

$$\Phi_{12} = \int_{A_1} B_2 \cdot d\vec{A} = B_2 A_2 = (\mu N_2 i_2) A_2$$

The flux linkage of solenoid 1 with total turns $N_1$ is

$$N_1 \Phi_{12} = (n_1 l) (\mu N_2 i_2) A_2 \quad \text{since} \quad N_1 = n_1 l$$

$$N_1 \Phi_{12} = (\mu N_1 n_2 A_2) i_2$$

$$\text{since} \quad N_1 \Phi_{12} = M_{12} i_2$$

Therefore, we get

$$M_{12} = (\mu N_1 n_2 A_2) i_2$$  \hspace{1cm} (4.23)

From equation (4.22) and (4.23), we can write

$$M_{12} = M_{21} = M$$  \hspace{1cm} (4.24)

In general, the mutual inductance between two long co-axial solenoids is given by

$$M = \mu N_2 A_2$$  \hspace{1cm} (4.25)

If a dielectric medium of relative permeability $\mu_r$ is present inside the solenoids, then

$$M = \mu N_2 A_2$$

(or) $$M = \mu N_2 A_2$$

**EXAMPLE 4.12**

The current flowing in the first coil changes from 2 A to 10 A in 0.4 sec. Find the mutual inductance between two coils if an emf of 60 mV is induced in the second coil. Also determine the induced emf in the second coil if the current in the first coil is changed from 4 A to 16 A in 0.03 sec. Consider only the magnitude of induced emf.

**Solution**

Case (i):

$$di_1 = 10 - 2 = 8 \text{ A}; \quad dt = 0.4 \text{ s}$$

$$\epsilon_2 = 60 \times 10^{-3} \text{ V}$$

Case (ii):

$$di_1 = 16 - 4 = 12 \text{ A}; \quad dt = 0.03 \text{ s}$$

(i) Mutual inductance of the second coil with respect to the first coil
**Solution**

\[ N_A = 200 \text{ turns}; \quad N_B = 1000 \text{ turns}; \]
\[ r_A = 20 \times 10^{-2} \text{ m}; \quad r_B = 2 \times 10^{-2} \text{ m}; \]
\[ dt = 0.04 \text{ s}; \quad i_A = 6-2 = 4 \text{ A} \]

Let \( i_A \) be the current flowing in coil \( A \), then the magnetic field \( B_A \) at the centre of the circular coil \( A \) is

\[ B_A = \frac{\mu N_A i_A}{2r_A} = \frac{4\pi \times 10^{-7} N_A i_A}{2r_A} = \frac{10^{-7} \times 2 \times 3.14 \times 200}{20 \times 10^{-2}} \times i_A = 6.28 \times 10^{-4} i_A \text{ Wb m}^{-2} \]

The magnetic flux linkage of coil \( B \) is

\[ N_B \Phi_B = N_B B_A A_B \]
\[ = 1000 \times 6.28 \times 10^{-4} i_A \times 3.14 \times (2 \times 10^{-2})^2 \]
\[ = 7.89 \times 10^{-4} i_A \text{ Wb turns} \]

The mutual inductance of the coil \( B \) with respect to coil \( A \) is

\[ M_{BA} = \frac{N_B \Phi_B}{i_A} = 7.89 \times 10^{-4} \text{ H} \]

Induced emf in coil \( B \) is

\[ \varepsilon_B = -M_{BA} \frac{di_A}{dt} \]

Considering only the magnitude,

\[ \varepsilon_B = \frac{7.89 \times 10^{-4} \times (6-2)}{0.04} \]
\[ = \frac{7.89 \times 10^{-4} \times (4)}{4 \times 10^{-2}} \]
\[ \varepsilon_B = 7.89 \times 10^{-2} \text{ V} \]
\[ \varepsilon_B = 78.9 \text{ mV} \]

The rate of change of magnetic flux of coil \( B \) is

\[ \frac{d(N_B \Phi_B)}{dt} = \varepsilon_B = 78.9 \text{ mWb s}^{-1} \]
4.4 METHODS OF PRODUCING INDUCED EMF

4.4.1 Introduction

Electromotive force is the characteristic of any energy source capable of driving electric charge around a circuit. We have already learnt that it is not actually a force. It is the work done in moving unit electric charge around the circuit. It is measured in $J C^{-1}$ or volt.

Some examples of energy source which provide emf are electrochemical cells, thermoelectric devices, solar cells and electrical generators. Of these, electrical generators are most powerful machines. They are used for large scale power generation.

According to Faraday’s law of electromagnetic induction, an emf is induced in a circuit when magnetic flux linked with it changes. This emf is called induced emf. The magnitude of the induced emf is given by

$$\varepsilon = \frac{d\Phi_B}{dt} \quad \text{or} \quad \varepsilon = \frac{d}{dt}(BA\cos\theta) \quad (4.26)$$

From the above equation, it is clear that induced emf can be produced by changing magnetic flux in any of the following ways.

(i) By changing the magnetic field $B$
(ii) By changing the area $A$ of the coil and
(iii) By changing the relative orientation $\theta$ of the coil with magnetic field

4.4.2 Induction of emf by changing the magnetic field

From Faraday’s experiments on electromagnetic induction, it was discovered that an emf is induced in a circuit on changing the magnetic flux of the field through it. The change in flux is brought about by (i) relative motion between the circuit and the magnet (First experiment) (ii) variation in magnet (Second experiment).

4.4.3 Induction of emf by changing the area of the coil

Consider a conducting rod of length $l$ moving with a velocity $v$ towards left on a rectangular metallic framework as shown in Figure 4.24. The whole arrangement is placed in a uniform magnetic field $\vec{B}$ whose magnetic lines are perpendicularly directed into the plane of the paper.

As the rod moves from $AB$ to $DC$ in a time $dt$, the area enclosed by the loop and hence the magnetic flux through the loop decreases.

From Figure 4.24, it can be observed that

$$\Phi_B = \frac{1}{2} l^2 B$$

and

$$\frac{d\Phi_B}{dt} = \frac{1}{2} l^2 \frac{dB}{dt}$$

Substituting this in (4.26), we get

$$\varepsilon = \frac{1}{2} l^2 \frac{dB}{dt}$$

for the magnitude of the induced emf.

**Figure 4.24** Induction of emf by changing the area enclosed by the loop
The change in magnetic flux in time $dt$ is

$$d\Phi_B = B \times \text{change in area}$$

$$= B \times \text{Area ABCD}$$

$$= Blv dt$$ since Area ABCD = $l(vdt)$

(or) $$\frac{d\Phi_B}{dt} = Blv$$

As a result of change in flux, an emf is generated in the loop. The magnitude of the induced emf is

$$\varepsilon = \frac{d\Phi_B}{dt}$$

$$\varepsilon = Blv \quad (4.27)$$

This emf is called **motional emf**. The direction of induced current is found to be clockwise from Fleming’s right hand rule.

**EXAMPLE 4.14**

A circular metal of area 0.03 m$^2$ rotates in a uniform magnetic field of 0.4 T. The axis of rotation passes through the centre and perpendicular to its plane and is also parallel to the field. If the disc completes 20 revolutions in one second and the resistance of the disc is 4 Ω, calculate the induced emf between the axis and the rim and induced current flowing in the disc.

**Solution**

$$A = 0.03 \text{ m}^2; B = 0.4 \text{ T}; f = 20 \text{ rps};$$

$$R = 4 \Omega$$

Area covered in 1 sec = Area of the disc $\times$ frequency

$$= 0.03 \times 20$$

$$= 0.6 \text{ m}^2$$

Induced emf, $\varepsilon = \text{Rate of change of flux}$

$$= \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt}$$

$$\varepsilon = \frac{0.4 \times 0.6}{1} = 0.24 \text{ V}$$

Induced current, $i = \frac{\varepsilon}{R} = \frac{0.24}{4} = 0.06 \text{ A}$

**Note** Emf can be induced by changing relative orientation between the coil and the magnetic field. This can be achieved either by rotating a coil in a magnetic field or by rotating a magnetic field within a stationary coil. Here rotating coil type is considered.

**4.4.4 Induction of emf by changing relative orientation of the coil with the magnetic field**

Consider a rectangular coil of $N$ turns kept in a uniform magnetic field $B$ as shown in Figure 4.25(a). The coil rotates in anti-clockwise direction with an angular velocity $\omega$ about an axis, perpendicular to the field.
linked with the coil has its maximum value \( \Phi_m = BA \) (where \( A \) is the area of the coil).

In a time \( t \) seconds, the coil is rotated through an angle \( \theta = \omega t \) in anti-clockwise direction. In this position, the flux linked is \( \Phi_m \cos \omega t \), a component of \( \Phi_m \) normal to the plane of the coil (Figure 4.25(b)). The component parallel to the plane (\( \Phi_m \sin \omega t \)) has no role in electromagnetic induction. Therefore, the flux linkage at this deflected position is

\[
N \Phi_n = N \Phi_m \cos \omega t
\]

According to Faraday’s law, the emf induced at that instant is

\[
E = \varepsilon_m \sin \omega t
\]

At time = 0, the plane of the coil is perpendicular to the field and the flux

\[
\Phi = \Phi_m \cos \omega t = 0
\]

When \( \omega t = \frac{\pi}{2} \),

\[
\Phi = \Phi_m \sin \frac{\pi}{2} = \varepsilon_m
\]

When \( \omega t = \pi \),

\[
\Phi = \Phi_m \sin \pi = 0
\]

When \( \omega t = \frac{3\pi}{2} \),

\[
\Phi = \Phi_m \sin \frac{3\pi}{2} = -\varepsilon_m
\]

When \( \omega t = 2\pi \),

\[
\Phi = \Phi_m \sin 2\pi = 0
\]
to a magnetic field of 0.4 Wb m$^{-2}$. If the coil completes 500 revolutions in a minute, calculate the instantaneous emf when the plane of the coil is (i) perpendicular to the field (ii) parallel to the field and (iii) inclined at 60° with the field.

**Solution**

$$A = 70 \times 10^{-4} \text{m}^2; \quad N = 600 \text{ turns}$$
$$B = 0.4 \text{ Wb m}^{-2}; \quad f = 500 \text{ rpm}$$

The instantaneous emf is

$$\varepsilon = \varepsilon_m \sin \omega t$$

since $\varepsilon_m = N \Phi_m \omega = N(BA)/(2\pi f)$

$$\varepsilon = NBA \times 2\pi f \times \sin \omega t$$

(i) When $\omega t = 0°$, 
$$\varepsilon = \varepsilon_m \sin 0 = 0$$

(ii) When $\omega t = 90°$, 
$$\varepsilon = \varepsilon_m \sin 90° = NBA \times 2\pi f \times 1$$
$$= 600 \times 0.4 \times 70 \times 10^{-4} \times 2 \times \frac{22}{7} \times \left( \frac{500}{60} \right)$$
$$= 88 \text{ V}$$

(iii) When $\omega t = 90° - 60° = 30°$, 
$$\varepsilon = \varepsilon_m \sin 30° = 88 \times \frac{1}{2} = 44 \text{ V}$$

**EXAMPLE 4.15**

A rectangular coil of area 70 cm$^2$ having 600 turns rotates about an axis perpendicular to a magnetic field of 0.4 Wb m$^{-2}$. If the coil completes 500 revolutions in a minute, calculate the instantaneous emf when the plane of the coil is (i) perpendicular to the field (ii) parallel to the field and (iii) inclined at 60° with the field.
4.5

AC GENERATOR

4.5.1 Introduction

AC generator or alternator is an energy conversion device. It converts mechanical energy used to rotate the coil or field magnet into electrical energy. Alternator produces a large scale electrical power for use in homes and industries. AC generator and its components are shown in Figure 4.27.

4.5.2 Principle

Alternators work on the principle of electromagnetic induction. The relative motion between a conductor and a magnetic field changes the magnetic flux linked with the conductor which in turn, induces an emf. The magnitude of the induced emf is given by Faraday’s law of electromagnetic induction and its direction by Fleming’s right hand rule.

Note

Alternating emf is generated by rotating a coil in a magnetic field or by rotating a magnetic field within a stationary coil. The first method is used for small AC generators while the second method is employed for large AC generators. The rotating-field method is the one which is mostly used in power stations.

4.5.3 Construction

Alternator consists of two major parts, namely stator and rotor. As their names suggest, stator is stationary while rotor rotates inside the stator. In any standard construction of commercial alternators, the armature winding is mounted on stator and the field magnet on rotor.

The construction details of stator, rotor and various other components involved in them are given below.

i) Stator

Figure 4.28 Stator and its components
The stationary part which has armature windings mounted in it is called stator. It has three components, namely stator frame, stator core and armature winding.

**Stator frame**

This is the outer frame used for holding stator core and armature windings in proper position. Stator frame provides best ventilation with the help of holes provided in the frame itself (Figure 4.28).

**Stator core**

Stator core or armature core is made up of iron or steel alloy. It is a hollow cylinder and is laminated to minimize eddy current loss. The slots are cut on inner surface of the core to accommodate armature windings.

**Armature winding**

Armature winding is the coil, wound on slots provided in the armature core. One or more than one coil may be employed, depending on the type of alternator.

Two types of windings are commonly used. They are i) single-layer winding and ii) double-layer winding. In single-layer winding, a slot is occupied by a coil as a single layer. But in double-layer winding, the coils are split into two layers such as top and bottom layers (Figure 4.29).

**ii) Rotor**

Rotor contains magnetic field windings. The magnetic poles are magnetized by DC source. The ends of field windings are connected to a pair of slip rings, attached to a common shaft about which rotor rotates. Slip rings rotate along with rotor. To maintain connection between the DC source and field windings, two brushes are used which continuously slide over the slip rings.

There are 2 types of rotors used in alternators i) salient pole rotor and ii) cylindrical pole rotor.

**Salient pole rotor**

![Figure 4.30 Salient 2-pole rotor](MN5C7X)
Construction of AC generator (Not for examination)

Alternator consists of two major parts, namely stator and rotor. (This box is given for better understanding of constructional details)

i) Stator
Stator has three components, namely stator frame, stator core and armature winding.

Figure(a): Stator core with empty slots

Figure(b): Stator core with rectangular loop

Figure(c): Stator core with armature windings

ii) Rotor
Rotor contains magnetic field windings, slip rings and brushes mounted on the same shaft.

Figure(d): Salient 6-pole rotor

Figure(e): Salient 6-pole rotor with field windings, slip rings and brushes

Figure(f): Stator core and rotor
The word salient means projecting. This rotor has a number of projecting poles having their bases riveted to the rotor. It is mainly used in low-speed alternators. The salient 2-pole rotor is shown in the Figure 4.30.

Cylindrical pole rotor

This rotor consists of a smooth solid cylinder. The slots are cut on the outer surface of the cylinder along its length. It is suitable for very high speed alternators (Figure 4.31).

The frequency of alternating emf induced is directly proportional to the rotor speed. In order to maintain the frequency constant, the rotor must run at a constant speed.

These are standard construction details of alternators. Based on the type of alternator being constructed, the details like number of poles, pole type, number of coils and type of windings would change from one another.

We will discuss the construction and working of two examples, namely single phase and three phase AC generators in the following sections.

4.5.4 Advantages of stationary armature-rotating field alternator

Alternators are generally high current and high voltage machines. The stationary armature-rotating field construction has many advantages. A few of them include:

1) The current is drawn directly from fixed terminals on the stator without the use of brush contacts.
2) The insulation of stationary armature winding is easier.
3) The number of sliding contacts (slip rings) is reduced. Moreover, the sliding contacts are used for low-voltage DC Source.
4) Armature windings can be constructed more rigidly to prevent deformation due to any mechanical stress.

4.5.5 Single phase AC generator

In a single phase AC generator, the armature conductors are connected in series so as to form a single circuit which generates a single-phase alternating emf and hence it is called single-phase alternator.
The simplified version of a AC generator is discussed here. Consider a stator core consisting of 2 slots in which 2 armature conductors PQ and RS are mounted to form single-turn rectangular loop PQRS as shown in Figure 4.33. Rotor has 2 salient poles with field windings which can be magnetized by means of DC source.

**Working**

The loop PQRS is stationary and is perpendicular to the plane of the paper. When field windings are excited, magnetic field is produced around it. The direction of magnetic field passing through the armature core is shown in Figure 4.33. Let the field magnet be rotated in clockwise direction by the prime mover. The axis of rotation is perpendicular to the plane of the paper.

Assume that initial position of the field magnet is horizontal. At that instant, the direction of magnetic field is perpendicular to the plane of the loop PQRS. The induced emf is zero (Refer case (iii) of section 4.4). This is represented by origin O in the graph between induced emf and time angle (Figure 4.34).

When field magnet rotates through 90°, magnetic field becomes parallel to PQRS. The induced emfs across PQ and RS would become maximum. Since they are connected in series, emfs are added up and the direction of total induced emf is given by Fleming’s right hand rule.

Care has to be taken while applying this rule; the thumb indicates the direction of the motion of the conductor with respect to field. For clockwise rotating poles, the conductor appears to be rotating anti-clockwise. Hence, thumb should point to the left. The direction of the induced emf is at right angles to the plane of the paper. For PQ, it is downwards and for RS upwards. Therefore, the current flows along PQRS. The point A in the graph represents this maximum emf.

For the rotation of 180° from the initial position, the field is again perpendicular to PQRS and the induced emf becomes zero. This is represented by point B.

The field magnet becomes again parallel to PQRS for 270° rotation of field magnet. The induced emf is maximum but the direction is reversed. Thus the current flows along SRQP. This is represented by point C.

On completion of 360°, the induced emf becomes zero and is represented by the point D. From the graph, it is clear that emf induced in PQRS is alternating in nature.

Therefore, when field magnet completes one rotation, induced emf in PQRS finishes one cycle. For this construction, the frequency of the induced emf depends on the speed at which the field magnet rotates.
4.5.6 Three-phase AC generator

Some AC generators may have more than one coil in the armature core and each coil produces an alternating emf. In these generators, more than one emf is produced. Thus they are called poly-phase generators.

If there are two alternating emfs produced in a generator, it is called two-phase generator. In some AC generators, there are three separate coils, which would give three separate emfs. Hence they are called three-phase AC generators.

In the simplified construction of three-phase AC generator, the armature core has 6 slots, cut on its inner rim. Each slot is 60° away from one another. Six armature conductors are mounted in these slots. The conductors 1 and 4 are joined in series to form coil 1. The conductors 3 and 6 form...
coil 2 while the conductors 5 and 2 form coil 3. So, these coils are rectangular in shape and are 120° apart from one another (Figure 4.35).

The initial position of the field magnet is horizontal and field direction is perpendicular to the plane of the coil 1. As it is seen in single phase AC generator, when field magnet is rotated from that position in clockwise direction, alternating emf $\varepsilon_1$ in coil 1 begins a cycle from origin O. This is shown in Figure 4.36.

The corresponding cycle for alternating emf $\varepsilon_2$ in coil 2 starts at point A after field magnet has rotated through 120°. Therefore, the phase difference between $\varepsilon_1$ and $\varepsilon_2$ is 120°. Similarly, emf $\varepsilon_3$ in coil 3 would begin its cycle at point B after 240° rotation of field magnet from initial position. Thus these emfs produced in the three phase AC generator have 120° phase difference between one another.

### 4.5.7 Advantages of three-phase alternator

Three-phase system has many advantages over single-phase system. Let us see a few of them.

1) For a given dimension of the generator, three-phase machine produces higher power output than a single-phase machine.
2) For the same capacity, three-phase alternator is smaller in size when compared to single phase alternator.
3) Three-phase transmission system is cheaper. A relatively thinner wire is sufficient for transmission of three-phase power.

## 4.6 TRANSFORMER

Transformer is a stationary device used to transform electrical power from one circuit to another without changing its frequency. The applied alternating voltage is either increased or decreased with corresponding decrease or increase of current in the circuit.
If the transformer converts an alternating current with low voltage into an alternating current with high voltage, it is called step-up transformer. On the contrary, if the transformer converts alternating current with high voltage into an alternating current with low voltage, then it is called step-down transformer.

### 4.6.1 Construction and working of transformer

**Principle**

The principle of transformer is the mutual induction between two coils. That is, when an electric current passing through a coil changes with time, an emf is induced in the neighbouring coil.

**Construction**

In the simple construction of transformers, there are two coils of high mutual inductance wound over the same transformer core. The core is generally laminated and is made up of a good magnetic material like silicon steel. Coils are electrically insulated but magnetically linked via transformer core (Figure 4.37).

The coil across which alternating voltage is applied is called primary coil $P$ and the coil from which output power is drawn out is called secondary coil $S$. The assembled core and coils are kept in a container which is filled with suitable medium for better insulation and cooling purpose.

**Working**

If the primary coil is connected to a source of alternating voltage, an alternating magnetic flux is set up in the laminated core. If there is no magnetic flux leakage, then whole of magnetic flux linked with primary coil is also linked with secondary coil. This means that rate at which magnetic flux changes through each turn is same for both primary and secondary coils.

As a result of flux change, emf is induced in both primary and secondary coils. The emf induced in the primary coil $\varepsilon_p$ is almost equal and opposite to the applied voltage $v_p$ and is given by

$$v_p = \varepsilon_p = -N_p \frac{d\Phi_p}{dt}$$  \hspace{1cm} (4.30)

The frequency of alternating magnetic flux in the core is same as the frequency of the applied voltage. Therefore, induced emf in secondary will also have same frequency as that of applied voltage. The emf induced in the secondary coil $\varepsilon_s$ is given by

$$\varepsilon_s = -N_s \frac{d\Phi_s}{dt}$$
4.6.2 Energy losses in a transformer

Transformers do not have any moving parts so that its efficiency is much higher than that of rotating machines like generators and motors. But there are many factors which lead to energy loss in a transformer.

i) Core loss or Iron loss

This loss takes place in transformer core. Hysteresis loss (Refer section 3.6) and eddy current loss are known as core loss or Iron loss. When transformer core is magnetized and demagnetized repeatedly by the alternating voltage applied across primary coil, hysteresis takes place due to which some energy is lost in the form of heat. Hysteresis loss is minimized by using steel of high silicon content in making transformer core.

Alternating magnetic flux in the core induces eddy currents in it. Therefore there is energy loss due to the flow of eddy current, called eddy current loss which is minimized by using very thin laminations of transformer core.

ii) Copper loss

Transformer windings have electrical resistance. When an electric current flows through them, some amount of energy is dissipated due to Joule heating. This energy loss is called copper loss which is minimized by using wires of larger diameter.

iii) Flux leakage

Flux leakage happens when the magnetic lines of primary coil are not completely linked with secondary coil. Energy loss due to this flux leakage is minimized by winding coils one over the other.
4.6.3 Advantages of AC in long distance power transmission

Electric power is produced in a large scale at electric power stations with the help of AC generators. These power stations are classified based on the type of fuel used as thermal, hydro electric and nuclear power stations. Most of these stations are located at remote places. Hence the electric power generated is transmitted over long distances through transmission lines to reach towns or cities where it is actually consumed. This process is called power transmission.

But there is a difficulty during power transmission. A sizable fraction of electric power is lost due to Joule heating \((i^2R)\) in the transmission lines which are hundreds of kilometer long. This power loss can be tackled either by reducing current \(i\) or by reducing resistance \(R\) of the transmission lines. The resistance \(R\) can be reduced with thick wires of copper or aluminium. But this increases the cost of production of transmission lines and other related expenses. So this way of reducing power loss is not economically viable.

Since power produced is alternating in nature, there is a way out. The most important property of alternating voltage that it can be stepped up and stepped down by using transformers could be exploited in reducing current and thereby reducing power losses to a greater extent.

At the transmitting point, the voltage is increased and the corresponding current is decreased by using step-up transformer (Figure 4.38). Then it is transmitted through transmission lines. This reduced current at high voltage reaches the destination without any appreciable loss. At the receiving point, the voltage is decreased and the current is increased to appropriate values by using step-down transformer and then it is given to consumers. Thus power transmission is done efficiently and economically.

**Illustration:**

An electric power of 2 MW is transmitted to a place through transmission lines of total resistance, say \(R = 40 \Omega\), at two different voltages. One is lower voltage (10 kV) and the other is higher (100 kV). Let us now calculate and compare power losses in these two cases.

**Case (i):**

\[ P = 2 \text{ MW}; \quad R = 40 \Omega; \quad V = 10 \text{ kV} \]

\[
\text{Power, } P = VI
\]

\[
\therefore \text{Current, } I = \frac{P}{V} = \frac{2 \times 10^6}{10 \times 10^3} = 200 \text{A}
\]

\[
\text{Power loss = Heat produced}
\]

\[
= I^2R = (200)^2 \times 40 = 1.6 \times 10^6 \text{W}
\]

\[
\% \text{ of power loss} = \frac{1.6 \times 10^6}{2 \times 10^6} \times 100\% = 80\%
\]
Power system at a glance (Not for Examination)

The generating stations present in a region are interconnected to form a common electrical network and are operated in parallel. This is to ensure uninterrupted power supply to a large number of consumers in the case of failure of any power station or a sudden increase of load beyond the capacity of the generating station.

The various elements such as generating stations, transmission lines, the substations and distributors etc are all tied together for continuous generation and consumption of electric energy. This is called power system. A part of power system consisting of the sub-stations and transmission lines is known as a grid.

In a power system, the transfer of electric power produced to the consumer is carried out in two stages which are further sub-divided into two as given below.

1) Transmission stage
   a) Primary transmission stage
   b) Secondary transmission stage

2) Distribution stage
   a) Primary distribution stage
   b) Secondary transmission stage

   and then it is supplied to individual consumers. These two stages of power transmission is presented in a single-line diagram shown in figure. The central system usually generates power at 11 kV which is stepped up to 132 kV and is transmitted through transmission lines. This is known as primary or high-voltage transmission.

   This high-voltage power reaches receiving station at the outskirts of the city where it is stepped down to 33 kV and is transmitted as secondary or low-voltage transmission to sub-stations situated within the city limits.

   In the primary distribution system, the voltage is reduced from 33 kV to 3.3kV at sub-stations and is given to distribution sub-stations. The voltage is finally brought down to 440V or 230V at distribution sub-station from where secondary distribution is done to factories (440V) and homes (230V) via distribution networks.
EXAMPLE 4.17

An inverter is common electrical device which we use in our homes. When there is no power in our house, inverter gives AC power to run a few electronic appliances like fan or light. An inverter has inbuilt step-up transformer which converts 12 V AC to 240 V AC. The primary coil has 100 turns and the inverter delivers 50 mA to the external circuit. Find the number of turns in the secondary and the primary current.

Solution

\[ V_p = 12 \text{ V}; \quad V_s = 240 \text{ V} \]
\[ I_s = 50 \text{ mA}; \quad N_p = 100 \text{ turns} \]

The transformation ratio, \( K = \frac{V_s}{V_p} = \frac{240}{12} = 20 \)

The number of turns in the secondary
\[ N_s = N_p \times K = 100 \times 20 = 2000 \]

Primary current,
\[ I_p = K \times I_s = 20 \times 50 \text{ mA} = 1 \text{ A} \]

4.7

ALTERNATING CURRENT

4.7.1 Introduction

In section 4.5, we have seen that when the orientation of the coil with the magnetic field is changed, an alternating emf is induced and hence an alternating current flows in the closed circuit. An alternating voltage is the voltage which changes polarity at regular intervals of time and the direction of the resulting alternating current also changes accordingly.
In the Figure 4.39(a), an alternating voltage source is connected to a resistor $R$ in which the upper terminal of the source is positive and lower terminal negative at an instant. Therefore, the current flows in clockwise direction. After a short time, the polarities of the source are reversed so that current now flows in anti-clockwise direction (Figure 4.39(b)). This current which flows in alternate directions in the circuit is called alternating current.

**Sinusoidal alternating voltage**

If the waveform of alternating voltage is a sine wave, then it is known as sinusoidal alternating voltage which is given by the relation.

$$\upsilon = V_m \sin \omega t$$  \hspace{1cm} (4.35)

where $\upsilon$ is the instantaneous value of alternating voltage; $V_m$ is the maximum value (amplitude) and $\omega$ is the angular frequency of the alternating voltage. When sinusoidal alternating voltage is applied to a closed circuit, the resulting alternating current is also sinusoidal in nature and its relation is

$$i = I_m \sin \omega t$$  \hspace{1cm} (4.36)

where $I_m$ is the maximum value (amplitude) of the alternating current. The direction of sinusoidal voltage or current is reversed after every half-cycle and its magnitude is also changing continuously as shown in Figure 4.40.

**Figure 4.40** (a) Sinusoidal alternating voltage (b) Sinusoidal alternating current

Interestingly, sine waves are very common in nature. The periodic motions like waves in water, swinging of pendulum are associated with sine waves. Thus sine wave seems to be nature’s standard. Also refer unit 11 of XI physics text book.

### 4.7.1 Mean or Average value of AC

The current and voltage in a DC system remain constant over a period of time so that there is no problem in specifying their magnitudes. However, an alternating current or voltage varies from time to time. Then a question arises how to express the magnitude of an alternating current or voltage. Though there are many ways of expressing it, we limit our discussion with two ways, namely mean value and RMS (Root Mean Square) value of AC.
Mean or Average value of AC

We have learnt that the magnitude of an alternating current in a circuit changes from one instant to other instant and its direction also reverses for every half cycle. During positive half cycle, current is taken as positive and during negative cycle it is negative. Therefore mean or average value of symmetrical alternating current over one complete cycle is zero.

Therefore the average or mean value is measured over one half of a cycle. These electrical terms, average current and average voltage can be used in both AC and DC circuit analysis and calculations.

The average value of alternating current is defined as the average of all values of current over a positive half-cycle or negative half-cycle.

The instantaneous value of sinusoidal alternating current is given by the equation $i = I_m \sin \theta$ or $i = I_m \sin \theta$ (where $\theta = \omega t$) whose graphical representation is given in Figure 4.41.

The sum of all currents over a half-cycle is given by area of positive half-cycle (or negative half-cycle). Therefore,

$$I_{av} = \frac{\text{Area of positive half-cycle}}{\text{Base length of half-cycle}}$$

(4.37)

Consider an elementary strip of thickness $d\theta$ in the positive half-cycle of the current wave (Figure 4.41). Let $i$ be the mid-ordinate of that strip.

Area of the elementary strip = $i \, d\theta$

Area of positive half-cycle

$$= \int_0^\pi i \, d\theta = \int_0^\pi I_m \sin \theta \, d\theta$$

$$= I_m [\cos \theta]_0^\pi = -I_m [\cos \pi - \cos 0] = 2I_m$$

Substituting this in equation (4.37), we get (The base length of half-cycle is $\pi$)

Average value of AC, $I_{av} = \frac{2I_m}{\pi}$

Hence the average value of AC is 0.637 times the maximum value $I_m$ of the alternating current. For negative half-cycle, $I_{av} = -0.637 I_m$.

**Note**

For example, if we consider $n$ currents in a half-cycle of AC, namely $i_1, i_2, ..., i_n$, then average value is given by

$$I_{av} = \frac{\text{Sum of all currents over half-cycle}}{\text{Number of currents}}$$

$$I_{av} = \frac{i_1 + i_2 + .... + i_n}{n}$$

4.7.2 RMS value of AC

The term RMS refers to time-varying sinusoidal currents and voltages and not used in DC systems.

The root mean square value of an alternating current is defined as the square root of the mean of the squares of all currents over one cycle. It is denoted by
For alternating voltages, the RMS value is given by \( V_{\text{RMS}} \).

The alternating current \( i = I_m \sin \omega t \) or \( i = I_m \sin \theta \), is represented graphically in Figure 4.42. The corresponding squared current wave is also shown by the dotted lines.

The sum of the squares of all currents over one cycle is given by the area of one cycle of squared wave. Therefore,

\[
I_{\text{RMS}} = \sqrt{\frac{\text{Area of one cycle of squared wave}}{\text{Base length of one cycle}}} \quad (4.39)
\]

An elementary area of thickness \( d\theta \) is considered in the first half-cycle of the squared current wave as shown in Figure 4.42. Let \( i^2 \) be the mid-ordinate of the element.

Area of the element = \( i^2 d\theta \)

Area of one cycle of squared wave = \( \int_0^{2\pi} i^2 d\theta \)

\[
I_{\text{RMS}} = \sqrt{\frac{\int_0^{2\pi} i^2 d\theta}{\pi}} \quad (4.40)
\]

Substituting this in equation (4.39), we get

\[
I_{\text{RMS}} = \sqrt{\frac{I_m^2 \pi}{2\pi}} = \frac{I_m}{\sqrt{2}} \quad [\text{Base length of one cycle is } 2\pi]
\]

Thus we find that for a symmetrical sinusoidal current RMS value of current is 70.7 % of its peak value.

Similarly for alternating voltage, it can be shown that

\[
V_{\text{RMS}} = 0.707 V_m \quad (4.42)
\]

Note: RMS value of alternating current is also called effective value and is represented as \( I_{\text{eff}} \). It is used to compare RMS current of AC to an equivalent steady current. RMS value is also defined as that value of the steady current which when flowing through a given circuit for a given time produces the same amount of heat as produced by the alternating current when flowing through the same circuit for the same time. The effective value of an alternating voltage is represented by \( V_{\text{eff}} \).
EXAMPLE 4.19

The equation for an alternating current is given by \( i = 77 \sin 314t \). Find the peak value, frequency, time period and instantaneous value at \( t = 2 \text{ ms} \).

**Solution**

\( i = 77 \sin 314t \); \( t = 2 \text{ ms} = 2 \times 10^{-3} \text{ s} \)

(i) Peak value, \( I_m = 77 \text{ A} \)

(ii) Frequency, \( f = \frac{\omega}{2\pi} = \frac{314}{2 \times 3.14} = 50 \text{ Hz} \)

(iii) Time period, \( T = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ s} \)

(iv) At \( t = 2 \text{ ms} \),

Instantaneous value,

\[ i = 77 \sin(314 \times 2 \times 10^{-3}) \]

\[ i = 45.24 \text{ A} \]

**Phasor and phasor diagram**

**Phasor**

A sinusoidal alternating voltage (or current) can be represented by a vector which rotates about the origin in anti-clockwise...
direction at a constant angular velocity $\omega$. Such a rotating vector is called a phasor. A phasor is drawn in such a way that
- the length of the line segment equals the peak value $V_m$ (or $I_m$) of the alternating voltage (or current)
- its angular velocity $\omega$ is equal to the angular frequency of the alternating voltage (or current)
- the projection of phasor on any vertical axis gives the instantaneous value of the alternating voltage (or current)
- the angle between the phasor and the axis of reference (positive x-axis) indicates the phase of the alternating voltage (or current).

The notion of phasors is introduced to analyse phase relationship between voltage and current in different AC circuits.

**Phasor diagram**

The diagram which shows various phasors and their phase relations is called phasor diagram. Consider a sinusoidal alternating voltage $v = V_m \sin \omega t$ applied to a circuit. This voltage can be represented by a phasor, namely $\overrightarrow{OA}$ as shown in Figure 4.43.

![Figure 4.43 Phasor diagram for an alternating voltage $v = V_m \sin \omega t$](image)

Here the length of $\overrightarrow{OA}$ equals the peak value ($V_m$), the angle it makes with x-axis gives the phase ($\omega t$) of the applied voltage. Its projection on y-axis provides the instantaneous value ($V_m \sin \omega t$) at that instant.

When $\overrightarrow{OA}$ rotates about $O$ with angular velocity $\omega$ in anti-clockwise direction, the

![Figure 4.44 Phasor diagram and wave diagram say that $i$ leads $v$ by $\phi$](image)
waveform of the voltage is generated. For one full rotation of $\overrightarrow{OA}$, one cycle of voltage is produced.

The alternating current in the same circuit may be given by the relation $i = I_m \sin (\omega t + \phi)$ which is represented by another phasor $\overrightarrow{OB}$. Here $\phi$ is the phase angle between voltage and current. In this case, the current leads the voltage by phase angle $\phi$ which is shown in Figure 4.44. If the current lags behind the voltage, then we write $i = I_m \sin (\omega t - \phi)$.

Consider a circuit containing a pure resistor of resistance $R$ connected across an alternating voltage source (Figure 4.45). The instantaneous value of the alternating voltage is given by

$$\upsilon = V_m \sin \omega t$$ (4.43)

An alternating current $i$ flowing in the circuit due to this voltage, develops a potential drop across $R$ and is given by

$$V_R = iR$$ (4.44)

Kirchhoff’s loop rule (Refer section 2.4) states that the algebraic sum of potential differences in a closed circuit is zero. For this resistive circuit,

$$\upsilon - V_R = 0$$

From equation (4.43) and (4.44),

$$V_m \sin \omega t = iR$$
$$i = \frac{V_m}{R} \sin \omega t$$
$$i = I_m \sin \omega t$$ (4.45)
where $\frac{V_m}{R} = I_m$, the peak value of alternating current in the circuit. From equations (4.43) and (4.45), it is clear that the applied voltage and the current are in phase with each other in a resistive circuit. It means that they reach their maxima and minima simultaneously. This is indicated in the phasor diagram (Figure 4.46). The wave diagram also depicts that current is in phase with the applied voltage (Figure 4.46).

### 4.7.4 AC circuit containing only an inductor

Consider a circuit containing a pure inductor of inductance $L$ connected across an alternating voltage source (Figure 4.47). The alternating voltage is given by the equation.

$$v = V_m \sin \omega t$$  \hspace{1cm} (4.46)

The alternating current flowing through the inductor induces a self-induced emf or back emf in the circuit. The back emf is given by

$$\text{Back emf}, \varepsilon = -L \frac{di}{dt}$$

By applying Kirchoff’s loop rule to the purely inductive circuit, we get

$$i = \frac{V_m}{L} \left(\sin \omega t \right) + \text{constant}$$

or $$i = I_m \sin \left(\omega t - \frac{\pi}{2}\right)$$  \hspace{1cm} (4.47)

where $\frac{V_m}{\omega L} = I_m$, the peak value of the alternating current in the circuit. From equation (4.46) and (4.47), it is evident that current lags behind the applied voltage by $\frac{\pi}{2}$ in an inductive circuit. This fact is depicted in the phasor diagram. In the wave diagram also, it is seen that current lags the voltage by 90° (Figure 4.48).

**Inductive reactance $X_L$**

The peak value of current $I_m$ is given by

$$I_m = \frac{V_m}{\omega L}$$

Let us compare this equation with

$$I_m = \frac{V_m}{R}$$ from resistive circuit. The quantity $\omega L$ plays the same role as the resistance in resistive circuit. This is the resistance offered by the inductor, called inductive reactance ($X_L$). It is measured in ohm.

$$X_L = \omega L$$
An inductor blocks AC but it allows DC. Why? and How?

An inductor $L$ is a closely wound helical coil. The steady DC current flowing through $L$ produces uniform magnetic field around it and the magnetic flux linked remains constant. Therefore there is no self-induction and self-induced emf (back emf). Since inductor behaves like a resistor, DC flows through an inductor.

The AC flowing through $L$ produces time-varying magnetic field which in turn induces self-induced emf (back emf). This back emf, according to Lenz’s law, opposes any change in the current. Since AC varies both in magnitude and direction, its flow is opposed in $L$. For an ideal inductor of zero ohmic resistance, the back emf is equal and opposite to the applied emf. Therefore $L$ blocks AC.

The inductive reactance ($X_L$) varies directly as the frequency.

\[ X_L = 2\pi f L \quad (4.48) \]

where $f$ is the frequency of the alternating current. For a steady current, $f = 0$. Therefore, $X_L = 0$. Thus an ideal inductor offers no resistance to steady DC current.

### 4.7.5 AC circuit containing only a capacitor

Consider a circuit containing a capacitor of capacitance $C$ connected across an alternating voltage source (Figure 4.49). The alternating voltage is given by

\[ V = V_m \sin \omega t \quad (4.49) \]

Figure 4.49 AC circuit with capacitance

\[ V = V_m \sin \omega t \]
Let \( q \) be the instantaneous charge on the capacitor. The emf across the capacitor at that instant is \( \frac{q}{C} \). According to Kirchoff’s loop rule,

\[
\nu - \frac{q}{C} = 0
\]

\( q = CV_m \sin \omega t \)

By the definition of current,

\[
i = \frac{dq}{dt} = \frac{d}{dt}(CV_m \sin \omega t)
\]

\( = CV_m \frac{d}{dt}(\sin \omega t) \)

\( = CV_m \omega \cos \omega t \)

or \( i = \frac{V_m}{C\omega} \sin(\omega t + \frac{\pi}{2}) \)

Instantaneous value of current,

\[
i = I_m \sin(\omega t + \frac{\pi}{2}) \tag{4.50}
\]

where \( \frac{V_m}{C\omega} = I_m \), the peak value of the alternating current. From equations (4.49) and (4.50), it is clear that current leads the applied voltage by \( \frac{\pi}{2} \) in a capacitive circuit. This is shown pictorially in Figure 4.50. The wave diagram for a capacitive circuit also shows that the current leads the applied voltage by 90°.

**Capacitive reactance \( X_c \)**

The peak value of current \( I_m \) is given by \( I_m = \frac{V_m}{C\omega} \). Let us compare this equation with \( I_m = \frac{V_m}{R} \) from resistive circuit. The quantity \( \frac{1}{C\omega} \) plays the same role as the resistance \( R \) in resistive circuit. This is the resistance offered by the capacitor, called capacitive reactance \( (X_c) \). It measured in ohm.

\[
X_c = \frac{1}{\omega C} \tag{4.51}
\]

The capacitive reactance \( (X_c) \) varies inversely as the frequency. For a steady current, \( f = 0 \).

\[
\therefore X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{0} = \infty
\]

Thus a capacitive circuit offers infinite resistance to the steady current. So that steady current cannot flow through the capacitor.

**What is ELI?**

ELI is an acronym which means that EMF (voltage) leads the current in an inductive circuit.

**What is ICE?**

ICE is an acronym which means that the current leads the EMF (voltage) current in a capacitive circuit.

It is easier to may remember the results of AC circuits with the mnemonic ‘ELI the ICE man’.
EXAMPLE 4.20

A 400 mH coil of negligible resistance is connected to an AC circuit in which an effective current of 6 mA is flowing. Find out the voltage across the coil if the frequency is 1000 Hz.

Solution

\[ L = 400 \times 10^{-3} \text{ H}; \quad I_{\text{eff}} = 6 \times 10^{-3} \text{ A} \]

\[ f = 1000 \text{ Hz} \]

Inductive reactance, \( X_L = L\omega = L \times 2\pi f \)

\[ = 2 \times 3.14 \times 1000 \times 0.4 \]

\[ = 2512 \Omega \]

Voltage across \( L \),

\[ V = I \times X_L = 6 \times 10^{-3} \times 2512 \]

\[ V = 15.072 \text{ V (RMS)} \]

EXAMPLE 4.21

A capacitor of capacitance \( \frac{10^5}{\pi} \mu F \) is connected across a 220 V, 50 Hz A.C. mains. Calculate the capacitive reactance, RMS value of current and write down the equations of voltage and current.

Solution

\[ C = \frac{10^5}{\pi} \times 10^{-6} \text{ F}, \quad V_{\text{RMS}} = 220 \text{ V}; \quad f = 50 \text{ Hz} \]

(i) Capacitive reactance,

\[ X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \]

\[ = \frac{1}{2 \times \pi \times 50 \times \frac{10^{-4}}{\pi}} = 100 \Omega \]

(ii) RMS value of current,

\[ I_{\text{RMS}} = \frac{V_{\text{RMS}}}{X_C} = \frac{220}{100} = 2.2 \text{ A} \]

(iii) \( V_m = 220 \times \sqrt{2} = 311 \text{ V} \)
A capacitor blocks DC but it allows AC. Why? and How? (Not for examination)

Capacitors have two parallel metallic plates placed close to each other and there is a gap between plates. Whenever a source of voltage (either DC voltage or AC voltage) is connected across a capacitor C, the electrons from the source will reach the plate and stop. They cannot jump across the gap between plates to continue its flow in the circuit. Therefore the electrons flowing in one direction (i.e. DC) cannot pass through the capacitor. But the electrons from AC source seem to flow through C. Let us see what really happens!

**DC cannot flow through a capacitor:**

Consider a parallel plate capacitor whose plates are uncharged (same amount of positive and negative charges). A DC source (battery) is connected across C as shown in Figure (a).

As soon as battery is connected, electrons start to flow from the negative terminal and are accumulated at the right plate, making it negative. Due to this negative potential, the electrons present in the nearby left plate are repelled and are moved towards positive terminal of the battery. When electrons leave the left plate, it becomes positively charged. This process is known as charging. The direction of flow of electrons is shown by arrows.

The charging of the plates continues till the level of the battery. Once C is fully charged and current will stop. At this time, we say that capacitor is blocking DC Figure (c).

**AC flows (?!?) through a capacitor:**

Now an AC source is connected across C. At an instant, the right side of the source is at negative potential, then the electrons flow from negative terminal to the right plate and from left plate to the positive terminal as shown in Figure (d) but no electron crosses the gap between the plates. These electron-flows are represented by arrows. Thus, the charging of the plates takes place and the plates become fully charged (Figure (e)).

After a short time, the polarities of AC source are reversed and the right side of the source is now positive. The electrons which were accumulated in the right plate start to flow to the positive terminal and the electrons from negative terminal flow to the left plate to neutralize the positive charges stored in it. As a result, the net charges present in the plates begin to decrease and this is called discharging. These electron-flows are represented by arrows as shown in Figure (f). Once the charges are exhausted, C will be charged again but with reversed polarities as shown in Figure (g).

Thus the electrons flow in one direction while charging the capacitor and its direction is reversed while discharging (the conventional current is also opposite in both cases). Though electrons flow in the circuit, no electron crosses the gap between the plates. In this way, AC flows through a capacitor.
The length of these phasors are

\[
OI = I_m, OA = I_m R, OB = I_m X_L; OC = I_m X_C
\]

The circuit is either effectively inductive or capacitive or resistive that depends on the value of \(V_L\) or \(V_C\). Let us assume that \(V_L > V_C\) so that net voltage drop across \(L-C\) combination is \(V_L - V_C\) which is represented by a phasor \(AD\).

By parallelogram law, the diagonal \(OE\) gives the resultant voltage \(V_R\) and \(VL - VC\) and its length \(OE\) is equal to \(V_m\).

Therefore,

\[
V = V_m \sin \omega t
\]

\[
I = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}}
\]

or

\[
I = \frac{V_m}{Z}
\]

where \(Z = \sqrt{R^2 + (X_L - X_C)^2}\)

\[ (4.53) \]

\[ (4.54) \]

\(Z\) is called impedance of the circuit which refers to the effective opposition to the circuit current by the series \(RLC\) circuit. The voltage triangle and impedance triangle are given in the Figure 4.53.

\[ (4.52) \]

\[ (4.53) \]

\[ (4.54) \]

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\[ (4.52) \]

\[ (4.53) \]

\[ (4.54) \]
If $XL = XC$, $\phi$ is zero. Therefore current and voltage are in the same phase and the circuit is resistive.

\[
\therefore \: \upsilon = V_m \sin \omega t; \: \: i = I_m \sin \omega t
\]

### 4.7.7 Resonance in series RLC Circuit

When the frequency of the applied alternating source ($\omega$) is equal to the natural frequency $\left(\dfrac{1}{\sqrt{LC}}\right)$ of the RLC circuit, the current in the circuit reaches its maximum value. Then the circuit is said to be in electrical resonance. The frequency at which resonance takes place is called resonant frequency.

Resonant angular frequency, $\omega_r = \dfrac{1}{\sqrt{LC}}$

or $f_r = \dfrac{1}{2\pi\sqrt{LC}}$ (4.56)

At series resonance,

$\omega_r = \dfrac{1}{\sqrt{LC}}$ or $\omega_r^2 = \dfrac{1}{LC}$

$\omega L = \dfrac{1}{\omega_r C}$ or

$X_L = X_C$ (4.57)

### Table 4.1 Summary of results of AC circuits

<table>
<thead>
<tr>
<th>Type of Impedance</th>
<th>Value of Impedance</th>
<th>Phase angle of current with voltage</th>
<th>Power factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance</td>
<td>$R$</td>
<td>$0^\circ$</td>
<td>1</td>
</tr>
<tr>
<td>Inductance</td>
<td>$X_L = \omega L$</td>
<td>$90^\circ$ lag</td>
<td>0</td>
</tr>
<tr>
<td>Capacitance</td>
<td>$X_C = \dfrac{1}{\omega C}$</td>
<td>$90^\circ$ lead</td>
<td>0</td>
</tr>
<tr>
<td>R- L - C</td>
<td>$\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$</td>
<td>Between $0^\circ$ and $90^\circ$ lag or lead</td>
<td>Between $0^\circ$ and $1$ or lead</td>
</tr>
</tbody>
</table>
Since $X_L$ and $X_C$ are frequency dependent, the resonance condition ($X_L = X_C$) can be achieved by varying the frequency of the applied voltage.

**Effects of series resonance**

When series resonance occurs, the impedance of the circuit is minimum and is equal to the resistance of the circuit. As a result of this, the current in the circuit becomes maximum. This is shown in the resonance curve drawn between current and frequency (Figure 4.54).

At resonance, the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R \text{ since } X_L = X_C$$

Therefore, the current in the circuit is

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$I_m = \frac{V_m}{R}$$

(4.58)

**Note**

The phenomenon of electrical resonance is possible when the circuit contains both $L$ and $C$. Only then the voltage across $L$ and $C$ cancel one another when $V_L$ and $V_C$ are $180^\circ$ out of phase and the circuit becomes purely resistive. This implies that resonance will not occur in a $RL$ and $RC$ circuits.

### Quality factor or $Q$–factor

The current in the series $RLC$ circuit becomes maximum at resonance. Due to the increase in current, the voltage across $L$ and $C$ are also increased. This magnification of voltages at series resonance is termed as $Q$–factor.
It is defined as the ratio of voltage across \( L \) or \( C \) to the applied voltage.

\[
\text{Q-factor} = \frac{\text{Voltage across } L \text{ or } C}{\text{Applied voltage}}
\]

At resonance, the circuit is purely resistive. Therefore, the applied voltage is equal to the voltage across \( R \).

\[
\text{Q-factor} = \frac{I_m X_L}{I_m R} = \frac{X_L}{R}
\]

The physical meaning is that Q-factor indicates the number of times the voltage across \( L \) or \( C \) is greater than the applied voltage at resonance.

**EXAMPLE 4.22**

Find the impedance of a series RLC circuit if the inductive reactance, capacitive reactance and resistance are 184 \( \Omega \), 144 \( \Omega \) and 30 \( \Omega \) respectively. Also calculate the phase angle between voltage and current.

**Solution**

\( X_L = 184 \Omega \), \( X_C = 144 \Omega \)

\( R = 30 \Omega \)

(i) The impedance is

\[
Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{30^2 + (184 - 144)^2} = 50 \Omega
\]

(ii) Phase angle is

\[
\tan \phi = \frac{X_L - X_C}{R} = \frac{184 - 144}{30} = 1.33
\]

\[
\phi = 53.1^\circ
\]

Since the phase angle is positive, voltage leads current by 53.1\(^\circ\) for this inductive circuit.

**EXAMPLE 4.23**

A 500 \( \mu \)H inductor, \( \frac{80}{\pi^2} \) pF capacitor and a 628 \( \Omega \) resistor are connected to form a series RLC circuit. Calculate the resonant frequency and Q-factor of this circuit at resonance.

**Solution**

\( L = 500 \times 10^{-6} \) H; \( C = \frac{80}{\pi^2} \times 10^{-12} \) F; \( R = 628 \Omega \)

(i) Resonant frequency is

\[
f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{500 \times 10^{-6} \times \frac{80}{\pi^2} \times 10^{-12}}} = \frac{1}{2 \sqrt{40,000 \times 10^{-18}}} = \frac{10,000 \times 10^3}{4} = 2500 \text{ KHz}
\]

(ii) Q-factor

\[
Q = \frac{\omega L}{R} = \frac{2 \times 3.14 \times 2500 \times 10^3 \times 500 \times 10^{-6}}{628} = 12.5
\]
In an inductive circuit, the voltage leads the current by $90^\circ$. Therefore,

\[ v = V_m \sin(\omega t + 90^\circ) \]
\[ v = 2.4 \sin(200t - 40^\circ + 90^\circ) \]
\[ v = 2.4 \sin(200t + 50^\circ) \text{ volt} \]

4.8

POWER IN AC CIRCUITS

4.8.1 Introduction of power in AC circuits

Power of a circuit is defined as the rate of consumption of electric energy in that circuit. It is given by the product of the voltage and current. In an AC circuit, the voltage and current vary continuously with time. Let us first calculate the power at an instant and then it is averaged over a complete cycle.

The alternating voltage and alternating current in the series RLC circuit at an instant are given by

\[ v = V_m \sin \omega t \quad \text{and} \quad i = I_m \sin(\omega t + \phi) \]

where $\phi$ is the phase angle between $v$ and $i$. The instantaneous power is then written as

\[ P = v i \]
\[ = V_m I_m \sin \omega t \sin(\omega t + \phi) \]
\[ = V_m I_m \sin \omega t \left[ \sin \omega t \cos \phi - \cos \omega t \sin \phi \right] \]
\[ P = V_m I_m \left[ \cos \phi \sin^2 \omega t - \sin \omega t \cos \omega t \sin \phi \right] \quad (4.61) \]
Here the average of \( \sin^2 \omega t \) over a cycle is \( \frac{1}{2} \) and that of \( \sin \omega t \cos \omega t \) is zero. Substituting these values, we obtain average power over a cycle.

\[
P_{av} = \frac{V_{av}}{\sqrt{2}} \cdot \frac{I_{av}}{\sqrt{2}} \cos \phi = \frac{V_{RMS}}{\sqrt{2}} \cdot \frac{I_{RMS}}{\sqrt{2}} \cos \phi
\]

\[
P_{av} = V_{RMS} I_{RMS} \cos \phi
\]

where \( V_{RMS} I_{RMS} \) is called apparent power and \( \cos \phi \) is power factor. The average power of an AC circuit is also known as the true power of the circuit.

**Special Cases**

(i) For a purely resistive circuit, the phase angle between voltage and current is zero and \( \cos \phi = 1 \).
\[
\therefore P_{av} = V_{RMS} I_{RMS}
\]

(ii) For a purely inductive or capacitive circuit, the phase angle is \( \pm \frac{\pi}{2} \) and \( \cos \left( \pm \frac{\pi}{2} \right) = 0 \).
\[
\therefore P_{av} = 0
\]

(iii) For series RLC circuit, the phase angle \( \phi = \tan^{-1} \left( \frac{X_L - X_C}{R} \right) \).
\[
\therefore P_{av} = V_{RMS} I_{RMS} \cos \phi
\]

(iv) For series RLC circuit at resonance, the phase angle is zero and \( \cos \phi = 1 \).
\[
\therefore P_{av} = V_{RMS} I_{RMS}
\]

**4.8.2 Wattless current**

Consider an AC circuit in which there is a phase angle of \( \phi \) between \( V_{RMS} \) and \( I_{RMS} \) and voltage is assumed to be leading the current by \( \phi \) as shown in the phasor diagram (Figure 4.55).

Now, \( I_{RMS} \) is resolved into two perpendicular components namely, \( I_{RMS} \cos \phi \) along \( V_{RMS} \) and \( I_{RMS} \sin \phi \) perpendicular to \( V_{RMS} \) as shown in Figure 4.56.

Now the component of current \( (I_{RMS} \cos \phi) \) which is in phase with the voltage is called active component. The power consumed by this current \( = V_{RMS} I_{RMS} \cos \phi \). So that it is also known as ‘Wattful’ current.

(ii) The other component \( (I_{RMS} \sin \phi) \) which has a phase angle of \( \frac{\pi}{2} \) with the voltage is called reactive component. The power consumed is zero. So that it is also known as ‘Wattless’ current.
The current in an AC circuit is said to be wattless current if the power consumed by it is zero. This wattless current happens in a purely inductive or capacitive circuit.

### 4.8.3 Power factor

The power factor of a circuit is defined in one of the following ways:

(i) \[ \text{Power factor} = \cos \phi = \text{cosine of the angle of lead or lag} \]

(ii) \[ \text{Power factor} = \frac{R}{Z} = \frac{\text{Resistance}}{\text{Impedance}} \]

(iii) \[ \text{Power factor} = \frac{VI \cos \phi}{VI} = \frac{\text{True power}}{\text{Apparent power}} \]

Some examples for power factors:

(i) Power factor = \cos 0^\circ = 1 for a pure resistive circuit because the phase angle \( \phi \) between voltage and current is zero.

(ii) Power factor = \cos \left( \frac{\pm \pi}{2} \right) = 0 for a purely inductive or capacitive circuit because the phase angle \( \phi \) between voltage and current is \( \pm \frac{\pi}{2} \).

(iii) Power factor lies between 0 and 1 for a circuit having \( R \), \( L \) and \( C \) in varying proportions.

### 4.8.4 Advantages and disadvantages of AC over DC

There are many advantages and disadvantages of AC system over DC system.

Advantages:

(i) The generation of AC is cheaper than that of DC.

(ii) When AC is supplied at higher voltages, the transmission losses are small compared to DC transmission.

(iii) AC can easily be converted into DC with the help of rectifiers.

Disadvantages:

(i) Alternating voltages cannot be used for certain applications e.g. charging of batteries, electroplating, electric traction etc.

(ii) At high voltages, it is more dangerous to work with AC than DC.

**Example 4.26**

A series RLC circuit which resonates at 400 kHz has 80 μH inductor, 2000 pF capacitor and 50 Ω resistor. Calculate (i) \( Q \)-factor of the circuit (ii) the new value of capacitance when the value of inductance is doubled and (iii) the new \( Q \)-factor.

**Solution**

\[ L = 80 \times 10^{-6} \text{H}; \quad C = 2000 \times 10^{-12} \text{F} \]

\[ R = 50 \Omega; \quad f_r = 400 \times 10^3 \text{Hz} \]

(i) \( Q \)-factor, \( Q_1 = \frac{L}{R \sqrt{C}} \)

\[ = \frac{1}{50} \sqrt{\frac{80 \times 10^{-6}}{200 \times 10^{-12}}} = 4 \]

(ii) When \( L_2 = 2 \) \( L \)

\[ = 2 \times 80 \times 10^6 \text{H} = 160 \times 10^6 \text{H}, \]

\[ C_2 = \frac{1}{4\pi^2 f_r^2 L_2} \]

\[ = \frac{1}{4 \times 3.14^2 \times (400 \times 10^3)^2 \times 160 \times 10^{-6}} \]

\[ \approx 1000 \times 10^{-12} \text{F} \]

\[ C_2 = 1000 \text{pF} \]

(iii) \( Q_2 = \frac{L_2}{R \sqrt{C_2}} = \frac{1}{50} \sqrt{\frac{160 \times 10^{-6}}{1000 \times 10^{-12}}} \]

\[ = \frac{1}{50} \sqrt{\frac{16 \times 10^{-3}}{10 \times 10^{-9}}} = \frac{4 \times 10^2}{50} = 8 \]
**Example 4.27**

A capacitor of capacitance $\frac{10^{-4}}{\pi} F$, an inductor of inductance $\frac{2}{\pi} H$ and a resistor of resistance $100 \, \Omega$ are connected to form a series $RLC$ circuit. When an AC supply of $220 \, V$, $50 \, Hz$ is applied to the circuit, determine (i) the impedance of the circuit (ii) the peak value of current flowing in the circuit (iii) the power factor of the circuit and (iv) the power factor of the circuit at resonance.

**Solution**

$$L = \frac{2}{\pi} H; C = \frac{10^{-4}}{\pi} F; R = 100 \, \Omega$$

$$V_{\text{RMS}} = 220 \, V; f = 50 \, Hz$$

$$X_L = 2\pi f L = 2\pi \times 50 \times \frac{2}{\pi} = 200 \, \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times \frac{10^{-4}}{\pi}} = 100 \, \Omega$$

(i) Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{100^2 + (200 - 100)^2} = 141.4 \, \Omega$$

(ii) Peak value of current,

$$I_m = \frac{V_m}{Z} = \frac{\sqrt{2} V_{\text{RMS}}}{Z}$$

$$= \frac{\sqrt{2} \times 220}{141.4} = 2.2 \, A$$

(iii) Power factor of the circuit,

$$\cos \phi = \frac{R}{Z} = \frac{100}{141.4} = 0.707$$

(iv) Power factor at resonance

$$\cos \phi = \frac{R}{Z} = \frac{R}{R} = 1$$

**4.9 Oscillation in LC Circuits**

### 4.9.1 Energy Conversion during LC Oscillations

We have learnt that energy can be stored in both inductors and capacitors (Refer sections 1.8.2 and 4.3.2). In inductors, the energy is stored in the form of magnetic field while in capacitors; it is stored as the electric field.

Whenever energy is given to a circuit containing a pure inductor of inductance $L$ and a capacitor of capacitance $C$, the energy oscillates back and forth between the magnetic field of the inductor and the electric field of the capacitor. Thus the electrical oscillations of definite frequency are generated. These oscillations are called $LC$ oscillations.

**Generation of LC Oscillations**

Let us assume that the capacitor is fully charged with maximum charge $Q_m$ at the initial stage. So that the energy stored in the capacitor is maximum and is given by $U_e = \frac{Q^2}{2C}$. As there is no current in the inductor, the energy stored in it is zero i.e., $U_L = 0$. Therefore, the total energy is wholly electrical. This is shown in Figure 4.57(a).

The capacitor now begins to discharge through the inductor that establishes current $i$ in clockwise direction. This current produces a magnetic field around the inductor and the energy stored in the inductor is given by $U_L = \frac{LI^2}{2}$. As the charge in the capacitor decreases, the energy stored in it also decreases and is given by
When the current in the circuit reduces to zero, the capacitor becomes fully charged in the opposite direction. The energy stored in the capacitor becomes maximum. Since the current is zero, the energy stored in the inductor is zero. The total energy is wholly electrical (Figure 4.57(e)).

The state of the circuit is similar to the initial state but the difference is that the capacitor is charged in opposite direction. The capacitor then starts to discharge through the inductor with anti-clockwise current. The total energy is the sum of electrical and magnetic energies (Figure 4.57(f)).

As already explained, the processes are repeated in opposite direction (Figure 4.57(g) and (h)). Finally, the circuit returns to the initial state (Figure 4.57(a)). Thus, when the circuit goes through these stages, an alternating current flows in the circuit. As this process is repeated again and again, the electrical oscillations of definite magnitude are obtained.
frequency are generated. These are known as LC oscillations.

In the ideal LC circuit, there is no loss of energy. Therefore, the oscillations will continue indefinitely. Such oscillations are called undamped oscillations.

**Note** But in practice, the Joule heating and radiation of electromagnetic waves from the circuit decrease the energy of the system. Therefore, the oscillations become damped oscillations.

### 4.9.2 Conservation of energy in LC oscillations

During LC oscillations in LC circuits, the energy of the system oscillates between the electric field of the capacitor and the magnetic field of the inductor. Although, these two forms of energy vary with time, the total energy remains constant. It means that LC oscillations take place in accordance with the law of conservation of energy.

Total energy, \( U = U_E + U_B = \frac{q^2}{2C} + \frac{1}{2} Li^2 \)

Let us consider 3 different stages of LC oscillations and calculate the total energy of the system.

**Case (i)** When the charge in the capacitor, \( q = Q_m \) and the current through the inductor, \( i = 0 \), the total energy is given by

\[
U = \frac{Q_m^2}{2C} + 0 = \frac{Q_m^2}{2C}
\]

The total energy is wholly electrical.

**Case (ii)** When charge = 0; current = \( I_m \), the total energy is

\[
U = 0 + \frac{1}{2} Li^2 = \frac{1}{2} Li_m^2
\]

\[
= \frac{L}{2} \left( \frac{Q_m^2}{LC} \right) \quad \text{since} \quad I_m = Q_m \omega = \frac{Q_m}{\sqrt{LC}}
\]

\[
= \frac{Q_m^2}{2C}
\]

The total energy is wholly magnetic.

**Case (iii)** When charge = \( q \); current = \( i \), the total energy is

\[
U = \frac{q^2}{2C} + \frac{1}{2} Li^2
\]

Since \( q = Q_m \cos \omega t \), \( i = -\frac{dq}{dt} = Q_m \cos \omega t \).

The negative sign in current indicates that the charge in the capacitor decreases with time.

\[
U = \frac{Q_m^2}{2C} \left( \cos^2 \omega t + \sin^2 \omega t \right)
\]

\[
= \frac{Q_m^2}{2C}
\]

From above three cases, it is clear that the total energy of the system remains constant.

### 4.9.3 Analogies between LC oscillations and simple harmonic oscillations

(i) **Qualitative treatment**

The electromagnetic oscillations of LC system can be compared with the mechanical oscillations of a spring-mass system.

There are two forms of energy involved in LC oscillations. One is electrical energy...
of the charged capacitor; the other magnetic energy of the inductor carrying current.

Likewise, the mechanical energy of the spring-mass system exists in two forms; the potential energy of the compressed or extended spring and the kinetic energy of the mass. The Table 4.3 lists these two pairs of energy.

By examining the Table 4.3, the analogies between the various quantities can be understood and these correspondences are given in the Table 4.4.

The angular frequency of oscillations of a spring-mass is given by (Refer equation 10.22 of section 10.4.1 of XI physics text book).

\[
\omega = \sqrt{\frac{k}{m}}
\]

From Table 4.4, \( k \to \frac{1}{C} \) and \( m \to L \). Therefore, the angular frequency of \( LC \) oscillations is given by

\[
\omega = \frac{1}{\sqrt{LC}} \quad (4.66)
\]

(ii) Quantitative treatment

The mechanical energy of the spring-mass system is given by

\[
E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \quad (4.67)
\]

The energy \( E \) remains constant for varying values of \( x \) and \( v \). Differentiating \( E \) with respect to time, we get

\[
\frac{dE}{dt} = \frac{1}{2} m \left( 2v \frac{dv}{dt} \right) + \frac{1}{2} k \left( 2x \frac{dx}{dt} \right) = 0
\]

or \( m \frac{d^2 x}{dt^2} + kx = 0 \) \quad (4.68)

This is the differential equation of the oscillations of the spring-mass system. The general solution of equation (4.68) is of the form

**Table 4.3 Energy in two oscillatory systems**

<table>
<thead>
<tr>
<th>LC oscillator</th>
<th>Spring-mass system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element</td>
<td>Energy</td>
</tr>
<tr>
<td>Capacitor</td>
<td>Electrical Energy = ( \frac{1}{2} \left( \frac{1}{C} \right) q^2 )</td>
</tr>
<tr>
<td>Inductor</td>
<td>Magnetic energy = ( \frac{1}{2} Li^2 ) i = ( \frac{dq}{dt} )</td>
</tr>
</tbody>
</table>

**Table 4.4 Analogies between electrical and mechanical quantities**

<table>
<thead>
<tr>
<th>Electrical system</th>
<th>Mechanical system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge ( q )</td>
<td>Displacement ( x )</td>
</tr>
<tr>
<td>Current ( i = \frac{dq}{dt} )</td>
<td>Velocity ( v = \frac{dx}{dt} )</td>
</tr>
<tr>
<td>Inductance ( L )</td>
<td>Mass ( m )</td>
</tr>
<tr>
<td>Reciprocal of capacitance ( \frac{1}{C} )</td>
<td>Force constant ( k )</td>
</tr>
<tr>
<td>Electrical energy</td>
<td>Potential energy</td>
</tr>
<tr>
<td>Magnetic energy</td>
<td>Kinetic energy</td>
</tr>
<tr>
<td>( E = \frac{1}{2} \left( \frac{1}{C} \right) q^2 )</td>
<td>( \frac{1}{2} k x^2 )</td>
</tr>
<tr>
<td>( \frac{1}{2} Li^2 )</td>
<td>( \frac{1}{2} mv^2 )</td>
</tr>
<tr>
<td>Electromagnetic energy</td>
<td>Mechanical energy</td>
</tr>
<tr>
<td>( \frac{1}{2} \frac{C}{L} q^2 + \frac{1}{2} Li^2 )</td>
<td>( E = \frac{1}{2} k x^2 + \frac{1}{2} mv^2 )</td>
</tr>
</tbody>
</table>
\[ x(t) = X_m \cos(\omega t + \phi) \]  \hspace{1cm} (4.69)

where \( X_m \) is the maximum value of \( x(t) \), \( \omega \) the angular frequency and \( \phi \) the phase constant.

Similarly, the electromagnetic energy of the LC system is given by

\[ U = \frac{1}{2} Li^2 + \frac{1}{2} \left( \frac{1}{C} \right) q^2 = \text{constant} \]  \hspace{1cm} (4.70)

Differentiating \( U \) with respect to time, we get

\[ \frac{dU}{dt} = \frac{1}{2} L \left( 2i \frac{di}{dt} \right) + \frac{1}{2C} \left( 2q \frac{dq}{dt} \right) = 0 \]

or

\[ L \frac{d^2q}{dt^2} + \frac{1}{C} q = 0 \]  \hspace{1cm} (4.71)

The general solution of equation (4.71) is of the form

\[ q(t) = Q_m \cos(\omega t + \phi) \]  \hspace{1cm} (4.72)

where \( Q_m \) is the maximum value of \( q(t) \), \( \omega \) the angular frequency and \( \phi \) the phase constant.

**Current in the LC circuit**

The current flowing in the \( LC \) circuit is obtained by differentiating \( q(t) \) with respect to time.

\[ i(t) = \frac{dq}{dt} = \frac{d}{dt} \left[ Q_m \cos(\omega t + \phi) \right] \]

\[ = -Q_m \omega \sin(\omega t + \phi) \quad \text{since} \quad I_m = Q_m \omega \]

or \( i(t) = -I_m \sin(\omega t + \phi) \)  \hspace{1cm} (4.73)

The equation (4.73) clearly shows that current varies as a function of time \( t \). In fact, it is a sinusoidally varying alternating current with angular frequency \( \omega \).

**Angular frequency of LC oscillations**

By differentiating equation (4.72) twice, we get

\[ \frac{d^2q}{dt^2} = -Q_m \omega^2 \cos(\omega t + \phi) \]  \hspace{1cm} (4.74)

Substituting equations (4.72) and (4.74) in equation (4.71), we obtain

\[ L[-Q_m \omega^2 \cos(\omega t + \phi)] + \frac{1}{C} Q_m \cos(\omega t + \phi) = 0 \]

Rearranging the terms, the angular frequency of LC oscillations is given by

\[ \omega = \frac{1}{\sqrt{LC}} \]  \hspace{1cm} (4.75)

This equation is the same as that obtained from qualitative analogy.

**Oscillations of electrical and magnetic energy**

The electrical energy of the \( LC \) oscillator is

\[ U_e = \frac{q^2}{2C} = \frac{Q_m^2}{2C} \cos^2(\omega t + \phi) \]  \hspace{1cm} (4.76)

The magnetic energy is

\[ U_b = \frac{1}{2} Li^2 = \frac{Q_m^2}{2C} \sin^2(\omega t + \phi) \]  \hspace{1cm} (4.77)

If the two energies are plotted with an assumption of \( \phi = 0 \), we obtain Figure 4.58.

![Figure 4.58](image-url)

**Figure 4.58** The variation of \( U_E \) and \( U_B \) as a function of time

From the graph, it can be noted that

(i) At any instant \( U_E + U_B = \frac{Q_m^2}{2C} = \text{constant} \)

(ii) The maximum values of \( U_E \) and \( U_B \) are both \( \frac{Q_m^2}{2C} \).

(iii) When \( U_E \) is Maximum, \( U_B \) is zero and vice versa.
Whenever the magnetic flux linked with a closed coil changes, an emf is induced and hence an electric current flows in the circuit. This phenomenon is known as electromagnetic induction.

Faraday's first law states that whenever magnetic flux linked with a closed circuit changes, an emf is induced in the circuit.

Faraday's second law states that the magnitude of induced emf in a closed circuit is equal to the time rate of change of magnetic flux linked with the circuit.

Lenz's law states that the direction of the induced current is such that it always opposes the cause responsible for its production.

Lenz's law is established on the basis of the law of conservation of energy.

Fleming's right hand rule states that if the index finger points the direction of the magnetic field and the thumb indicates the direction of motion of the conductor, then the middle finger will indicate the direction of the induced current.

Even for a conductor in the form of a sheet or a plate, an emf is induced when magnetic flux linked with it changes. The induced currents flow in concentric circular paths called Eddy currents or Foucault currents.

Inductor is a device used to store energy in a magnetic field when an electric current flows through it.

If the flux linked with the coil is changed, an emf is induced in that same coil. This phenomenon is known as self-induction. The emf induced is called self-induced emf.

When an electric current passing through a coil changes with time, an emf is induced in the neighbouring coil. This phenomenon is known as mutual induction and the emf is called mutually induced emf.

AC generator or alternator is an energy conversion device. It converts mechanical energy used to rotate the coil or field magnet into electrical energy.

In some AC generators, there are three separate coils, which would give three separate emfs. Hence they are called three-phase AC generators.

Transformer is a stationary device used to transform AC electric power from one circuit to another without changing its frequency.

The efficiency of a transformer is defined as the ratio of the useful output power to the input power.

An alternating voltage is a voltage which changes polarity at regular intervals of time and the resulting alternating current changes direction accordingly.

The average value of alternating current is defined as the average of all values of current over a positive half-circle or negative half-circle.

The root mean square value or effective value of an alternating current is defined as the square root of the mean of the squares of all currents over one cycle.
A sinusoidal alternating voltage (or current) can be represented by a vector which rotates about the origin in anti-clockwise direction at a constant angular velocity. Such a rotating vector is called a phasor.

When the frequency of the applied alternating source is equal to the natural frequency of the RLC circuit, the current in the circuit reaches its maximum value. Then the circuit is said to be in electrical resonance.

The magnification of voltages at series resonance is termed as Q–factor.

Power of a circuit is defined as the rate of consumption of electric energy in that circuit. It depends on the components of the circuit.

Whenever energy is given to a LC circuit, the electrical oscillations of definite frequency are generated. These oscillations are called LC oscillations.

During LC oscillations, the total energy remains constant. It means that LC oscillations take place in accordance with the law of conservation of energy.
Electromagnetic induction and Alternating currents

- Electromagnetic induction
- Alternating currents

Faraday's law
Lenz's law
Fleming's RH rule

- Methods of inducing emf
  - By changing magnetic field
  - By changing area
  - By changing orientation

- Transformer
- Three-phase AC generator
- Single-phase AC generator

Power in AC circuits
Wattless current
Power factor

RMS value
Resistive circuit
Capacitive circuit
Inductive circuit
Capacitive circuit
Inductive circuit
Transformer

AC generator

Merits and Demerits of AC

Energy conservation
Mechanical analogies
LC oscillations
I Multiple Choice Questions

1. An electron moves on a straight line path XY as shown in the figure. The coil abcd is adjacent to the path of the electron. What will be the direction of current, if any, induced in the coil?

(a) The current will reverse its direction as the electron goes past the coil
(b) No current will be induced
(c) abcd
(d) adcb

2. A thin semi-circular conducting ring (PQR) of radius r is falling with its plane vertical in a horizontal magnetic field B, as shown in the figure.

The potential difference developed across the ring when its speed v, is

(a) Zero
(b) \( \frac{Bv\pi r^2}{2} \)
(c) \( \pi rBv \)
(d) \( 2rBv \)

3. The flux linked with a coil at any instant t is given by \( \Phi_\theta = 10r^2 - 50r + 250 \). The induced emf at \( t = 3s \) is

(a) \(-190 V\) 
(b) \(-10 V\)
(c) \(10 V\)
(d) \(190 V\)

4. When the current changes from +2A to -2A in 0.05 s, an emf of 8 V is induced in a coil. The co-efficient of self-induction of the coil is

(a) 0.2 H 
(b) 0.4 H
(c) 0.8 H 
(d) 0.1 H

5. The current \( i \) flowing in a coil varies with time as shown in the figure. The variation of induced emf with time would be (NEET – 2011)

The induced emf is

(a) \( T/4 \)
(b) \( T/2 \)
(c) \( 3T/4 \)
(d) \( T \)
6. A circular coil with a cross-sectional area of 4 cm² has 10 turns. It is placed at the centre of a long solenoid that has 15 turns/cm and a cross-sectional area of 10 cm². The axis of the coil coincides with the axis of the solenoid. What is their mutual inductance?
(a) 7.54 µH  (b) 8.54 µH  (c) 9.54 µH  (d) 10.54 µH

7. In a transformer, the number of turns in the primary and the secondary are 410 and 1230 respectively. If the current in primary is 6 A, then that in the secondary coil is
(a) 2 A  (b) 18 A  (c) 12 A  (d) 1 A

8. A step-down transformer reduces the supply voltage from 220 V to 11 V and increase the current from 6 A to 100 A. Then its efficiency is
(a) 1.2  (b) 0.83  (c) 0.12  (d) 0.9

9. In an electrical circuit, R, L, C and AC voltage source are all connected in series. When L is removed from the circuit, the phase difference between the voltage and current in the circuit is \( \pi/3 \). Instead, if C is removed from the circuit, the phase difference is again \( \pi/3 \). The power factor of the circuit is
(a) \( \frac{1}{2} \)  (b) \( \frac{1}{\sqrt{2}} \)  (c) 1  (d) \( \frac{\sqrt{3}}{2} \)

10. In a series RL circuit, the resistance and inductive reactance are the same. Then the phase difference between the voltage and current in the circuit is
(a) \( \frac{\pi}{4} \)  (b) \( \frac{\pi}{2} \)  (c) \( \frac{\pi}{6} \)  (d) zero

11. In a series resonant RLC circuit, the voltage across 100 Ω resistor is 40 V. The resonant frequency \( \omega \) is 250 rad/s. If the value of C is 4 µF, then the voltage across L is
(a) 600 V  (b) 4000 V  (c) 400 V  (d) 1 V

12. An inductor 20 mH, a capacitor 50 µF and a resistor 40 Ω are connected in series across a source of emf \( v = 10 \sin 340t \). The power loss in AC circuit is
(a) 0.76 W  (b) 0.89 W  (c) 0.46 W  (d) 0.67 W

13. The instantaneous values of alternating current and voltage in a circuit are
\[ i = \frac{1}{\sqrt{2}} \sin(100\pi t) \text{ A and} \]
\[ v = \frac{1}{\sqrt{2}} \sin \left(100\pi t + \frac{\pi}{3}\right) \text{ V.} \]
The average power in watts consumed in the circuit is (IIT Main 2012)
(a) \( \frac{1}{4} \)  (b) \( \frac{\sqrt{3}}{4} \)  (c) \( \frac{1}{2} \)  (d) \( \frac{1}{8} \)

14. In an oscillating LC circuit, the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic fields is
(a) \( \frac{Q}{2} \)  (b) \( \frac{Q}{\sqrt{3}} \)  (c) \( \frac{Q}{\sqrt{2}} \)  (d) Q
15. \( \frac{20}{\pi^2} \) inductor is connected to a capacitor of capacitance C. The value of C in order to impart maximum power at 50 Hz is
(a) 50 μF (b) 0.5 μF (c) 500 μF (d) 5 μF

**Answers**

1) a  2) d  3) b  4) d  5) a  6) a  7) a  8) b  9) c  10) a  11) c  12) c  13) d  14) c  15) d

**II Short Answer Questions**

1. What is meant by electromagnetic induction?
5. State Lenz’s law.
6. State Fleming’s right hand rule.
7. How is Eddy current produced? How do they flow in a conductor?
8. Mention the ways of producing induced emf.
9. What for an inductor is used? Give some examples.
10. What do you mean by self-induction?
11. What is meant by mutual induction?
13. List out the advantages of stationary armature-rotating field system of AC generator.
14. What are step-up and step-down transformers?
15. Define average value of an alternating current.

16. How will you define RMS value of an alternating current?
17. What are phasors?
18. Define electric resonance.
19. What do you mean by resonant frequency?
20. How will you define Q-factor?
21. What is meant by wattles current?
22. Give any one definition of power factor.
23. What are LC oscillations?

**III Long Answer Questions**

1. Establish the fact that the relative motion between the coil and the magnet induces an emf in the coil of a closed circuit.
2. Give an illustration of determining direction of induced current by using Lenz’s law.
3. Show that Lenz’s law is in accordance with the law of conservation of energy.
4. Obtain an expression for motional emf from Lorentz force.
5. Using Faraday’s law of electromagnetic induction, derive an equation for motional emf.
6. Give the uses of Foucault current.
7. Define self-inductance of a coil in terms of (i) magnetic flux and (ii) induced emf.
8. How will you define the unit of inductance?
10. Assuming that the length of the solenoid is large when compared to
its diameter, find the equation for its inductance.

11. An inductor of inductance L carries an electric current i. How much energy is stored while establishing the current in it?

12. Show that the mutual inductance between a pair of coils is same \( M_{12} = M_{21} \).

13. How will you induce an emf by changing the area enclosed by the coil?

14. Show mathematically that the rotation of a coil in a magnetic field over one rotation induces an alternating emf of one cycle.

15. Elaborate the standard construction details of AC generator.

16. Explain the working of a single-phase AC generator with necessary diagram.

17. How are the three different emfs generated in a three-phase AC generator? Show the graphical representation of these three emfs.

18. Explain the construction and working of transformer.

19. Mention the various energy losses in a transformer.

20. Give the advantage of AC in long distance power transmission with an example.

21. Find out the phase relationship between voltage and current in a pure inductive circuit.

22. Derive an expression for phase angle between the applied voltage and current in a series RLC circuit.

23. Define inductive and capacitive reactance. Give their units.

24. Obtain an expression for average power of AC over a cycle. Discuss its special cases.

25. Show that the total energy is conserved during LC oscillations.

26. Prove that energy is conserved during electromagnetic induction.

27. Compare the electromagnetic oscillations of LC circuit with the mechanical oscillations of block-spring system to find the expression for angular frequency of LC oscillators mathematically.

IV. Numerical problems

1. A square coil of side 30 cm with 500 turns is kept in a uniform magnetic field of 0.4 T. The plane of the coil is inclined at an angle of 30° to the field. Calculate the magnetic flux through the coil.

2. A straight metal wire crosses a magnetic field of flux 4 mWb in a time 0.4 s. Find the magnitude of the emf induced in the wire.

3. The magnetic flux passing through a coil perpendicular to its plane is a function of time and is given by \( \Phi_B = (2t^3 + 4t^2 + 8t + 8) \text{ Wb} \). If the resistance of the coil is 5 Ω, determine the induced current through the coil at a time \( t = 3 \text{ second} \).

4. A closely wound coil of radius 0.02 m is placed perpendicular to the magnetic field. When the magnetic field is changed from 8000 T to 2000 T in 6 s, an emf of 44 V is induced. Calculate the number of turns in the coil.
5. A rectangular coil of area 6 cm$^2$ having 3500 turns is kept in a uniform magnetic field of 0.4 T. Initially, the plane of the coil is perpendicular to the field and is then rotated through an angle of 180°. If the resistance of the coil is 35 Ω, find the amount of charge flowing through the coil.

   (Ans: $48 \times 10^{-3}$ C)

6. An induced current of 2.5 mA flows through a single conductor of resistance 100 Ω. Find out the rate at which the magnetic flux is cut by the conductor.

   (Ans: 250 mWb$^{-1}$)

7. A fan of metal blades of length 0.4 m rotates normal to a magnetic field of $4 \times 10^{-3}$ T. If the induced emf between the centre and edge of the blade is 0.02 V, determine the rate of rotation of the blade.

   (Ans: 9.95 revolutions/second)

8. A bicycle wheel with metal spokes of 1 m long rotates in Earth’s magnetic field. The plane of the wheel is perpendicular to the horizontal component of Earth’s field of $4 \times 10^{-3}$ T. If the emf induced across the spokes is 31.4 mV, calculate the rate of rotation of the wheel.

   (Ans: 250 revolutions/second)

9. Determine the self-inductance of 4000 turn air-core solenoid of length 2m and diameter 0.04 m.

   (Ans: 12.62 mH)

10. A coil of 200 turns carries a current of 4 A. If the magnetic flux through the coil is $6 \times 10^{-3}$ Wb, find the magnetic energy stored in the medium surrounding the coil.

    (Ans: 0.024 J)

11. A 50 cm long solenoid has 400 turns per cm. The diameter of the solenoid is 0.04 m. Find the magnetic flux of a turn when it carries a current of 1 A.

    (Ans: 1.26 Wb)

12. A coil of 200 turns carries a current of 0.4 A. If the magnetic flux of 4 mWb is linked with the coil, find the inductance of the coil.

    (Ans: 2H)

13. Two air core solenoids have the same length of 80 cm and same cross-sectional area 5 cm$^2$. Find the mutual inductance between them if the number of turns in the first coil is 1200 turns and that in the second coil is 400 turns.

    (Ans: 0.38 mH)

14. A long solenoid having 400 turns per cm carries a current 2A. A 100 turn coil of cross-sectional area 4 cm$^2$ is placed co-axially inside the solenoid so that the coil is in the field produced by the solenoid. Find the emf induced in the coil if the current through the solenoid reverses its direction in 0.04 sec.

    (Ans: 0.20 V)

15. A 200 turn coil of radius 2 cm is placed co-axially within a long solenoid of 3 cm radius. If the turn density of the solenoid is 90 turns per cm, then calculate mutual inductance of the coil.

    (Ans: 2.84 mH)

16. The solenoids $S_1$ and $S_2$ are wound on an iron-core of relative permeability 900. The area of their cross-section and their length are the same and are 4 cm$^2$ and 0.04 m respectively. If the number of turns in $S_1$ is 200 and that in $S_2$ is 800, calculate the mutual inductance between the coils. The current in solenoid 1 is increased from 2A to 8A
in 0.04 second. Calculate the induced emf in solenoid 2.

(Ans: 1.81H; 271.5 V)

17. A step-down transformer connected to main supply of 220 V is made to operate 11V,88W lamp. Calculate (i) Transformation ratio and (ii) Current in the primary.

(Ans: 1/20 and 0.4A)

18. A 200V/120V step-down transformer of 90% efficiency is connected to an induction stove of resistance 40 Ω. Find the current drawn by the primary of the transformer.

(Ans: 2A)

19. The 300 turn primary of a transformer has resistance 0.82 Ω and the resistance of its secondary of 1200 turns is 6.2 Ω. Find the voltage across the primary if the power output from the secondary at 1600V is 32 kW. Calculate the power losses in both coils when the transformer efficiency is 80%.

(Ans: 8.2 kW and 2.48 kW)

20. Calculate the instantaneous value at 60°, average value and RMS value of an alternating current whose peak value is 20 A. (Ans: 17.32A, 12.74A, 14.14 A)

**IV Conceptual Questions**

1. A graph between the magnitude of the magnetic flux linked with a closed loop and time is given in the figure. Arrange the regions of the graph in ascending order of the magnitude of induced emf in the loop.

2. Using Lenz’s law, predict the direction of induced current in conducting rings 1 and 2 when current in the wire is steadily decreasing.

3. A flexible metallic loop abcd in the shape of a square is kept in a magnetic field with its plane perpendicular to the field. The magnetic field is directed into the paper normally. Find the direction of the induced current when the square loop is crushed into an irregular shape as shown in the figure.

4. Predict the polarity of the capacitor in a closed circular loop when two bar magnets are moved as shown in the figure.
5. In series LC circuit, the voltages across L and C are 180° out of phase. Is it correct? Explain.

6. When does power factor of a series RLC circuit become maximum?

7. Draw graphs showing the distribution of charge in a capacitor and current through an inductor during LC oscillations with respect to time. Assume that the charge in the capacitor is maximum initially.

**BOOK FOR REFERENCES**

1. H.C. Verma, Concepts of Physics, Volume 1 and 2, Bharathi Bhawan publishers.
In this activity you will be able to
(1) understand electromagnetic induction.
(2) verify Faraday's laws in virtual lab.

**STEPS:**

- Open the browser and type “phet.colorado.edu” in the address bar. Click play with simulation tab. Search Faraday’s electromagnetic lab in the search box.
- Select ‘pick coil’ tab. Move the magnet through the coil. Note what happens when the magnetic field linked with the coil changes. Change the loop area, flux change and observe the intensity of current with the help of glowing bulb.
- Select ‘Electromagnet’ tab. Change the current flowing through the coil and observe the change in magnetic flux generated.
- Select ‘Generator’ tab. Observe induced emf in the coil if you change the angular velocity of the coil.

**Note:**
Install Java application if it is not in your system. You can download all the phet simulation and works in off line from [https://phet.colorado.edu/en/offline-access](https://phet.colorado.edu/en/offline-access).

**URL:**
[https://phet.colorado.edu/en/simulation/legacy/faraday](https://phet.colorado.edu/en/simulation/legacy/faraday)

* Pictures are indicative only.
* If browser requires, allow Flash Player or Java Script to load the page.
In this unit, the student is exposed to
- the displacement current
- Maxwell’s correction to Ampere’s circuital law
- Maxwell’s equation in integral form
- production and properties of electromagnetic waves – Hertz’s experiment
- sources of electromagnetic waves
- electromagnetic spectrum

**5.1 INTRODUCTION**

![Visible spectrum – rainbow and lightning](image)

We see the world around us through light. Light from the Sun is one of the sources of energy without which we human beings cannot survive in this planet. Light plays crucial role in understanding the structure and properties of various things from atom to universe. Without light, even our eyes cannot see objects. What is light?. This puzzle made many physicists sleepless until middle of 19th century. Earlier, many scientists thought that optics and electromagnetism are two different branches of physics. But from the work of James Clerk Maxwell, who actually enlightened the concept of light from his theoretical prediction is that light is an electromagnetic wave which moves with the speed equal to $3 \times 10^8$ m/s (in free space or vacuum). Later, it was confirmed that light is just only small portion of electromagnetic spectrum, which ranges from gamma rays to radio waves.

In the unit 4, we studied that time varying magnetic field produces an electric field (Faraday’s law of electromagnetic induction). Maxwell strongly believed that nature must possess symmetry and he asked...
the following question, “when the time varying magnetic field produces an electric field, why not the time varying electric field produce a magnetic field?”

Later he proved that indeed it exists, which is often known as Maxwell’s law of induction. In 1888, H. Hertz experimentally verified Maxwell’s predication and hence, this understanding resulted in new technological invention, especially in wireless communication, LASER (Light Amplification by Stimulated Emission of Radiation) technology, RADAR (Radio Detection And Ranging), etc.

In today’s digital world, cell phones (Figure 5.2 (a)) have greater influence in our day to day life. It is a faster and more effective mode of transferring information from one place to another. It works on the basis that light is an electromagnetic wave. In hospitals, the location of bone fracture can be detected using X-rays as shown in Figure 5.2 (b), which is also an electromagnetic wave. For cooking microwave oven is used. The microwave is also an electromagnetic wave. There are plenty of applications of electromagnetic waves in engineering, medical (example LASER surgery, etc), defence (example, RADAR signals) and also in fundamental scientific research. In this unit, basics of electromagnetic waves are covered.

### 5.1.1 Displacement current and Maxwell’s correction to Ampere’s circuital law

In unit 4, we studied Faraday’s law of electromagnetic induction which states that the change in magnetic field produces an electric field. Mathematically

\[
\oint E \cdot dl = - \frac{\partial}{\partial t} \Phi_B + \frac{\partial}{\partial t} \oint B \cdot dS \tag{5.1}
\]

where \( \Phi_B \) is the magnetic flux and \( \frac{\partial}{\partial t} \) is the partial derivative with respect to time. Equation (5.1) means that the electric field \( \vec{E} \) is induced along a closed loop by the changing magnetic flux \( \Phi_B \) in the region encircled by the loop. Now the question asked by James Clerk Maxwell is ‘Is converse of this statement true?’ Answer is ‘yes’. He
showed that the change in electric field also produces magnetic field which is

\[ \oint \vec{B} \cdot d\vec{l} = -\frac{\partial}{\partial t} \Phi_E = -\frac{\partial}{\partial t} \oint \vec{B} \cdot d\vec{S} \]  

(5.2)

\[ \oint \vec{E} \cdot d\vec{l} = \frac{\partial}{\partial t} \Phi_E = \frac{\partial}{\partial t} \oint \vec{B} \cdot d\vec{S} \]  

where \( \Phi_E \) is the electric flux. This is known as Maxwell's law of induction, which explains that the magnetic field \( \vec{B} \) induced along a closed loop by the changing electric flux \( \Phi_E \) in the region encircled by that loop. This in turn, explains the existence of radio waves, gamma rays, infrared rays, etc.

In order to understand how the changing electric field produces magnetic field, let us consider a situation of 'charging a parallel plate capacitor' shown in Figure 5.3 Assume that the medium in between the capacitor plates is a non-conducting medium.

Figure 5.3 charging of capacitor

The electric current passing through the wire is the conduction current \( I_C \). This current generates magnetic field around the wire (refer Unit 3) connected across the capacitor. Therefore, when a magnetic needle is kept near the wire, deflection is observed. In order to compute the strength of magnetic field at a point, we use Ampere's circuital law (from Unit 3) which states that 'the line integral of the magnetic field \( \vec{B} \) around any closed loop is equal to \( \mu_0 \) times the net current \( I \) threading through the area enclosed by the loop.' Ampere’s law in equation form is

\[ \oint \vec{B} \cdot d\vec{l} = \mu_0 I(t) \]  

(5.3)

where \( \mu_0 \) is the permeability of free space.

Figure 5.4 Applying Ampere’s circuital law - loop enclosing surface

To calculate the magnetic field at a point \( P \) near the wire as shown in Figure 5.4, let us draw an amperian loop (circular loop) which encloses the surface \( S_1 \) (circular surface). Therefore, using Ampere’s circuital law (equation 5.3), we get

\[ \oint \vec{B} \cdot d\vec{l} = \mu_0 I_C \]  

(5.4)

where \( I_C \) is the conduction current.

Suppose the same loop is enclosed by balloon shaped surface \( S_2 \) as shown in Figure 5.5. This means that the boundaries of two surfaces \( S_1 \) and \( S_2 \) are same but shape of the enclosing surfaces are different (first surface \( S_1 \) is circular in shape and second one is balloon shaped surface \( S_2 \)). As the Ampere's law applied for a given closed loop does not depend on shape of the enclosing surface, the integrals will give the same answer. But by applying Ampere’s circuital law (equation 5.3), we get
between the plates of the capacitor also produces a current known as displacement current.

\[
\Phi_E = \int\int \mathbf{E} \cdot d\mathbf{A} = EA = \frac{q}{\varepsilon_0}
\]

where \( A \) is the area of the plates of capacitor. The change in electric flux is

\[
\frac{d\Phi_E}{dt} = \frac{1}{\varepsilon_0} \frac{dq}{dt} \Rightarrow \frac{dq}{dt} = I_d = \varepsilon_0 \frac{d\Phi_E}{dt}
\]

where \( I_d \) is known as displacement current.

The displacement current can be defined as the current which comes into play in the region in which the electric field and the electric flux are changing with time. In other words, whenever the change in electric field takes place, displacement current is produced. Maxwell modified Ampere's law as

\[
\oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 I = \mu_0 (I_c + I_d)
\]

**Displacement current**

The name stuck because Maxwell named it. The word displacement is poorly chosen because nothing is being displaced here.
where \( I = I_c + I_d \) which means the total current enclosed by the surface is sum of conduction current and displacement current. When a constant current is applied, displacement current \( I_d = 0 \) and hence \( I_c = I \). Between the plates, the conduction current \( I_c = 0 \) and hence \( I_d = I \).

**EXAMPLE 5.1**

Consider a parallel plate capacitor which is maintained at potential of 200 V. If the separation distance between the plates of the capacitor and area of the plates are 1 mm and 20 cm². Calculate the displacement current for the time in \( \mu s \).

**Solution**

Potential difference between the plates of the capacitor, \( V = 200 \text{ V} \)

The distance between the plates,

\[ d = 1 \text{ mm} = 1 \times 10^{-3} \text{ m} \]

Area of the plates of the capacitor,

\[ A = 20 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2 \]

Time is given in micro-second, \( \mu s = 10^{-6} \text{ s} \)

Displacement current

\[ I_d = \varepsilon \frac{d\Phi}{dt} \Rightarrow I_d = \epsilon \frac{EA}{t} \]

But electric field, \( E = \frac{V}{d} \)

Therefore,

\[ I = \frac{V}{d} I_d = \epsilon \frac{E A}{t d} = 8.85 \times 10^{-12} \times \frac{200 \times 20 \times 10^{-4}}{1 \times 10^{-6} \times 1 \times 10^{-3}} \]

\[ = 35400 \times 10^{-7} = 3.5 \text{ mA} \]

**5.1.3 Maxwell’s equations in integral form**

Electrodynamics can be summarized into four basic equations, known as Maxwell’s equations. These equations are analogous to Newton’s equations in mechanics. Maxwell’s equations completely explain the behaviour of charges, currents and properties of electric and magnetic fields. These equations can be written in integral form (or integration form) or derivative form (or differentiation form). The differential form of Maxwell’s equation is beyond higher secondary level because we need to learn additional mathematical operations like curl of vector fields and divergence of vector fields. So we focus here only in integral form of Maxwell’s equations:

1. First equation is nothing but the Gauss’s law. It relates the net electric flux to net electric charge enclosed in a surface. Mathematically, it is expressed as

\[ \oint E \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0} \quad \text{(Gauss’s law)} \quad \text{(5.7)} \]

where \( \vec{E} \) is the electric field and \( Q_{enclosed} \) is the charge enclosed. This equation is true for both discrete or continuous distribution of charges. It also indicates that the electric field lines start from positive charge and terminate at negative charge. This implies that the electric field lines do not form a continuous closed path. In other words, it means that isolated positive charge or negative charge can exist.

2. Second equation has no name. But this law is similar to Gauss’s law in electrostatics. So this law can also be called as Gauss’s law in magnetism. The
where $B$ is the magnetic field. This equation implies that the magnetic lines of force form a continuous closed path. In other words, it means that no isolated magnetic monopole exists.

3. Third equation is Faraday's law of electromagnetic induction. This law relates electric field with the changing magnetic flux which is mathematically written as

$$\oint E \cdot d\ell = -\frac{d}{dt} \Phi_B \text{ (Faraday’s law)} \quad (5.9)$$

where $E$ is the electric field. This equation implies that the line integral of the electric field around any closed path is equal to the rate of change of magnetic flux through the closed path bounded by the surface. Our modern technological revolution is due to Faraday’s laws of electromagnetic induction. The electrical energy supplied to our houses from electricity board by using Faraday’s law of induction.

4. Fourth equation is modified Ampere’s circuital law. This is also known as Ampere – Maxwell’s law. This law relates the magnetic field around any closed path to the conduction current and displacement current through that path.

$$\oint B \cdot d\ell = \mu_0 I_{\text{enclosed}} + \mu_0 \varepsilon_0 \frac{d}{dt} \int E \cdot dA \quad \text{(Ampere-Maxwell’s law)} \quad (5.10)$$

where $\dot{B}$ is the magnetic field. This equation shows that both conduction and also displacement current produces magnetic field. These four equations are known as Maxwell’s equations in electrodynamics. This equation ensures the existence of electromagnetic waves. The entire communication system in the world depends on electromagnetic waves. In fact our understanding of stars, galaxy, planets etc come by analysing the electromagnetic waves emitted by these astronomical objects.

5.2 ELECTROMAGNETIC WAVES

Electromagnetic waves are non-mechanical waves which move with speed equals to the speed of light (in vacuum). It is a transverse wave. In the following subsections, we discuss the production of electromagnetic waves and its properties, sources of electromagnetic waves and also classification of electromagnetic spectrum.

5.2.1 Production and properties of electromagnetic waves - Hertz experiment

Maxwell’s prediction was experimentally confirmed by Heinrich Rudolf Hertz (Figure 5.7 (a)) in 1888. The experimental set up used is shown in Figure 5.7 (b).

It consists of two metal electrodes which are made of small spherical metals as shown in Figure 5.7. These are connected to larger spheres and the ends of them are connected to induction coil with very large number of turns. This is to produce very high electromotive force (emf). Since the
The electric and magnetic fields are in the y and z directions respectively and the direction of propagation is along x direction. This is shown in Figure 5.8.

4. Electromagnetic waves travel with speed which is equal to the speed of light in vacuum or free space, \( c = \frac{1}{\sqrt{\varepsilon \mu}} = 3 \times 10^8 \text{ m s}^{-1} \), where \( \varepsilon_0 \) is the permittivity of free space or vacuum and \( \mu_0 \) is the permeability of free space or vacuum (refer Unit 1 for permittivity and Unit 3 for permeability).

5. In a medium with permittivity \( \varepsilon \) and permeability \( \mu \), the speed of electromagnetic wave is less than speed in free space or vacuum, that is, \( v < c \). In a medium of refractive index, 
\[
\mu = \frac{c}{v} = \frac{1}{\sqrt{\varepsilon \mu}} \Rightarrow \mu = \sqrt{\varepsilon \mu} \text{'}, where \( \varepsilon \) is the relative permittivity of the medium (also known as dielectric constant) and \( \mu \) is the relative permeability of the medium.

6. Electromagnetic waves are not deflected by electric field or magnetic field.

Properties of electromagnetic waves
1. Electromagnetic waves are produced by any accelerated charge.
2. Electromagnetic waves do not require any medium for propagation. So electromagnetic wave is a non-mechanical wave.
3. Electromagnetic waves are transverse in nature. This means that the oscillating electric field vector, oscillating magnetic field vector and propagation vector (gives direction of propagation) are mutually perpendicular to each other.

Figure 5.7 Hertz experiment (a) Heinrich Rudolf Hertz (1857 – 1894) (b) Hertz apparatus
7. Electromagnetic waves can show interference, diffraction and can also be polarized.
8. The energy density (energy per unit volume) associated with an electromagnetic wave propagating in vacuum or free space is 
\[ u = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2 \]
where, \( \frac{1}{2} \varepsilon_0 E^2 = u_E \) is the energy density in an electric field and \( \frac{1}{2\mu_0} B^2 = u_B \) is the energy density in a magnetic field.

Since, \( E = Bh \) \( \Rightarrow u_B = u_E \).

The energy density of the electromagnetic wave is 
\[ u = \varepsilon_0 E^2 = \frac{1}{\mu_0} B^2 \]

9. The average energy density for electromagnetic wave, 
\[ \langle u \rangle = \frac{1}{2} \varepsilon_0 E^2 = \frac{1}{2\mu_0} B^2 \].

10. The energy crossing per unit area per unit time and perpendicular to the direction of propagation of electromagnetic wave is called the intensity.

Intensity, \[ I = \langle u \rangle c \]
\[ I = \frac{\text{total electromagnetic energy} (U)}{\text{Surface area} (A) \times \text{time} (t)} \]
\[ = \frac{\text{Power} (P)}{\text{Surface area} (A)} \]

11. Like other waves, electromagnetic waves also carry energy and momentum. For the electromagnetic wave of energy \( U \) propagating with speed \( c \) has linear momentum which
is given by \( \frac{\text{Energy}}{\text{speed}} = \frac{U}{c} \). The force exerted by an electromagnetic wave on unit area of a surface is called radiation pressure.

12. If the electromagnetic wave incident on a material surface is completely absorbed, then the energy delivered is \( U \) and momentum imparted on the surface is \( p = \frac{U}{c} \).

13. If the incident electromagnetic wave of energy \( U \) is totally reflected from the surface, then the momentum delivered to the surface is \( \Delta p = \frac{U}{c} - \left( -\frac{U}{c} \right) = 2\frac{U}{c} \).

14. The rate of flow of energy crossing a unit area is known as pointing vector for electromagnetic waves, which is \( \vec{S} = \frac{1}{\mu_0} \left( \vec{E} \times \vec{B} \right) = c^2 \varepsilon_0 \left( \vec{E} \times \vec{B} \right) \). The unit for pointing vector is \( \text{W m}^{-2} \). The pointing vector at any point gives the direction of energy transport from that point.

15. Electromagnetic waves carries not only energy and momentum but also angular momentum.

**EXAMPLE 5.2**

The relative magnetic permeability of the medium is 2.5 and the relative electrical permittivity of the medium is 2.25. Compute the refractive index of the medium.

**Solution**

Dielectric constant (relative permeability of the medium) is \( \varepsilon_r = 2.25 \)

Magnetic permeability is \( \mu_r = 2.5 \)

Refractive index of the medium,

\[
\eta = \sqrt{\varepsilon_r \mu_r} = \sqrt{2.25 \times 2.5} = 2.37
\]
where, $E_o$ and $B_o$ are amplitude of oscillating electric and magnetic field, $k$ is a wave number, $\omega$ is the angular frequency of the wave and $\vec{k}$ (unit vector, here it is called propagation vector) denotes the direction of propagation of electromagnetic wave.

Note that both electric field and magnetic field oscillate with a frequency (frequency of electromagnetic wave) which is equal to the frequency of the source (here, oscillating charge is the source for the production of electromagnetic waves). In free space or in vacuum, the ratio between $E_o$ and $B_o$ is equal to the speed of electromagnetic wave, which is equal to speed of light $c$.

$$c = \frac{E_o}{B_o}$$

In any medium, the ratio of $E_o$ and $B_o$ is equal to the speed of electromagnetic wave in that medium, mathematically, it can be written as

$$v = \frac{E_o}{B_o} < c$$

Further, the energy of electromagnetic waves comes from the energy of the oscillating charge.

**EXAMPLE 5.3**

Compute the speed of the electromagnetic wave in a medium if the amplitude of electric and magnetic fields are $3 \times 10^4$ N C$^{-1}$ and $2 \times 10^{-4}$ T, respectively.

**Solution**

The amplitude of the electric field, $E_o = 3 \times 10^4$ N C$^{-1}$

The amplitude of the magnetic field, $B_o = 2 \times 10^{-4}$ T. Therefore, speed of the electromagnetic wave in a medium is

$$v = \frac{3 \times 10^4}{2 \times 10^{-4}} = 1.5 \times 10^8 \text{ m s}^{-1}$$

Electromagnetic spectrum is an orderly distribution of electromagnetic waves in terms of wavelength or frequency as shown in Figure 5.10.

**Radio waves**

It is produced by oscillators in electric circuits. The wavelength range is $1 \times 10^{-1}$ m to $1 \times 10^4$ m and frequency range is $3 \times 10^9$ Hz to $3 \times 10^4$ Hz. It obeys reflection and diffraction. It is used in radio and television communication systems and also in cellular phones to transmit voice communication in the ultra high frequency band.

**Microwaves**

It is produced by electromagnetic oscillators in electric circuits. The wavelength range is $1 \times 10^{-3}$ m to $3 \times 10^{-1}$ m and frequency range is $3 \times 10^{11}$ Hz to $1 \times 10^9$ Hz. It obeys reflection and polarization. It is used in radar system for aircraft navigation, speed of the vehicle, microwave oven for cooking and very long distance wireless communication through satellites.

**Figure 5.10 Electromagnetic spectrum** -

Electromagnetic spectrum is an orderly distribution of electromagnetic waves in terms of wavelength or frequency as shown in Figure 5.10.
Measuring the speed of light using the microwave oven

Nowadays the microwave oven is very commonly used to heat the food items. Micro waves of wavelengths 1 mm to 30 cm are produced in these ovens. Such waves form the standing waves between the interior walls of the oven. It is interesting to note that the speed of light can be measured using micro wave oven.

We studied about the standing waves in XI physics, Volume 2, Unit 11. The standing waves have nodes and antinodes at fixed points. At node point, the amplitude of the wave is zero and at antinodes point, the amplitude is maximum. In other words, the maximal energy of microwaves is located at antinode points. When we keep some food items like chappathi or choclate (after removing the rotating platform) inside the oven, we can notice that at antinode locations, chappathi will be burnt more than other locations. It is shown in the Figure (c) and (d). The distance between two successive burnt spots will give the wavelength of microwave. The frequency of microwave is printed in the panel of oven. By knowing wavelength and frequency of microwaves, using the formula \( v \lambda = c \), we can calculate the speed of light \( c \).

**Infrared radiation**

It is produced from hot bodies (also known as heat waves) and also when the molecules undergo rotational and vibrational transitions. The wavelength range is \( 8 \times 10^{-7} \text{ m} \) to \( 5 \times 10^{-3} \text{ m} \) and frequency range are \( 4 \times 10^{14} \text{ Hz} \) to \( 6 \times 10^{10} \text{ Hz} \). It provides electrical energy to satellites by means of solar cells. It is used to produce dehydrated fruits, in green houses to keep the plants warm, heat therapy for muscular pain or sprain, TV remote as a signal carrier, to look through haze fog or mist and used in night vision or infrared photography.

**Visible light**

It is produced by incandescent bodies and also it is radiated by excited atoms in gases. The wavelength range is \( 4 \times 10^{-7} \text{ m} \) to \( 7 \times 10^{-7} \text{ m} \) and frequency range are \( 7 \times 10^{14} \text{ Hz} \) to \( 4 \times 10^{14} \text{ Hz} \). It obeys the laws of reflection, refraction, interference, diffraction, polarization, photo-electric effect and photographic action. It can be used to study the structure of molecules, arrangement of electrons in external shells of atoms and sensation of our eyes.

**Ultraviolet radiation**

It is produced by Sun, arc and ionized gases. The wavelength range is \( 6 \times 10^{-10} \text{ m} \) to \( 4 \times 10^{-7} \text{ m} \) and frequency range are \( 5 \times 10^{17} \text{ Hz} \) to \( 7 \times 10^{14} \text{ Hz} \). It has less penetrating power. It can be absorbed by atmospheric ozone and harmful to human body. It is used to destroy bacteria, sterilizing the surgical instruments, burglar alarm, detect the invisible writing, finger prints and also in the study of molecular structure.
**Table 5.1** visible region, frequency and wavelength of different types of radiation

<table>
<thead>
<tr>
<th>Type of Radiation</th>
<th>Frequency Range (Hz)</th>
<th>Wavelength Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>gamma-rays</td>
<td>$10^{20}$-$10^{24}$</td>
<td>&lt;10$^{-12}$m</td>
</tr>
<tr>
<td>x-rays</td>
<td>$10^{12}$-$10^{16}$</td>
<td>1 nm - 1 pm</td>
</tr>
<tr>
<td>ultraviolet</td>
<td>$10^{15}$-$10^{17}$</td>
<td>400 nm - 1 nm</td>
</tr>
<tr>
<td>visible</td>
<td>$4 \cdot 7.5 \times 10^{14}$</td>
<td>750 nm - 400nm</td>
</tr>
<tr>
<td>near-infrared</td>
<td>$1 \times 10^{14}$-$4 \times 10^{14}$</td>
<td>2.5 μm - 750nm</td>
</tr>
<tr>
<td>infrared</td>
<td>$10^{15}$-$10^{14}$</td>
<td>25 μm - 2.5μm</td>
</tr>
<tr>
<td>microwaves</td>
<td>$3 \times 10^{11}$-$10^{13}$</td>
<td>1 mm - 25 μm</td>
</tr>
<tr>
<td>radio waves</td>
<td>&lt; $3 \times 10^{11}$</td>
<td>&gt; 1 mm</td>
</tr>
</tbody>
</table>

**X-rays**

It is produced when there is a sudden deceleration of high speed electrons at high-atomic number target, and also by electronic transitions among the innermost orbits of atoms. The wavelength range $10^{-13}$ m to $10^{-8}$ m and frequency range are $3 \times 10^{21}$ Hz to $1 \times 10^{16}$ Hz. X-rays have more penetrating power than ultraviolet radiation. X-rays are used extensively in studying structures of inner atomic electron shells and crystal structures. It is used in detecting fractures, diseased organs, formation of bones and stones, observing the progress of healing bones. Further, in a finished metal product, it is used to detect faults, cracks, flaws and holes.

**Gamma rays**

It is produced by transitions of atomic nuclei and decay of certain elementary particles. They produce chemical reactions on photographic plates, fluorescence, ionisation, diffraction. The wavelength range is $1 \times 10^{-14}$ m to $1 \times 10^{-10}$ m and frequency range are $3 \times 10^{22}$ Hz to $3 \times 10^{18}$ Hz. Gamma rays have high penetrating power than X-rays and ultraviolet radiations; it has no charge but harmful to human body. Gamma rays provide information about the structure of atomic nuclei. It is used in radio therapy for the treatment of cancer and tumour, in food industry to kill pathogenic microorganism.

**Example 5.4**

A magnetron in a microwave oven emits electromagnetic waves (em waves) with frequency $f = 2450$ MHz. What magnetic field strength is required for electrons to move in circular paths with this frequency?

**Solution**

Frequency of the electromagnetic waves given is $f = 2450$ MHz

The corresponding angular frequency is

$$\omega = 2\pi f = 2 \times 3.14 \times 2450 \times 10^6$$

$$= 15,386 \times 10^6 \text{ Hz}$$

$$= 1.54 \times 10^{10} \text{ s}^{-1}$$

The magnetic field $B = \frac{m_e \omega}{|q|}$

Mass of the electron, $m_e = 9.22 \times 10^{-31}$ kg

Charge of the electron $q = -1.60 \times 10^{-19} C \Rightarrow |q| = 1.60 \times 10^{-19} C$

$$B = \frac{(9.22 \times 10^{-31})(1.54 \times 10^{10})}{(1.60 \times 10^{-19})} = 8.87425 \times 10^{-7} T$$

$$B = 0.0887 \text{ T}$$

This magnetic field can be easily produced with a permanent magnet. So, electromagnetic waves of frequency 2450 MHz can be used for heating and cooking food because they are strongly absorbed by water molecules.
5.3 TYPES OF SPECTRUM—EMISSION AND ABSORPTION SPECTRUM—FRAUNHOFER LINES

Blackbody radiation curves

Figure 5.11 Black body radiation spectrum – variation with temperature

When an object burns, it emits colours. That is, it emits electromagnetic radiation which depends on temperature. If the object becomes hot then it glows in red colour. If the temperature of the object is further increased then it glows in reddish-orange colour and becomes white when it is hottest. The spectrum in Figure 5.11 usually called as black body spectrum (refer plus one volume two Unit 8). It is a continuous frequency (or wavelength) curve and depends on the body’s temperature.

Suppose we allow a beam of white light to pass through the prism as shown in Figure 5.12, it is split into its seven constituent colours which can be viewed on the screen as continuous spectrum. This phenomenon is known as dispersion of light and the definite pattern of colours obtained on the screen after dispersion is called as spectrum. The plural for spectrum is spectra. The spectra can be broadly classified into two categories:

(a) Emission spectra

When the spectrum of self luminous source is taken, we get emission spectrum. Each source has its own characteristic emission spectrum. The emission spectrum can be divided into three types:

(i) Continuous emission spectra (or continuous spectra)

Figure 5.13 continuous emission spectra

If the light from incandescent lamp (filament bulb) is allowed to pass through prism (simplest spectroscope), it splits into seven colours. Thus, it consists of wavelengths containing all the visible colours ranging from violet to red (Figure 5.13). Examples: spectrum obtained from carbon arc, incandescent solids, liquids gives continuous spectra.

(ii) Line emission spectrum (or line spectrum):
When light from the incandescent lamp is passed through cold gas (medium), the spectrum obtained through the dispersion due to prism is line absorption spectrum (Figure 5.15). Similarly, if the light from the carbon arc is made to pass through sodium vapour, a continuous spectrum of carbon arc with two dark lines in the yellow region of sodium vapour is obtained.

(iii) Band absorption spectrum

When the white light is passed through the iodine vapour, dark bands on continuous bright background is obtained. This type of band is also obtained when white light is passed through diluted solution of blood or chlorophyll or through certain solutions of organic and inorganic compounds.

Fraunhofer lines

When the spectrum obtained from the Sun is examined, it consists of large number of dark lines (line absorption spectrum). These dark lines in the solar spectrum are known as Fraunhofer lines (Figure 5.16). The absorption spectra for various materials are compared with the Fraunhofer lines in the solar spectrum, which helps in identifying elements present in the Sun's atmosphere.

(b) Absorption spectra

When light is allowed to pass through a medium or an absorbing substance then the spectrum obtained is known as absorption spectrum. It is the characteristic of absorbing substance. Absorption spectrum is classified into three types:

(i) Continuous absorption spectrum

When the light is passed through a medium, it is dispersed by the prism, we get continuous absorption spectrum. For instance, when we pass white light through a blue glass plate, it absorbs everything except blue. This is an example of continuous absorption spectrum.

(ii) Line absorption spectrum

When light from the hot gas is allowed to pass through prism, line spectrum is observed (Figure 5.14). Line spectra are also known as discontinuous spectra. The line spectra are sharp lines of definite wavelengths or frequencies. Such spectra arise due to excited atoms of elements. These lines are the characteristics of the element which means it is different for different elements. Examples: spectra of atomic hydrogen, helium, etc.

(3) Band emission spectrum (or band spectrum)

Band spectrum consists of several number of very closely spaced spectral lines which overlapped together forming specific bands which are separated by dark spaces, known as band spectra. This spectrum has a sharp edge at one end and fades out at the other end. Such spectra arise when the molecules are excited. Band spectrum is the characteristic of the molecule hence, the structure of the molecules can be studied using their band spectra. Examples, spectra of hydrogen gas, ammonia gas in the discharge tube etc.
Displacement current can be defined as ‘the current which comes into play in the region in which the electric field and the electric flux are changing with time’

Maxwell modified Ampere’s law as
\[ \oint \mathbf{B} \cdot d\mathbf{S} = \mu \int (I_r + I_d) \]

An electromagnetic wave is radiated by an accelerated charge which propagates through space as coupled electric and magnetic fields, oscillating perpendicular to each other and to the direction of propagation of the wave

Electromagnetic wave is a transverse wave. They are non-mechanical wave and do not require any medium for propagation

The instantaneous magnitude of the electric and magnetic field vectors in electromagnetic wave are related by \[ \mathbf{E} = \mathbf{B}c \]

Electromagnetic waves are transverse in nature. This means that the oscillating electric field vector, oscillating magnetic field vector and propagation vector are (gives direction of propagation) mutually perpendicular to each other

Electromagnetic waves can show interference, diffraction and also can be polarized

The average energy density \[ \langle u \rangle = 2u_c = 2u_m = \varepsilon \varepsilon_0 E^2 = \frac{1}{\mu_0} B^2 \]

The energy crossing per unit area per unit time and perpendicular to the direction of propagation of electromagnetic wave is called the intensity, which is \[ I = \langle u \rangle c \]

If the electromagnetic wave incident on a material surface is completely absorbed, then the energy delivered is \( U \) and momentum imparted on the surface is \( p = \frac{U}{c} \)

If the incident electromagnetic wave of energy \( U \) is totally reflected from the surface, then the momentum delivered to the surface is \( \Delta p = \frac{U}{c} - \frac{U}{c} = 2 \frac{U}{c} \)

The rate of flow of energy crossing a unit area is known as poynting vector for electromagnetic waves, which is \( \mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B}) = c^2 \varepsilon_0 (\mathbf{E} \times \mathbf{B}) \).

Electromagnetic waves carry not only energy and momentum but also angular momentum.

Types of spectrum – emission and absorption

When the spectrum of self luminous source is taken, we get emission spectrum. Each source has its own characteristic emission spectrum. The emission spectrum can be divided into three types: continuous, line and band.

The spectrum obtained from the Sun is examined, it consists of large number of dark lines (line absorption spectrum). These dark lines in the solar spectrum are known as Fraunhofer lines.
Electromagnetic waves

- Displacement Current
- Maxwell’s equations
- Sources
  - Production
- EM Spectrum
- Absorption Spectrum
  - Continuous Absorption Spectrum
  - Band Absorption Spectrum
    - Franhofer Lines
- Emission Spectrum
  - Line Absorption Spectrum
  - Band Emission Spectrum
  - Continuous Emission Spectrum
  - Line Emission Spectrum
**I Multiple choice questions**

1. The dimension of $\frac{1}{\mu_0 \varepsilon_0}$ is
   (a) [L T$^{-1}$]    (b) [L$^2$ T$^{-2}$]
   (c) [L$^{-1}$ T]    (d) [L$^{-2}$ T$^2$]

2. If the amplitude of the magnetic field is $3 \times 10^{-6}$ T, then amplitude of the electric field for a electromagnetic waves is
   (a) 100 V m$^{-1}$    (b) 300 V m$^{-1}$
   (c) 600 V m$^{-1}$    (d) 900 V m$^{-1}$

3. Which of the following electromagnetic radiation is used for viewing objects through fog
   (a) microwave    (b) gamma rays
   (c) X-rays    (d) infrared

4. Which of the following are false for electromagnetic waves
   (a) transverse
   (b) mechanical waves
   (c) longitudinal
   (d) produced by accelerating charges

5. Consider an oscillator which has a charged particle and oscillates about its mean position with a frequency of 300 MHz. The wavelength of electromagnetic waves produced by this oscillator is
   (a) 1 m    (b) 10 m
   (c) 100 m    (d) 1000 m

6. The electric and the magnetic field, associated with an electromagnetic wave, propagating along X axis can be represented by
   (a) $\vec{E} = E \hat{j}$ and $\vec{B} = B \hat{k}$
   (b) $\vec{E} = E \hat{k}$ and $\vec{B} = B \hat{j}$
   (c) $\vec{E} = E \hat{i}$ and $\vec{B} = B \hat{j}$
   (d) $\vec{E} = E \hat{j}$ and $\vec{B} = B \hat{i}$

7. In an electromagnetic wave in free space the rms value of the electric field is 3 V m$^{-1}$. The peak value of the magnetic field is
   (a) $1.414 \times 10^{-8}$ T    (b) $1.0 \times 10^{-8}$ T
   (c) $2.828 \times 10^{-8}$ T    (d) $2.0 \times 10^{-8}$ T

8. During the propagation of electromagnetic waves in a medium:
   (a) electric energy density is double of the magnetic energy density
   (b) electric energy density is half of the magnetic energy density
   (c) electric energy density is equal to the magnetic energy density
   (d) both electric and magnetic energy densities are zero

9. If the magnetic monopole exists, then which of the Maxwell’s equation to be modified?
   (a) $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$
   (b) $\oint \vec{E} \cdot d\vec{A} = 0$
   (c) $\oint \vec{E} \cdot d\vec{A} = \mu_0 I_{enclosed} + \mu_0 \oint \frac{d}{dt} \oint \vec{E} \cdot d\vec{A}$
   (d) $\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \oint \Phi_B$

10. A radiation of energy $E$ falls normally on a perfectly reflecting surface. The momentum transferred to the surface is
    (a) $\frac{E}{c}$    (b) $2 \frac{E}{c}$
    (c) $Ec$    (d) $\frac{E}{c^2}$
11. Which of the following is an electromagnetic wave?
   (a) α - rays (b) β - rays (c) γ - rays (d) all of them

12. Which one of them is used to produce a propagating electromagnetic wave?
   (a) an accelerating charge (b) a charge moving at constant velocity (c) a stationary charge (d) an uncharged particle

13. Let \( E = E_o \sin[10^6 x - \omega t] \) be the electric field of plane electromagnetic wave, the value of \( \omega \) is
   (a) \( 0.3 \times 10^{-14} \text{ rad s}^{-1} \) (b) \( 3 \times 10^{-14} \text{ rad s}^{-1} \) (c) \( 0.3 \times 10^{14} \text{ rad s}^{-1} \) (d) \( 3 \times 10^{14} \text{ rad s}^{-1} \)

14. Which of the following is NOT true for electromagnetic waves?
   (a) it transport energy (b) it transport momentum (c) it transport angular momentum (d) in vacuum, it travels with different speeds which depend on their frequency

15. The electric and magnetic fields of an electromagnetic wave are
   (a) in phase and perpendicular to each other (b) out of phase and not perpendicular to each other (c) in phase and not perpendicular to each other (d) out of phase and perpendicular to each other

**Answers**

1) b 2) d 3) d 4) c 5) a 6) b 7) a 8) c 9) b 10) b 11) c 12) a 13) d 14) d 15) a

**II Short answer questions**

1. What is displacement current?
2. What are electromagnetic waves?
3. Write down the integral form of modified Ampere's circuital law.
4. Explain the concept of intensity of electromagnetic waves.
5. What is meant by Fraunhofer lines?

**III Long answer questions**

1. Write down Maxwell equations in integral form.
2. Write short notes on (a) microwave (b) X-ray (c) radio waves (d) visible spectrum
3. Discuss briefly the experiment conducted by Hertz to produce and detect electromagnetic spectrum.
4. Explain the Maxwell’s modification of Ampere’s circuital law.
5. Write down the properties of electromagnetic waves.
6. Discuss the source of electromagnetic waves.
7. What is emission spectra? Give their types.
8. What is absorption spectra? Give their types.
IV Numerical problems

1. Consider a parallel plate capacitor whose plates are closely spaced. Let $R$ be the radius of the plates and the current in the wire connected to the plates is 5 A, calculate the displacement current through the surface passing between the plates by directly calculating the rate of change of flux of electric field through the surface.

   Answer: $I_d = I_c = 5$ A

2. A transmitter consists of LC circuit with an inductance of 1 $\mu$H and a capacitance of 1 $\mu$F. What is the wavelength of the electromagnetic waves it emits?

   Answer: $18.84 \times 10^{-6}$ m

3. A pulse of light of duration $10^{-6}$ s is absorbed completely by a small object initially at rest. If the power of the pulse is $60 \times 10^{-3}$ W, calculate the final momentum of the object.

   Answer: $20 \times 10^{-17}$ kg m s$^{-1}$

4. Let an electromagnetic wave propagate along the x direction, the magnetic field oscillates at a frequency of $10^{10}$ Hz and has an amplitude of $10^{-5}$ T, acting along the y - direction. Then, compute the wavelength of the wave. Also write down the expression for electric field in this case.

   Answer: $\lambda = 3 \times 10^{-18}$ m and $\vec{E}(x,t) = 3 \times 10^4 \sin(2.09 \times 10^{18} x - 6.28 \times 10^9 t) \hat{j} \text{ NC}^{-1}$

5. If the relative permeability and relative permittivity of the medium is 1.0 and 2.25, respectively. Find the speed of the electromagnetic wave in this medium.

   Answer: $v = 2$ m s$^{-1}$

BOOKS FOR REFERENCE:

1. H. C. Verma, *Concepts of Physics – Volume 2*, Bharati Bhawan Publisher
Electromagnetic waves

In this activity you will be able to how do microwaves heat food?

### STEPS:

- Open the browser and type “phet.colorado.edu/en/simulation/microwaves” in the address bar. Run the simulation.
- Select ‘one molecule’ tab. Turn on the microwave using the button in the right control panel. The arrows indicate the strength and direction of the force that would be exerted by the microwave on the water molecules present in food. Observe the response of water molecule in response to this force?
- Observe how do microwaves heat food by rotating water molecule?
- Change amplitude and frequency of microwave and discuss how fast the water molecules are rotating?

**Note:**
Install Java application if it is not in your system. You can download all the phet simulation and works in offline from [https://phet.colorado.edu/en/offline-access](https://phet.colorado.edu/en/offline-access).

**URL:**
[https://phet.colorado.edu/en/simulation/microwaves](https://phet.colorado.edu/en/simulation/microwaves)

* Pictures are indicative only.
* If browser requires, allow Flash Player or Java Script to load the page.

Discuss the relationship between rotating speed of the molecule with cooking time.
LIST OF EXPERIMENTS

1. Determination of the specific resistance of the material of the given coil using metre bridge.

2. Determination of the value of the horizontal component of the Earth's magnetic field using tangent galvanometer.

3. Determination of the magnetic field at a point on the axis of a circular coil.

4. Determination of the refractive index of the material of the prism by finding angle of prism and angle of minimum deviation using spectrometer.

5. Determination of the wavelength of a composite light by normal incidence method using diffraction grating and spectrometer (The number of lines per metre length of the grating is given).

6. Investigation of the voltage-current (V-I) characteristics of PN junction diode.

7. Investigation of the voltage-current (V-I) characteristics of Zener diode.

8. Investigation of the static characteristics of a NPN Junction transistor in common emitter configuration.

9. Verification of the truth table of the basic logic gates using integrated circuits.

10. Verification of De Morgan's theorems using integrated circuits.
1. SPECIFIC RESISTANCE OF THE MATERIAL OF THE COIL USING METRE BRIDGE

AIM
To determine the specific resistance of the material of the given coil using metre bridge.

APPARATUS REQUIRED
Meter bridge, galvanometer, key, resistance box, connecting wires, Lechlanche cell, jockey and high resistance.

FORMULA
\[ \rho = \frac{X \pi r^2}{L} \quad (\Omega m) \]
where,
- \( X \to \) Resistance of the given coil (\( \Omega \))
- \( R \to \) Known resistance (\( \Omega \))
- \( L \to \) Length of the coil (m)
- \( r \to \) Radius of the wire (m)

CIRCUIT DIAGRAM

PROCEDURE
- A resistance box \( R \) is connected in the left gap and the unknown resistance \( X \) in the right gap.
- A Lechlanche cell is connected across the wire of length 1 m through a key.
- A sensitive galvanometer \( G \) is connected between the central strip and the jockey through a high resistance (HR).
- With a suitable resistance included in the resistance box, the circuit is switched on.
- To check the circuit connections, the jockey is pressed near one end of the wire, say A. The galvanometer will show deflection in one direction. When the jockey is pressed near the other end of the wire B, the galvanometer will show deflection in the opposite directions. This ensures that the circuit connections are correct.
• By moving the jockey over the wire, the point on the wire at which the galvanometer shows null deflection i.e., balancing point \( J \) is found.

• The balancing length \( AJ = l \) is noted.

• The unknown resistance \( X_1 \) is found using the formula \( X_1 = \frac{R(100-l)}{l} \).

• The experiment is repeated for different values of \( R \).

• The same procedure is repeated after interchanging \( R \) and \( X \).

• The unknown resistance \( X_2 \) is found using the formula \( X_2 = \frac{Rl}{(100-l)} \).

• The experiment is repeated for same values of \( R \) as before.

• The resistance of the given coil is found from the mean value of \( X_1 \) and \( X_2 \).

• The radius of the wire \( r \) is found using screw gauge.

• The length of the coil \( L \) is measured using meter scale.

• From the values of \( X \), \( r \) and \( L \), the specific resistance of the material of the wire is determined.

**OBSERVATION**

length of the coil \( L = \) ______________________________ cm.

| S.No. | Resistance \( R \) (Ω) | Before interchanging | After interchanging | Mean  
\( X = \frac{X_1 + X_2}{2} \) (Ω) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( X_1 = \frac{R(100-l)}{l} ) (Ω)</td>
<td>( X_2 = \frac{Rl}{(100-l)} ) (Ω)</td>
<td>( X = \frac{X_1 + X_2}{2} ) (Ω)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2  To find the radius of the wire

Zero error  =  
Zero correction  =  

<table>
<thead>
<tr>
<th>Sl.No.</th>
<th>PSR (mm)</th>
<th>HSC (div.)</th>
<th>Total Reading  =  PSR + (HSC × LC) (mm)</th>
<th>Corrected Reading  =  TR ± ZC (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mean diameter 2r = ............ cm  
Radius of the wire r = ............ cm
r = ............ m  

CALCULATION

(i)  \[ \rho = \frac{X \pi r^2}{L} = \]

RESULT

The specific resistance of the material of the given coil = ____________________(Ωm)

Note:

i)  To check the circuit connections:

The meter bridge wire is touched near one end (say, end A) with jockey, galvanometer shows a deflection in any one direction. Now the other end (say, end B) is touched. If the galvanometer shows a deflection in the opposite direction, then the circuit connections are correct.

ii)  The usage of high resistance (HR):

The galvanometer is a very sensitive device. If any high current flows through the galvanometer, its coil gets damaged. Therefore in order to protect the galvanometer, a high resistance (HR) is used. When HR is connected in series with the galvanometer, the current through it is reduced so that the galvanometer is protected. But the balancing length is not accurate.

iii)  To find the accurate balancing length:

The HR is first included in the circuit (that is, the plug key in HR is removed), the approximate balancing length is found. Now HR is excluded in the circuit (that is, the plug key in HR is closed), then the accurate balancing length is found.
2. HORIZONTAL COMPONENT OF EARTH’S MAGNETIC FIELD USING TANGENT GALVANOMETER

AIM
To determine the horizontal component of the Earth’s magnetic field using tangent galvanometer.

APPARATUS REQUIRED
Tangent galvanometer (TG), commutator, battery, rheostat, ammeter, key and connecting wires.

FORMULA
\[ B_h = \frac{\mu_0 n k}{2r} \text{(Tesla)} \]
\[ k = \frac{I}{\tan \theta} \text{(A)} \]

where,  
\( B_h \rightarrow \) Horizontal component of the Earth’s magnetic field (T)  
\( \mu_0 \rightarrow \) Permeability of free space \((4\pi \times 10^{-7} \text{ H m}^{-1})\)  
\( n \rightarrow \) Number of turns of TG in the circuit (No unit)  
\( k \rightarrow \) Reduction factor of TG (A)  
\( r \rightarrow \) Radius of the coil (m)

CIRCUIT DIAGRAM

Figure (a) Tangent Galvanometer  
Figure (b) Number of turns  
(c) Circuit diagram
PROCEDURE

- The preliminary adjustments are carried out as follows.
  a. The leveling screws at the base of TG are adjusted so that the circular turn table is horizontal and the plane of the circular coil is vertical.
  b. The circular coil is rotated so that its plane is in the magnetic meridian i.e., along the north-south direction.
  c. The compass box alone is rotated till the aluminium pointer reads $0^\circ - 0^\circ$.
- The connections are made as shown in Figure (c).
- The number of turns $n$ is selected and the circuit is switched on.
- The range of current through TG is chosen in such a way that the deflection of the aluminium pointer lies between $30^\circ - 60^\circ$.
- A suitable current is allowed to pass through the circuit, the deflections $\theta_1$ and $\theta_2$ are noted from two ends of the aluminium pointer.
- Now the direction of current is reversed using commutator C, the deflections $\theta_3$ and $\theta_4$ in the opposite direction are noted.
- The mean value $\theta$ of $\theta_1$, $\theta_2$, $\theta_3$ and $\theta_4$ is calculated and tabulated.
- The reduction factor $k$ is calculated for each case and it is found that $k$ is a constant.
- The experiment is repeated for various values of current and the readings are noted and tabulated.
- The radius of the circular coil is found by measuring the circumference of the coil using a thread around the coil.
- From the values of $r$, $n$ and $k$, the horizontal component of Earth’s magnetic field is determined.

**Commutator:**

It is a kind of switch employed in electrical circuits, electric motors and electric generators. It is used to reverse the direction of current in the circuit.
OBSERVATION

Number of turns of the coil \( n = \)
Circumference of the coil \( (2\pi r) = \)
Radius of the coil \( r = \)

<table>
<thead>
<tr>
<th>S.No</th>
<th>Current I (A)</th>
<th>Deflection in TG (degree)</th>
<th>Mean ( \theta ) (degree)</th>
<th>( \tan \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>( \theta_1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>( \theta_2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>( \theta_3 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>( \theta_4 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

CALCULATION

\[ B_{h} = \frac{\mu_{0}nki}{2r} \]

RESULT

The horizontal component of Earth’s magnetic field is found to be ________

Note:

i) The magnetic materials and magnets present in the vicinity of TG should be removed.

ii) The readings from the ends of the aluminium pointer should be taken without parallax error.

iii) The deflections of TG is restricted between 30° and 60°. It is because, the TG is most sensitive for deflection around 45° and is least sensitive around 0° and 90°. We know that

\[ I = k \tan \theta \]

or \( dI = k \sec^2 \theta \ d\theta \)

\[ \frac{d\theta}{dI} = \frac{\sin 2\theta}{2I} \]

For given current, sensitivity \( \frac{d\theta}{dI} \) maximum for \( \sin 2\theta = 1 \) or \( \theta = 45° \)
3. MAGNETIC FIELD ALONG THE AXIS OF A CIRCULAR COIL - DETERMINATION OF $B_H$

**AIM**
To determine the horizontal component of Earth's magnetic field using current carrying circular coil and deflection magnetometer.

**APPARATUS REQUIRED**
Circular coil apparatus, compass box, rheostat, battery or power supply, ammeter, commutator, key and connecting wires.

**FORMULA**

$$B_H = \frac{\mu_0 nI r^2}{2\left(r^2 + x^2\right)^{3/2}} \left(\frac{1}{\tan \theta}\right) \text{(Tesla)}$$

where,
- $B_H \rightarrow$ Horizontal component of Earth's magnetic field ($T$)
- $\mu_0 \rightarrow$ Permeability of the free space ($4\pi \times 10^{-7} \text{ H m}^{-1}$)
- $n \rightarrow$ Number of turns included in the circuit (No unit)
- $I \rightarrow$ Current flowing through the coil (A)
- $r \rightarrow$ Radius of the circular coil (m)
- $x \rightarrow$ Distance between center of compass box and centre of the coil (m)

**CIRCUIT DIAGRAM**

![Circuit Diagram]

**PROCEDURE**
- The preliminary adjustments are carried out as follows.
  - i. The leveling screws are adjusted so that the circular coil is vertical.
  - ii. The wooden bench is adjusted to be along the magnetic east-west direction i.e., along aluminium pointer.
  - iii. The circular coil is rotated so that its plane is in magnetic meridian i.e., along the north-south direction.
  - iv. A compass box is placed with its centre coinciding with the axis of the coil.
  - v. The compass box alone is rotated till the aluminium pointer reads 0° – 0°
- Electrical connections are made as shown in the circuit diagram.
- The compass box is placed along its axis, with its centre at a distance $x$ from the centre of the coil on one side.
A suitable current (1A) is passed through the coil by adjusting rheostat so that the deflection of the aluminium pointer lies between 30° and 60°.

The value of the current I is noted from ammeter.

Two readings $\theta_1$ and $\theta_2$ corresponding to two ends of the pointer are noted.

Now the direction of the current is reversed using commutator, two more readings $\theta_3$ and $\theta_4$ are noted.

Now the compass box is taken to the other side and is kept at the same distance $x$.

Four more readings $\theta_5$, $\theta_6$, $\theta_7$, and $\theta_8$ are taken as done before.

These eight readings and their average value are tabulated.

The experiment is repeated for another value of current, say 1.5 A by keeping the compass box at the same distance $x$.

The radius of the circular coil is found by measuring the circumference of the coil using a thread around the coil.

The number of turns $n$ of the coil is noted.

From the values of $n$, $r$, $x$ and $I/\tan\theta$, the horizontal component of Earth's magnetic field is now found using the formula.

**OBSERVATION**

Number of turns in the coil $n = \quad$

Circumference of the coil $(2\pi r) = \quad$

Radius of the coil $r = \quad$

To find horizontal component of the Earth's magnetic field

<table>
<thead>
<tr>
<th>S.No</th>
<th>Distance $x$ (cm)</th>
<th>Current I (A)</th>
<th>Deflection for eastern side</th>
<th>Deflection for western side</th>
<th>Mean $\theta$</th>
<th>$I/\tan\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\theta_1$</td>
<td>$\theta_2$</td>
<td>$\theta_3$</td>
<td>$\theta_4$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**CALCULATION**

$$B_H = \frac{\mu_0 n I r^2}{2(r^2 + x^2)^{3/2}} \left( \frac{1}{\tan\theta} \right) =$$

**RESULT**

Horizontal component of the Earth's magnetic field at a place = _________ T
4. REFRACTIVE INDEX OF THE MATERIAL OF THE PRISM

AIM

To determine the refractive index of the material of a prism using spectrometer.

APPARATUS REQUIRED

Spectrometer, prism, prism clamp, sodium vapour lamp, spirit level.

FORMULA

$$\mu = \frac{\sin \left(\frac{A + D}{2}\right)}{\sin \left(\frac{A}{2}\right)}$$ (No unit)

where,

- $\mu \rightarrow$ Refractive index of the material of the prism (No unit)
- $A \rightarrow$ Angle of the prism (degree)
- $D \rightarrow$ Angle of minimum deviation (degree)

DIAGRAMS

Figure (a) Angle of the prism

Figure (b) Angle of the prism

Figure (c) Angle of minimum deviation
PROCEDURE

1) Initial adjustments of the spectrometer

- Eye-piece: The eye-piece of the telescope is adjusted so that the cross-wires are seen clearly.
- Slit: The slit of the collimator is adjusted such that it is very thin and vertical.
- Base of the spectrometer: The base of the spectrometer is adjusted to be horizontal using leveling screws.
- Telescope: The telescope is turned towards a distant object and is adjusted till the clear inverted image of the distant object is seen. Now the telescope is adjusted to receive parallel rays.
- Collimator: The telescope is brought in line with the collimator. Collimator is adjusted until a clear image of the slit is seen in the telescope. Now the collimator gives parallel rays.
- Prism table: Using a spirit level, the prism table is adjusted to be horizontal with the three leveling screws provided in the prism table.

2) Determination of angle of the prism (A)

- The slit is illuminated by yellow light from sodium vapour lamp.
- The given equilateral prism is placed on the prism table in such a way that refracting edge of the prism is facing the collimator.
- The light emerging from the collimator is incident on both reflecting faces of the prism and is reflected.
- The telescope is rotated towards left to obtain reflected image of the slit from face 1 of the prism and is fixed.
- Using tangential screws, the telescope is adjusted until the vertical cross-wire coincides with the reflected image of the slit.
- The main scale reading and vernier coincidence are noted from both vernier scales.
- The telescope is now rotated towards right to obtain the reflected image from face 2 of the prism. As before, the readings are taken.
- The difference between the two readings gives 2A from which the angle of the prism A is calculated.

3) Determination of angle of minimum deviation (D)

- The prism table is rotated such that the light emerging from the collimator is incident on one of the refracting faces of the prism, gets refracted and emerges out from the other refracting face.
- The telescope is turned to view the refracted image.
- Looking through the telescope, the prism table is rotated in such a direction that the image moves towards the direct ray.
At one particular position, the refracted ray begins to retrace its path. The position where the refracted image returns is the position of minimum deviation.

The telescope is fixed in this position and is adjusted until the vertical cross-wire coincides with the refracted image of the slit.

The readings are taken from both vernier scales.

The prism is now removed and the telescope is rotated to obtain the direct ray image and the readings are taken.

The readings are tabulated and the difference between these two readings gives the angle of minimum deviation D.

From the values of A and D, the refractive index of the material of the glass prism is determined.

### Least count

1 MSD = 30′

Number of vernier scale divisions = 30

For spectrometer, 30 vernier scale divisions will cover 29 main scale divisions.

\[
\therefore 30 \text{ VSD} = 29 \text{ MSD}
\]

Or 1 VSD = \(29 / 30\) MSD

Least count (LC) = 1 MSD – 1 VSD

\[
= 1 / 30 \text{ MSD}
\]

\[
= 1′
\]

### OBSERVATION

**Table 1** To find the angle of the prism (A)

<table>
<thead>
<tr>
<th>Image</th>
<th>Vernier A (Degree)</th>
<th>Vernier B (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSR</td>
<td>VSC</td>
</tr>
<tr>
<td>Reflected image from face 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflected image from face 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difference 2A</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mean 2A =

Mean A =
Table 2 To find the angle of minimum deviation (D)

<table>
<thead>
<tr>
<th>Image</th>
<th>Vernier A (Degree)</th>
<th>Vernier B (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSR</td>
<td>VSC</td>
</tr>
<tr>
<td>Refracted image</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct image</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difference D</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mean D =

RESULT

1. Angle of the Prism (A) = ............ (degree)
2. Angle of the minimum deviation of the prism (D) = ............ (degree)
3. Refractive index of the material of the Prism (μ) = ............ (No unit)

Note:

i) Once initial adjustments are done, spectrometer should not be disturbed.

ii) Total reading TR = MSR + (VSC × LC)

Where

MSR → Main Scale Reading
VSC → Vernier Scale Coincidence
LC → Least count (= 1')
5. WAVELENGTH OF THE CONSTITUENT COLOURS OF A COMPOSITE LIGHT USING DIFFRACTION GRATING AND SPECTROMETER

AIM
To find the wavelength of the constituent colours of a composite light using diffraction grating and spectrometer.

APPARATUS REQUIRED
Spectrometer, mercury vapour lamp, diffraction grating, grating table, and spirit level.

FORMULA
\[ \lambda = \frac{\sin \theta}{nN} \lambda \]

where,
\( \lambda \rightarrow \) Wavelength of the constituent colours of a composite light (Å)
\( N \rightarrow \) Number of lines per metre length of the given grating (No unit) (the value of N for the grating is given)
\( n \rightarrow \) Order of the diffraction (No unit)
\( \theta \rightarrow \) Angle of diffraction (degree)

DIAGRAMS

![Diagram 1: Normal incidence](image1)

![Diagram 2: Angle of diffraction](image2)

PROCEDURE
1) Initial adjustments of the spectrometer
   - Eye-piece: The eye-piece of the telescope is adjusted so that the cross-wires are seen clearly.
   - Slit: The slit of the collimator is adjusted such that it is very thin and vertical.
   - Base of the spectrometer: The base of the spectrometer is adjusted to be horizontal using leveling screws.
Telescope: The telescope is turned towards a distant object and is adjusted till the clear image of the distant object is seen. Now the telescope is adjusted to receive parallel rays.

Collimator: The telescope is brought in line with the collimator. Collimator is adjusted until a clear image of the slit is seen in the telescope. Now the collimator gives parallel rays.

Grating table: Using a spirit level, the grating table is adjusted to be horizontal with the three leveling screws provided in the grating table.

2) Adjustment of the grating for normal incidence

- The slit is illuminated with a composite light (white light) from mercury vapour lamp.
- The telescope is brought in line with the collimator. The vertical cross-wire is made to coincide with the image of the slit (Figure (a)1).
- The vernier disc alone is rotated till the vernier scale reads 0° - 180° and is fixed. This is the reading for the direct ray.
- The telescope is then rotated (anti-clockwise) through an angle of 90° and fixed (Figure (a)2).
- Now the plane transmission grating is mounted on the grating table.
- The grating table alone is rotated so that the light reflected from the grating coincides with vertical cross-wire of the telescope. The reflected image is white in colour (Figure (a)3).
- Now the vernier disc is released. The vernier disc along with grating table is rotated through an angle of 45° in the appropriate direction such that the light from the collimator is incident normally on the grating (Figure (a)4).

3) Determination of wave length of the constituent colours of the mercury spectrum

- The telescope is released and is brought in line with the collimator to receive central direct image. This undispersed image is white in colour.
- The diffracted images of the slit are observed on either side of the direct image.
- The diffracted image consists of the prominent colours of mercury spectrum in increasing order of wavelength.
- The telescope is turned to any one side (say left) of direct image to observe first order diffracted image.
- The vertical cross-wire is made to coincide with the prominent spectral lines (violet, blue, yellow and red) and the readings of both vernier scales for each case are noted.
- Now the telescope is rotated to the right side of the direct image and the first order image is observed.
- The vertical cross-wire is made to coincide with the same prominent spectral lines and the readings of both vernier scales for each case are again noted.
- The readings are tabulated.
- The difference between these two readings gives the value of 2θ for the particular spectral line.
- The number of lines per metre length of the given grating N is noted from the grating.
- From the values of N, n and θ, the wave length of the prominent colours of the mercury light is determined using the given formula.
OBSERVATION

To find the wave length of prominent colours of the mercury spectrum

<table>
<thead>
<tr>
<th>Colour of Light</th>
<th>Diffracted Ray Reading (Degree)</th>
<th>Difference 2θ (Degree)</th>
<th>θ (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Left</td>
<td>Right</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Vernier A MSR VSC TR</td>
<td>Vernier A MSR VSC TR</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Vernier B MSR VSC TR</td>
<td>Vernier B MSR VSC TR</td>
<td></td>
</tr>
<tr>
<td>Blue</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Green</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yellow</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Red</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

CALCULATION

(i) For blue, \( \lambda = \frac{\sin \theta}{nN} \),

(ii) For green, \( \lambda = \frac{\sin \theta}{nN} \)

(iii) For yellow, \( \lambda = \frac{\sin \theta}{nN} \),

(iv) For red, \( \lambda = \frac{\sin \theta}{nN} \)

RESULT

1. The wavelength of blue line = --------------m
2. The wavelength of green line = --------------m
3. The wavelength of yellow line = --------------m
4. The wavelength of red line = --------------m

Note:

i) Once initial adjustments are done, spectrometer should not be disturbed.

ii) Total reading \( TR = MSR + (VSC \times LC) \)

Where

MSR \rightarrow Main Scale Reading
VSC \rightarrow Vernier Scale Coincidence
LC \rightarrow Least count (= 1')
6. VOLTAGE-CURRENT CHARACTERISTICS OF A PN JUNCTION DIODE

AIM
To draw the voltage-current (V-I) characteristics of the PN junction diode and to determine its knee voltage and forward resistance.

APPARATUS REQUIRED
PN junction diode (IN4007), variable DC power supply, milli-ammeter, micro-ammeter, voltmeter, resistance and connecting wires.

FORMULA
\[ R_F = \frac{\Delta V_F}{\Delta I_F} \ (\Omega) \]

where,
- \( R_F \rightarrow \) Forward resistance of the diode (Ω)
- \( \Delta V_F \rightarrow \) The change in forward voltage (volt)
- \( \Delta I_F \rightarrow \) The change in forward current (mA)

CIRCUIT DIAGRAM

Figure (a) PN junction diode and its symbol (Silver ring denotes the negative terminal of the diode)

Figure (b) PN junction diode in forward bias

Figure (c) PN junction diode in reverse bias

Precaution
Care should be taken to connect the terminals of ammeter, voltmeter, dc power supply and the PN junction diode with right polarity.
PROCEDURE

i) Forward bias characteristics
- In the forward bias, the P-region of the diode is connected to the positive terminal and N-region to the negative terminal of the DC power supply.
- The connections are given as per the circuit diagram.
- The voltage across the diode can be varied with the help of the variable DC power supply.
- The forward voltage \((V_f)\) across the diode is increased from 0.1 V in steps of 0.1 V up to 0.8 V and the forward current \((I_f)\) through the diode is noted from the milli-ammeter. The readings are tabulated.
- The forward voltage \(V_f\) and the forward current \(I_f\) are taken as positive.
- A graph is drawn taking the forward voltage \((V_f)\) along the x-axis and the forward current \((I_f)\) along the y-axis.
- The voltage corresponding to the dotted line in the forward characteristics gives the knee voltage or threshold voltage or turn-on voltage of the diode.
- The slope in the linear portion of the forward characteristics is calculated. The reciprocal of the slope gives the forward resistance of the diode.

ii) Reverse bias characteristics
- In the reverse bias, the polarity of the DC power supply is reversed so that the P-region of the diode is connected to the negative terminal and N-region to the positive terminal of the DC power supply.
- The connections are made as given in the circuit diagram.
- The voltage across the diode can be varied with the help of the variable DC power supply.
- The reverse voltage \((V_r)\) across the diode is increased from 1 V in steps of 1 V up to 5 V and the reverse current \((I_r)\) through the diode is noted from the micro-ammeter. The readings are tabulated.
- The reverse voltage \(V_r\) and reverse current \(I_r\) are taken as negative.
- A graph is drawn taking the reverse bias voltage \((V_r)\) along negative x-axis and the reverse bias current \((I_r)\) along negative y-axis.
**OBSERVATION**

Table 1 Forward bias characteristic curve

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Forward bias voltage $V_F$ (volt)</th>
<th>Forward bias current $I_F$ (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 Reverse bias characteristic curve

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Reverse bias voltage $V_R$ (volt)</th>
<th>Reverse bias current $I_R$ (µA)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

**CALCULATION**

(i) Forward resistance $R_F$ =

(ii) Knee voltage =

**RESULT**

The V-I characteristics of the PN junction diode are studied.

i) Knee voltage of the PN junction diode =………………..V

ii) Forward resistance of the diode =………………..Ω

**Practical Tips**

- The DC power supply voltage should be increased only up to the specified range in the forward (0 – 2V) and reverse (0 – 15V) directions. Forward bias offers very low resistance and hence an external resistance of 470Ω is connected as a safety measure.
- The voltage applied beyond this limit may damage the resistance or the diode.
- In the forward bias, the current flow will be almost zero till it crosses the junction potential or knee voltage (approximately 0.7 V). Once knee voltage is crossed, the current increases with the applied voltage.
- The diode voltage in the forward direction should be increased in steps of 0.1 V to a maximum of 0.8 V after the threshold voltage to calculate the forward resistance.
- The diode voltage in the reverse direction is increased in steps of 1 V to a maximum of 5 V. The current must be measured using micro-ammeter as the strength of current in the reverse direction is very less. This is due to the flow of the minority charge carriers called the leakage current.
7. VOLTAGE-CURRENT CHARACTERISTICS OF A ZENER DIODE

AIM
To draw the voltage-current (V-I) characteristic curves of a Zener diode and to determine its knee voltage, forward resistance and reverse breakdown voltage.

APPARATUS REQUIRED
Zener diode IZ5.6V, variable dc power supply (0 – 15V), milli ammeter, volt meter, 470 \( \Omega \) resistance, and connecting wires.

FORMULA

\[
R_F = \frac{\Delta V_F}{\Delta I_F} \quad (\Omega)
\]

where,

- \( R_F \rightarrow \) Forward resistance of the diode (\( \Omega \))
- \( \Delta V_F \rightarrow \) The change in forward voltage (volt)
- \( \Delta I_F \rightarrow \) The change in forward current (mA)

CIRCUIT DIAGRAM

Figure (a) Zener diode and its symbol (The black colour ring denotes the negative terminal of the Zener diode)

Precaution

Care should be taken to connect the terminals of ammeter, voltmeter, dc power supply and the Zener diode with right polarity.
PROCEDURE

i) Forward bias characteristics

- In the forward bias, the P-region of the diode is connected to the positive terminal and N-region to the negative terminal of the DC power supply.
- The connections are given as per the circuit diagram.
- The voltage across the diode can be varied with the help of the variable DC power supply.
- The forward voltage ($V_f$) across the diode is increased from 0.1V in steps of 0.1V up to 0.8V and the forward current ($I_f$) through the diode is noted from the milli-ammeter. The readings are tabulated.
- The forward voltage and the forward current are taken as positive.
- A graph is drawn taking the forward voltage along the x-axis and the forward current along the y-axis.
- The voltage corresponding to the dotted line in the forward characteristics gives the knee voltage or threshold voltage or turn-on voltage of the diode.
- The slope in the linear portion of the forward characteristics is calculated. The reciprocal of the slope gives the forward resistance of the diode.

ii) Reverse bias characteristics

- In the reverse bias, the polarity of the DC power supply is reversed so that the P-region of the diode is connected to the negative terminal and N-region to the positive terminal of the DC power supply.
- The connections are made as given in the circuit diagram.
- The voltage across the diode can be varied with the help of the variable DC power supply.
- The reverse voltage ($V_r$) across the diode is increased from 0.5V in steps of 0.5V up to 6V and the reverse current ($I_r$) through the diode is noted from the milli-ammeter. The readings are tabulated.
- Initially, the voltage is increased in steps of 0.5V. When the breakdown region is approximately reached, then the input voltage may be raised in steps of, say 0.1V to find the breakdown voltage.
- The reverse voltage and reverse current are taken as negative.
- A graph is drawn taking the reverse bias voltage along negative x-axis and the reverse bias current along negative y-axis.
- In the reverse bias, Zener breakdown occurs at a particular voltage called Zener voltage $V_Z$ (~5.6 to 5.8V) and a large amount of current flows through the diode which is the characteristics of a Zener diode.
- The breakdown voltage of the Zener diode is determined from the graph as shown.
OBSERVATION

Table 1 Forward bias characteristic curve

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Forward bias voltage $V_F$ (volt)</th>
<th>Forward bias current $I_F$ (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

Table 2 Reverse bias characteristic curve

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Reverse bias voltage $V_R$ (volt)</th>
<th>Reverse bias current $I_R$ (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

CALCULATION

(i) Forward resistance $R_F =$

(ii) Knee voltage =

(iii) The breakdown voltage of the Zener diode $V_Z =$ ----V
RESULT
The V-I characteristics of the Zener diode are studied.

(i) Forward resistance $R_F = $

(ii) Knee voltage =

(iii) The breakdown voltage of the Zener diode $V_Z = ----V$

Practical Tips

• The DC power supply voltage should be increased only up to the specified range in the forward (0 – 2 V) and reverse (0 – 15 V) directions.
• The voltage applied beyond this limit may damage the resistor or the diode.
• Zener diode functions like an ordinary PN junction diode in the forward direction. Hence the forward characteristic is the same for both PN junction diode and Zener diode. Therefore, knee voltage and forward resistance can be determined as explained in the previous experiment.
• Unlike ordinary PN junction diode, the reverse current in Zener diode is measured using milli-ammeter due to the large flow of current.
**8 CHARACTERISTICS OF A NPN-JUNCTION TRANSISTOR IN COMMON EMITTER CONFIGURATION**

**AIM**
To study the characteristics and to determine the current gain of a NPN junction transistor in common emitter configuration.

**APPARATUS REQUIRED**
Transistor - BC 548/BC107, bread board, micro ammeter, milli ammeter, voltmeters, variable DC power supply and connecting wires.

**FORMULA**

\[ r_i = \left[ \frac{\Delta V_{BE}}{\Delta I_B} \right]_{I_B} (\Omega), \quad r_o = \left[ \frac{\Delta V_{CE}}{\Delta I_C} \right]_{I_C} (\Omega), \quad \beta = \left[ \frac{\Delta I_C}{\Delta I_B} \right]_{V_{CC}} \]

Where,
- \( r_i \rightarrow \) Input impedance (\( \Omega \))
- \( \Delta V_{BE} \rightarrow \) The change in base-emitter voltage (volt)
- \( \Delta I_B \rightarrow \) The change in base current (µA)
- \( r_o \rightarrow \) Output impedance (\( \Omega \))
- \( \Delta V_{CE} \rightarrow \) The change in collector-emitter voltage (volt)
- \( \Delta I_C \rightarrow \) The change in collector current (mA)
- \( \beta \rightarrow \) Current gain of the transistor (No unit)

**CIRCUIT DIAGRAM**

Figure (a) NPN - Junction transistor and its symbol (Transistor is held with the flat surface facing us)

Figure (b) NPN junction transistor in CE configuration

A resistor is connected in series with the base to prevent excess current flowing into the base.

**Note**
Precautions

- Care should be taken to connect the terminals of ammeters, voltmeters, and dc power supplies with right polarity.
- The collector and emitter terminals of the transistor must not be interchanged.

PROCEDURE

- The connections are given as shown in the diagram.
- The current and voltage at the input and output regions can be varied by adjusting the DC power supply.

(i) **Input characteristic curve: \( V_{BE} \) vs \( I_B \) (\( V_{CE} \) constant)

- The collector-emitter voltage \( V_{CE} \) is kept constant.
- The base-emitter voltage \( V_{BE} \) is varied in steps of 0.1V and the corresponding base current \( I_B \) is noted. The readings are taken till \( V_{CE} \) reaches a constant value.
- The same procedure is repeated for different values of \( V_{CE} \). The readings are tabulated.
- A graph is plotted by taking \( V_{BE} \) along x-axis and \( I_B \) along y-axis for both the values of \( V_{CE} \).
- The curves thus obtained are called the input characteristics of a transistor.
- The reciprocal of the slope of these curves gives the input impedance of the transistor.

<table>
<thead>
<tr>
<th>S. No</th>
<th>( V_{CE} = 1V )</th>
<th>( V_{CE} = 2V )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( V_{BE} ) (V)</td>
<td>( I_B ) (( \mu A ))</td>
</tr>
</tbody>
</table>

\[
\text{Slope} = \frac{\Delta I_B}{\Delta V_{BE}}
\]

\[
\text{Input impedance} = \frac{1}{\text{slope}}
\]
(ii) Output characteristic curve: $V_{CE}$ vs $I_C$ ($I_B$ constant)

- The base current $I_B$ is kept constant.
- $V_{CE}$ is varied in steps of 1 V and the corresponding collector current $I_C$ is noted. The readings are taken till the collector current becomes almost constant.
- Initially $I_B$ is kept at 0 mA and the corresponding collector current is noted. This current is the reverse saturation current $I_{CEO}$.
- The experiment is repeated for various values of $I_B$. The readings are tabulated.
- A graph is drawn by taking $V_{CE}$ along x-axis and $I_C$ along y-axis for various values of $I_B$.
- The set of curves thus obtained is called the output characteristics of a transistor.
- The reciprocal of the slope of the curve gives output impedance of the transistor.

<table>
<thead>
<tr>
<th>S. No</th>
<th>$I_B = 20 \mu A$</th>
<th>$I_B = 40 \mu A$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_{CE}$ (V)</td>
<td>$V_{CE}$ (V)</td>
</tr>
<tr>
<td></td>
<td>$I_C$ (mA)</td>
<td>$I_C$ (mA)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$20 \mu A$</th>
<th>$40 \mu A$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>20 µA</td>
<td>40 µA</td>
</tr>
<tr>
<td></td>
<td>60 µA</td>
<td>60 µA</td>
</tr>
</tbody>
</table>

\[ \text{Slope} = \frac{\Delta I_C}{\Delta V_{CE}} \]

Output impedance = \frac{1}{\text{slope}}

(iii) Transfer characteristic curve: $I_B$ vs $I_C$ ($V_{CE}$ constant)

- The collector-emitter voltage $V_{CE}$ is kept constant.
- The base current $I_B$ is varied in steps of 10 µA and the corresponding collector current $I_C$ is noted.
- This is repeated by changing the value of $V_{CE}$. The readings are tabulated.
- The transfer characteristics is a plot between the input current $IB$ along x-axis and the output current $I_C$ along y-axis keeping $V_{CE}$ constant.
- The slope of the transfer characteristics plot gives the current gain $\beta$ can be calculated.
RESULT

i) The input, output and transfer characteristics of the NPN junction in common emitter mode are drawn.

ii) (a) Input impedance = ________Ω
    (b) Output impedance = ________Ω
    (c) Current gain $\beta = ____$(no unit)
9. VERIFICATION OF TRUTH TABLES OF LOGIC GATES USING INTEGRATED CIRCUITS

AIM
To verify the truth tables of AND, OR, NOT, EX-OR, NAND and NOR gates using integrated circuits

COMPONENTS REQUIRED
AND gate (IC 7408), NOT gate (IC 7404), OR gate (IC 7432), NAND gate (IC 7400), NOR gate (IC 7402), X-OR gate (IC 7486), Power supply, Digital IC trainer kit, connecting wires.

BOOLEAN EXPRESSIONS
(i) AND gate \( Y = A \cdot B \)  
(ii) OR gate \( Y = A + B \)  
(iii) NOT gate \( Y = \overline{A} \)  
(iv) Ex OR gate \( Y = \overline{A} \cdot B + A \cdot \overline{B} \)  
(v) NAND gate \( Y = \overline{A} \cdot B \)  
(vi) NOR gate \( Y = A + B \)

CIRCUIT DIAGRAM
Pin Identification

Note:
The chip must be inserted in the bread board in such a way that the identification mark should be on our left side. In this position, pin numbers are counted as marked in the picture above. Pin identification is the same for all chips that are mentioned below.

AND Gate:

OR Gate:
NOT Gate:

**SYMBOL**:  

**PIN DIAGRAM-IC 7404**:  

**TRUTH TABLE**:  

<table>
<thead>
<tr>
<th>A</th>
<th>Y = \overline{A}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

X-OR Gate:

**SYMBOL**:  

**PIN DIAGRAM-IC 7486**:  

**TRUTH TABLE**:  

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Y = \overline{A} + \overline{B}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
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</tr>
</tbody>
</table>

NAND Gate:

**SYMBOL**:  

**PIN DIAGRAM-IC 7400**:  

**TRUTH TABLE**:  

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Y = \overline{A} + \overline{B}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
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<td>1</td>
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<tr>
<td>1</td>
<td>0</td>
<td>0</td>
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<tr>
<td>1</td>
<td>1</td>
<td>0</td>
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</tbody>
</table>

NOR Gate:

**SYMBOL**:  

**PIN DIAGRAM-IC 7402**:  

**TRUTH TABLE**:  

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Y = \overline{A} + \overline{B}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
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<tr>
<td>0</td>
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<td>1</td>
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</tbody>
</table>

PROCEDURE

- To verify the truth table of a logic gate, the suitable IC is taken and the connections are given using the circuit diagram.
- For all the ICs, 5V is applied to the pin 14 while the pin 7 is connected to the ground.
- The logical inputs of the truth table are applied and the corresponding output is noted.
- Similarly the output is noted for all other combinations of inputs.
- In this way, the truth table of a logic gate is verified.

RESULT

The truth table of logic gates AND, OR, NOT, Ex-OR, NAND and NOR using integrated circuits is verified.

Precautions

(i) $V_{CC}$ and ground pins must not be interchanged while making connections. Otherwise the chip will be damaged. (ii) The pin configuration for NOR gate is different from other gates.
10. VERIFICATION OF DE MORGAN’S THEOREMS

AIM: To verify De Morgan’s first and second theorems.

COMPONENTS REQUIRED: Power Supply (0 – 5V), IC 7400, 7408, 7432, 7404, and 7402, Digital IC trainer kit, connecting wires.

FORMULA

De Morgan’s first theorem \( A + \bar{B} = \bar{A} \cdot B \)

De Morgan’s second theorem \( A \cdot \bar{B} = \bar{A} + B \)

CIRCUIT DIAGRAM:

De Morgan’s first theorem

\[
\begin{align*}
A & \quad \rightarrow \quad A + B \\
B & \quad \rightarrow \quad A + B
\end{align*}
\]

De Morgan’s second theorem

\[
\begin{align*}
A & \quad \rightarrow \quad A \cdot B \\
B & \quad \rightarrow \quad A \cdot B
\end{align*}
\]

PROCEDURE:

i) Verification of De Morgan’s first theorem

- The connections are made for LHS \( A + \bar{B} \) of the theorem as shown in the circuit diagram using appropriate ICs.
- The output is noted and tabulated for all combinations of logical inputs of the truth table.
- The same procedure is repeated for RHS \( A \cdot \bar{B} \) of the theorem.
- From the truth table, it can be shown that \( A + B = \bar{A} \cdot B \).
ii) Verification of De Morgan’s second theorem

- The connections are made for LHS \( \overline{A \cdot B} \) of the theorem as shown in the circuit diagram using appropriate ICs.
- The output is noted and tabulated for all combinations of logical inputs of the truth table.
- The same procedure is repeated for RHS \( \overline{A + B} \) of the theorem.
- From the truth table, it can be shown that \( \overline{A \cdot B} = \overline{A} + \overline{B} \).

**OBSERVATION**

**De-Morgan’s first theorem**

**Truth Table**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>( A + B )</th>
<th>( \overline{A \cdot B} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>1</td>
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<td>1</td>
<td>1</td>
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<td>0</td>
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</tbody>
</table>

**De-Morgan’s second theorem**

**Truth Table**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>( \overline{A \cdot B} )</th>
<th>( \overline{A + B} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
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<td>0</td>
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<tr>
<td>1</td>
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</tbody>
</table>

**RESULT**

De Morgan’s first and second theorems are verified.

**Note**

The pin diagram for IC 7408, IC 7432 and IC 7404 can be taken from previous experiment.

**Precautions**

\( V_{cc} \) and ground pins must not be interchanged while making connections. Otherwise the chip will be damaged.

For the ICs used, 5V is applied to the pin 14 while the pin 7 is connected to the ground.
SUGGESTED QUESTIONS FOR THE PRACTICAL EXAMINATION

1. Determine the resistance of a given wire using metre bridge. Also find the radius of the wire using screw gauge and hence determine the specific resistance of the material of the wire. Take at least 4 readings.

2. Determine the value of the horizontal component of the Earth’s magnetic field, using tangent galvanometer. Take at least 4 readings.

3. Determine the value of the horizontal component of the Earth’s magnetic field using the magnetic field produced along the axial line of the carrying-current circular coil. Take at least 2 readings.

4. Using the spectrometer, measure the angle of the given prism and angle of minimum deviation. Hence calculate the refractive index of the material of the prism.

5. Adjust the grating for normal incidence using the spectrometer. Determine the wavelength of green, blue, yellow and red lines of mercury spectrum (The number of lines per metre length of the grating can be noted from the grating).

6. Draw the V-I characteristics of PN junction diode and determine its forward resistance and knee voltage from forward characteristics.

7. Draw the V-I characteristics of Zener diode and determine its forward resistance and knee voltage from forward characteristics. Also find breakdown voltage of the Zener diode from reverse characteristics.

8. Draw the input and transfer characteristic curves of the given NPN junction transistor in CE mode. Find the input impedance from input characteristics and current gain from transfer characteristics.

9. Draw the output and transfer characteristic curves of the given NPN junction transistor in CE mode. Find the output impedance from output characteristics and current gain from transfer characteristics.

10. Verify the truth table of logic gates AND, NOT, Ex-OR and NOR gates using integrated circuits.

11. Verify the truth table of logic gates OR, NOT, Ex-OR and NOR gates using integrated circuits.

12. Verify De Morgan’s first and second theorems.
| 1. Absorption spectra | உட்கெர் நி்றமபாலை |
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